NATIONAL SCIENCE CENTRE
POLAND
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IMPACT OF SELF-SCATTERING ON DARK MATTER RELIC DENSITY

Andrzej Hryczuk

NCBJ

based on:

A.H. & M. Laletin 2204.07078

and **T. Binder, T. Bringmann, M. Gustafsson & A.H.** 1706.07433, 2103.01944

andrzej.hryczuk@ncbj.gov.pl

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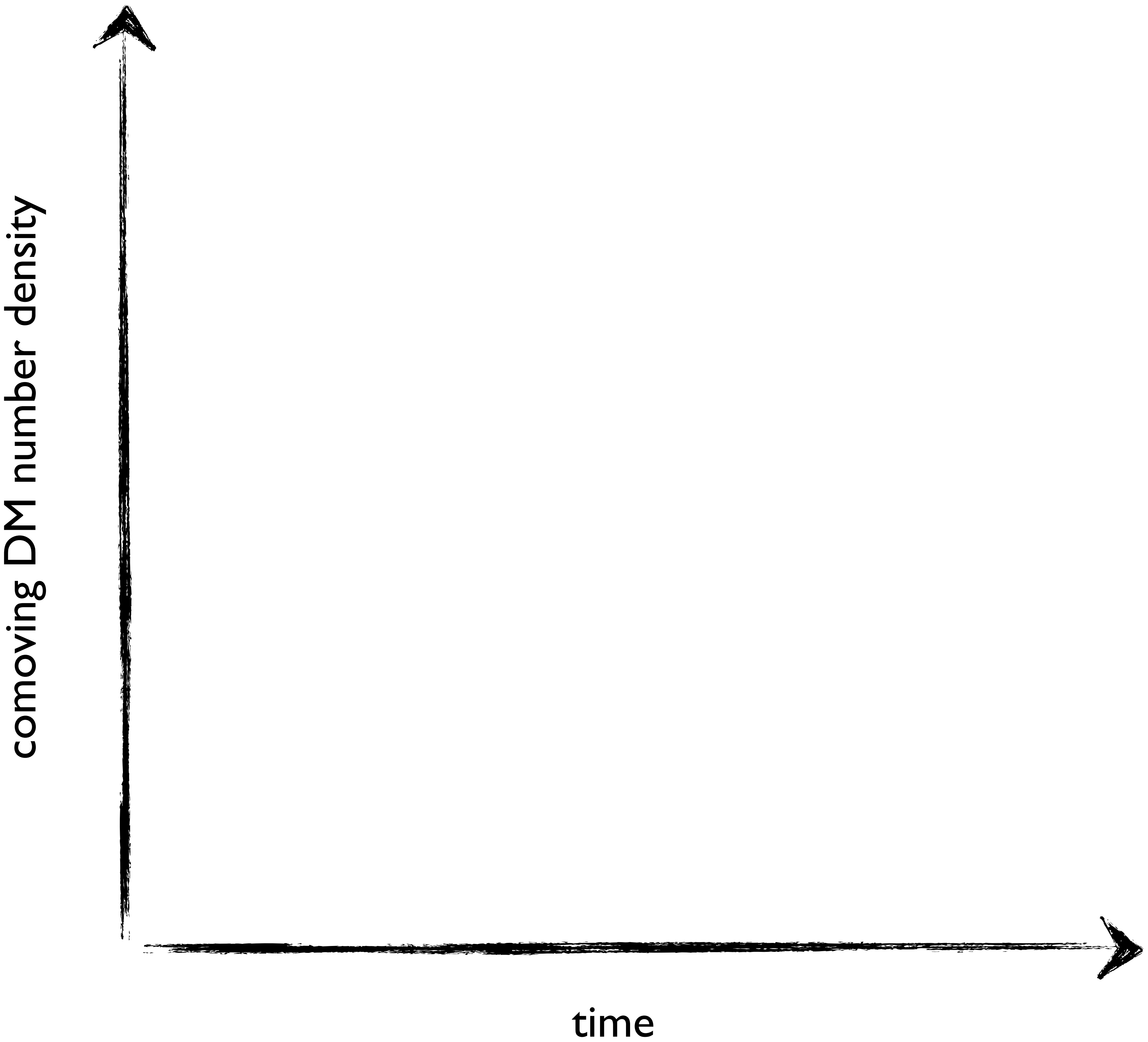

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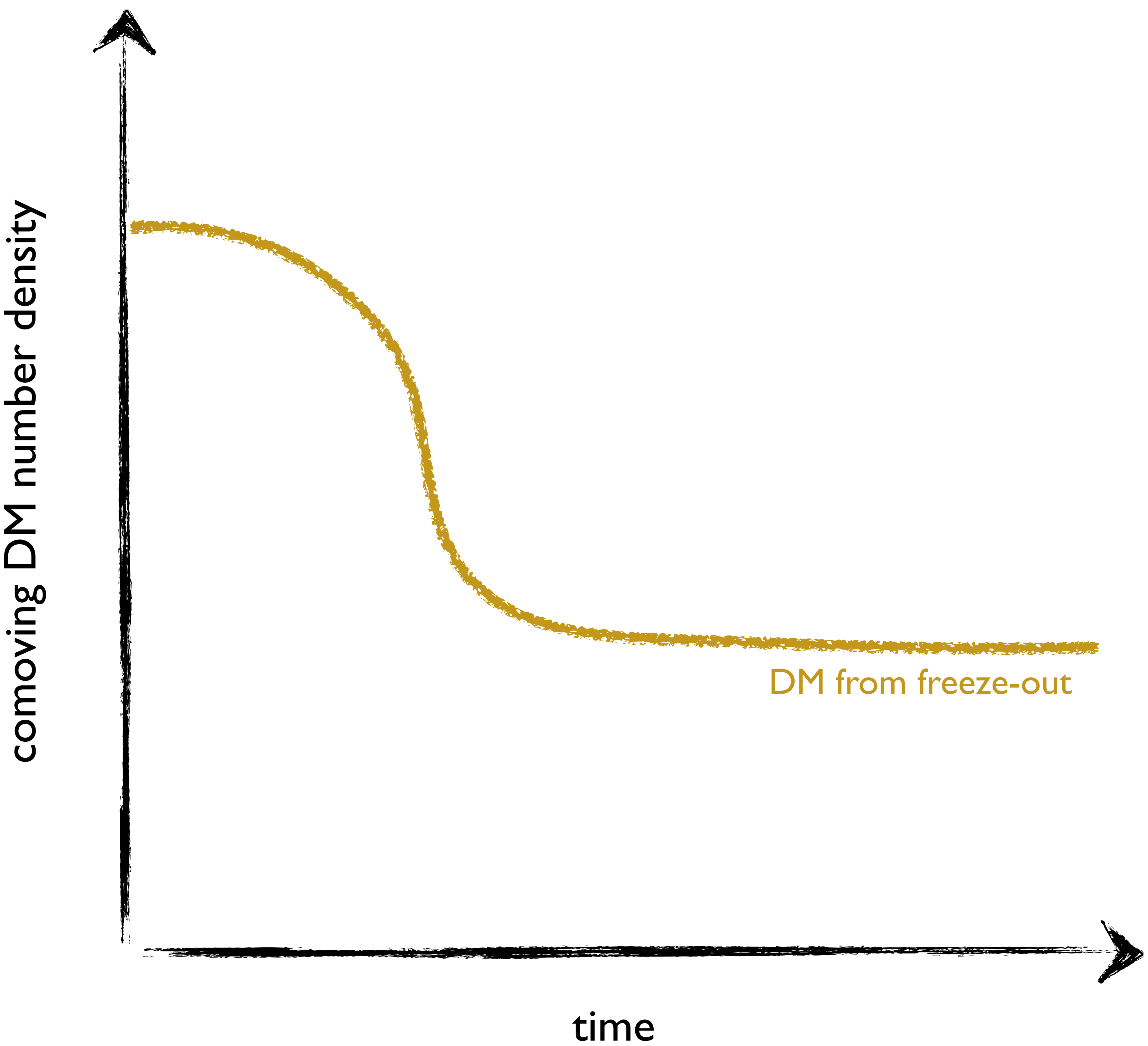

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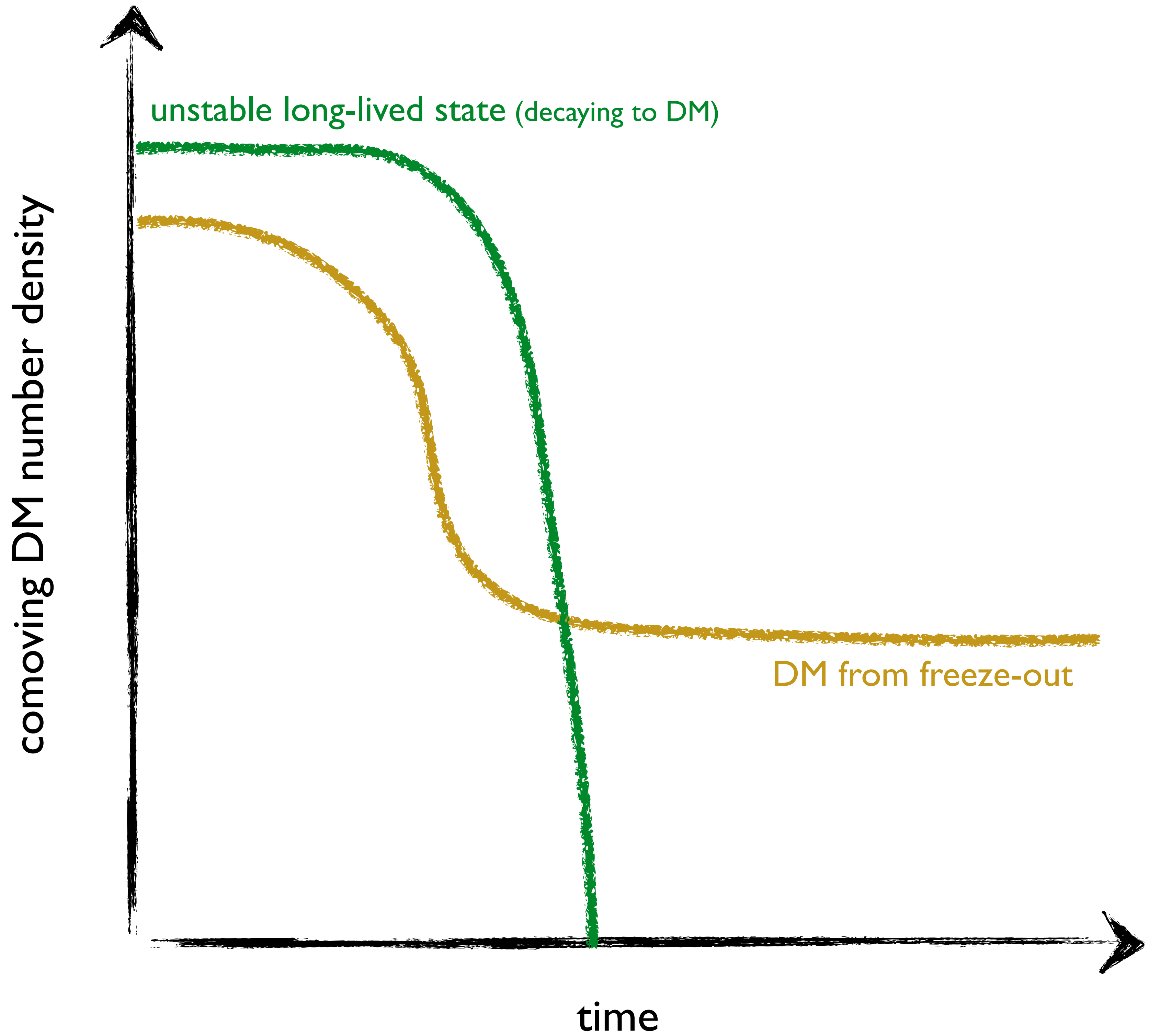

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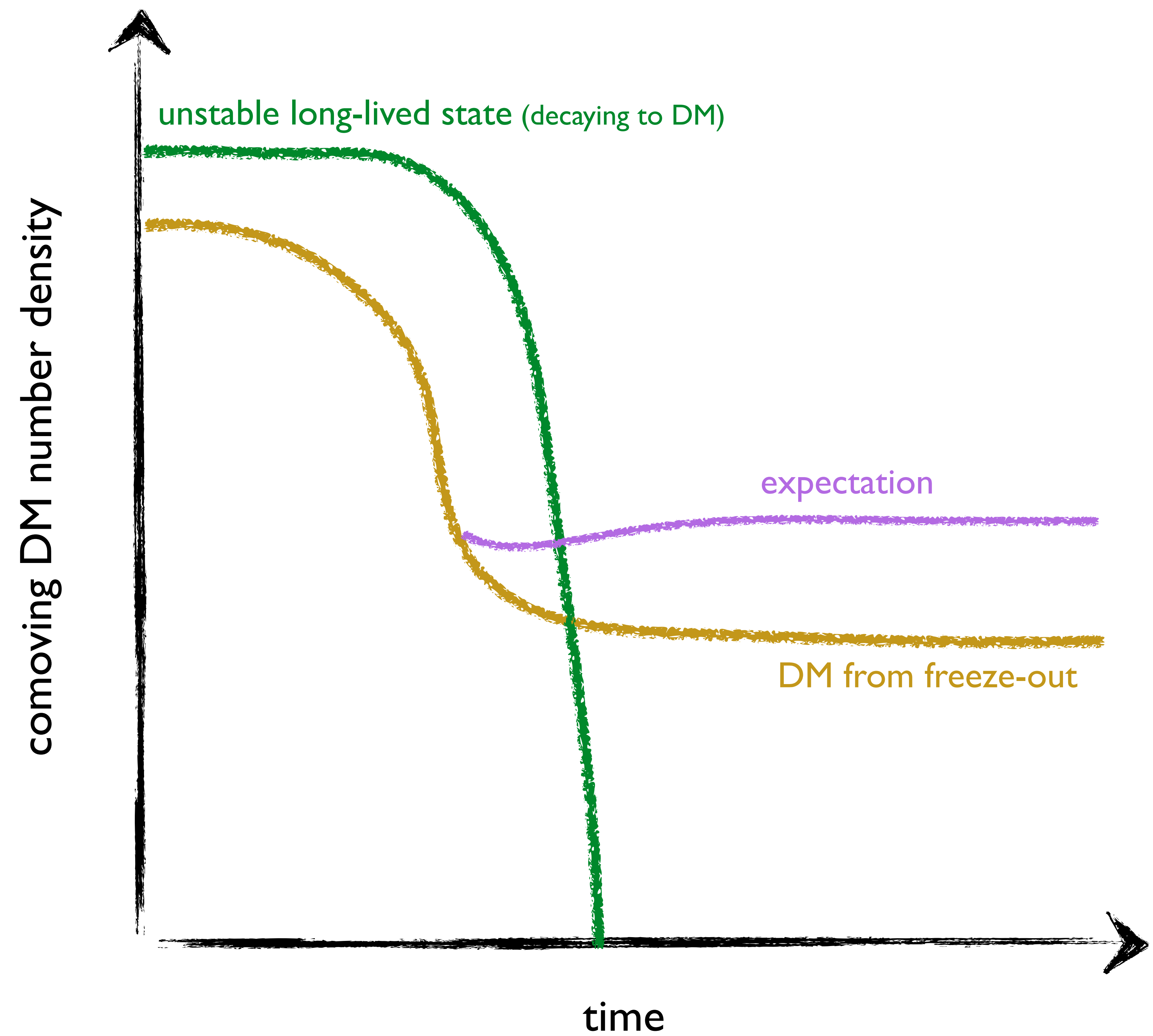

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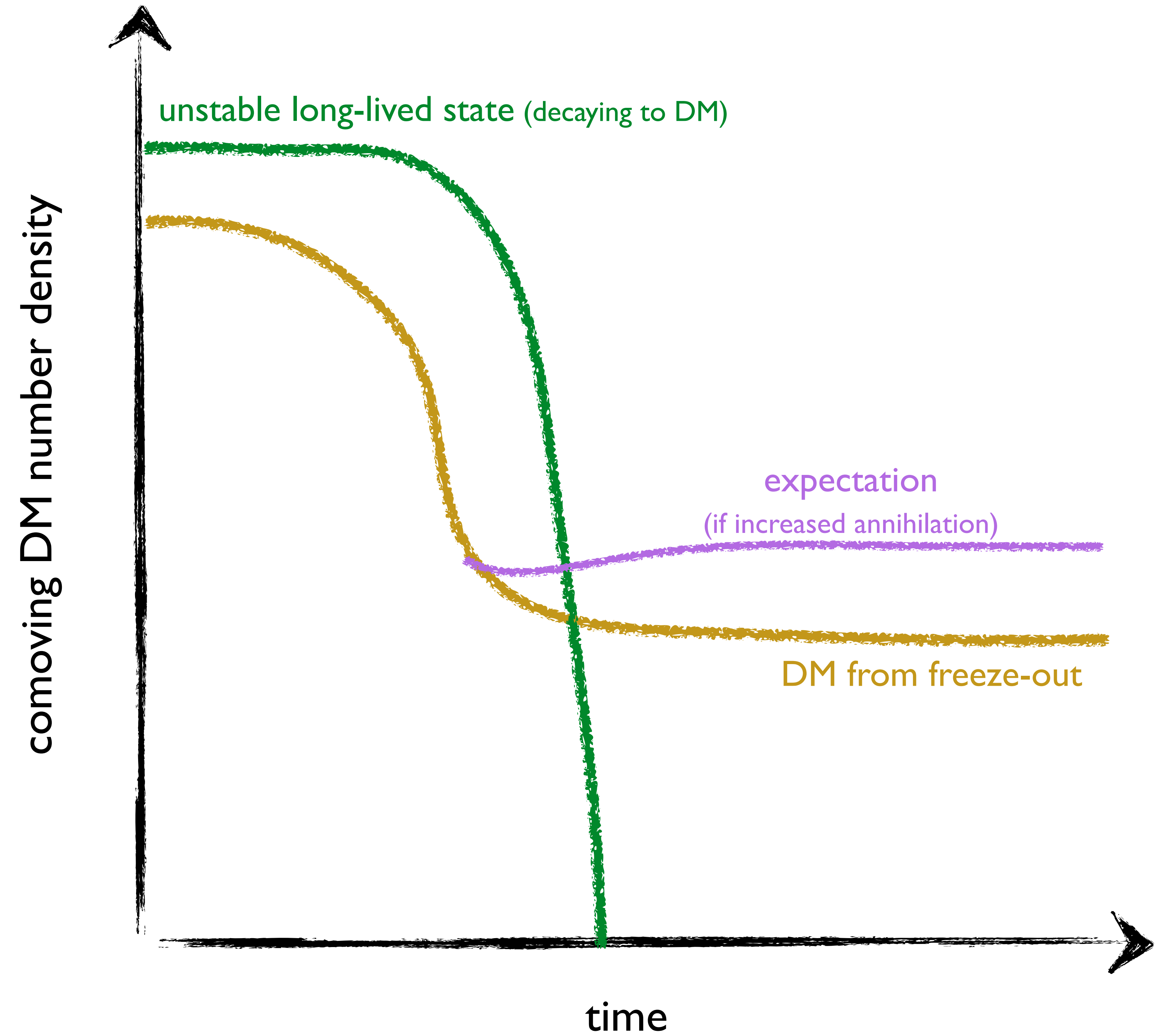

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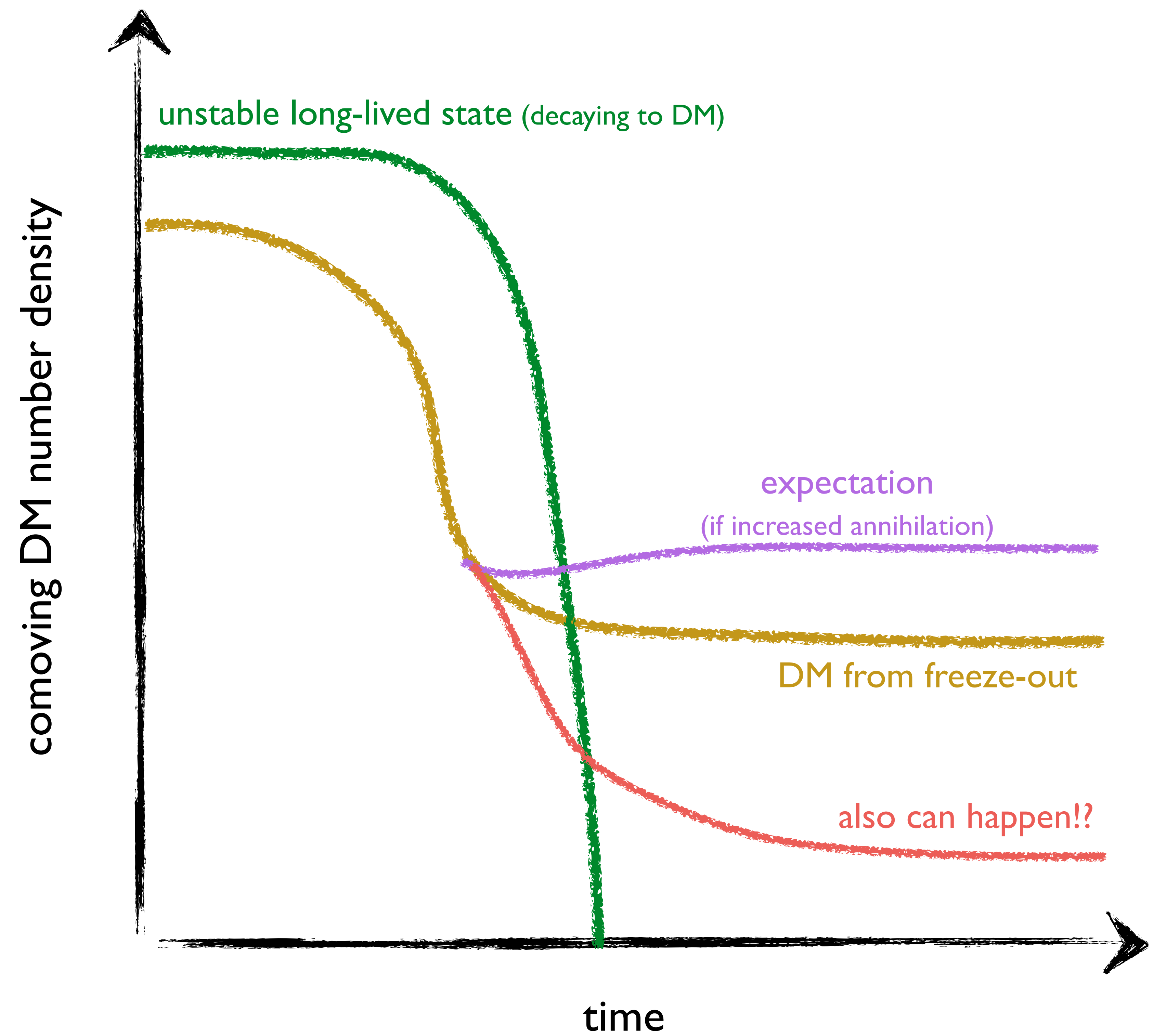

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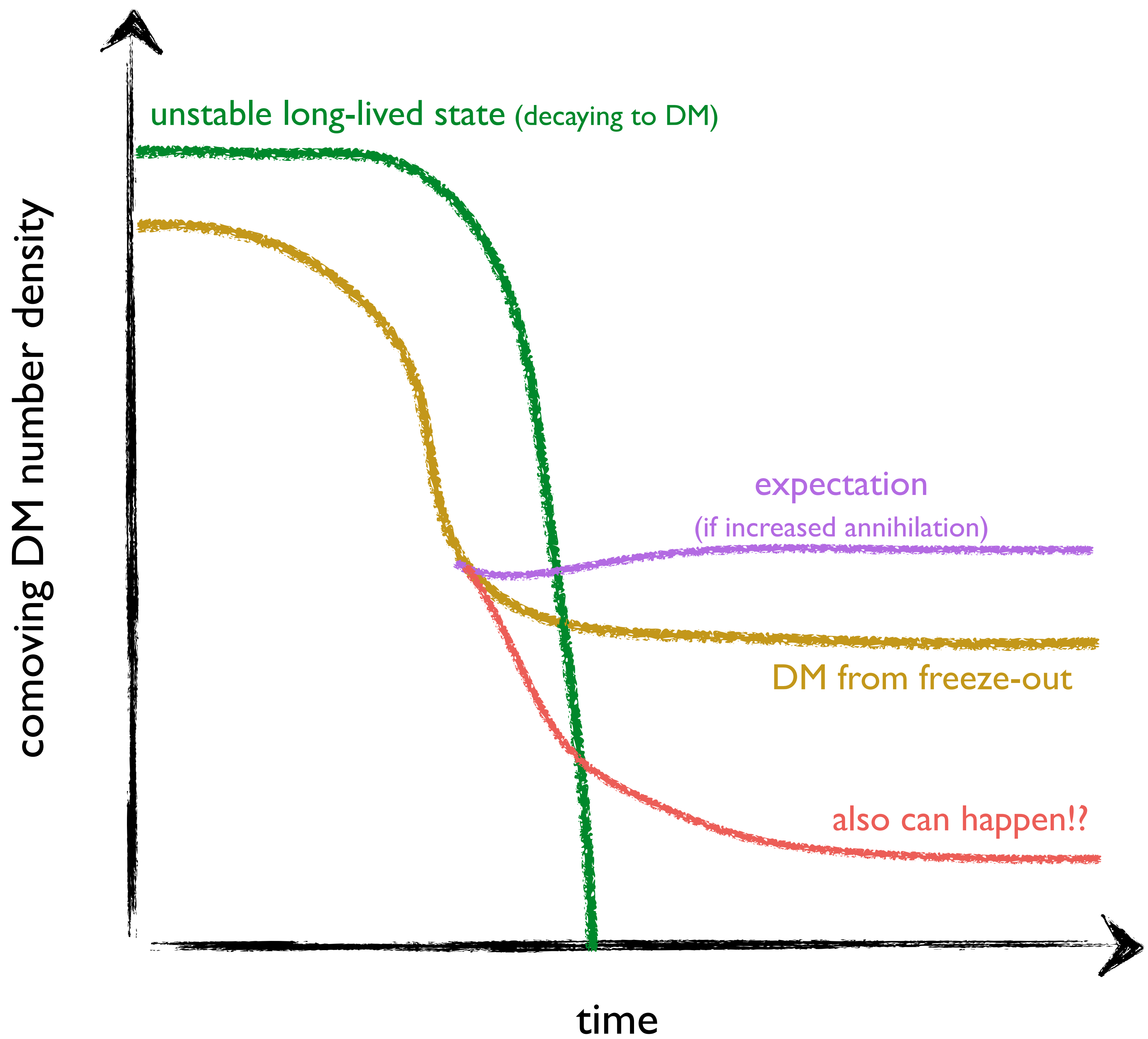

TO SEE WHY AND LEARN MORE STAY TUNED :)

THERMAL RELIC DENSITY

STANDARD SCENARIO

T

$\Gamma_{\rm ann} > H$	DM in full equilibrium (chemical and kinetic)
$\Gamma_{\rm ann} \sim H$	chemical decoupling
$\Gamma_{\rm ann} < H$	freeze-out
$\Gamma_{\rm scatt} \sim H$	kinetic decoupling

time

Thermal Relic Density

STANDARD SCENARIO

(chemical and kinetic)

T		time
	$\Gamma_{\rm ann} > H$ — DM in full equilibrium	
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non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$\boxed{E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right)} f_\chi = \mathcal{C}[f_\chi]$$

Liouville operator in FRW background

the collision term

THERMAL RELIC DENSITY
STANDARD APPROACH

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq: classical limit, molecular chaos,...

…for derivation from thermal QFT see e.g., 1409.3049

Thermal Relic Density
standard approach

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$\Downarrow$ integrate over p (i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

where the thermally averaged cross section:

$$\langle\sigma_{\chi\bar{\chi}\to ij}v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}}\int\frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\,\sigma_{\chi\bar{\chi}\to ij}v_{\rm rel}\; f_\chi^{\rm eq} f_{\bar{\chi}}^{\rm eq}$$

THERMAL RELIC DENSITY

STANDARD APPROACH

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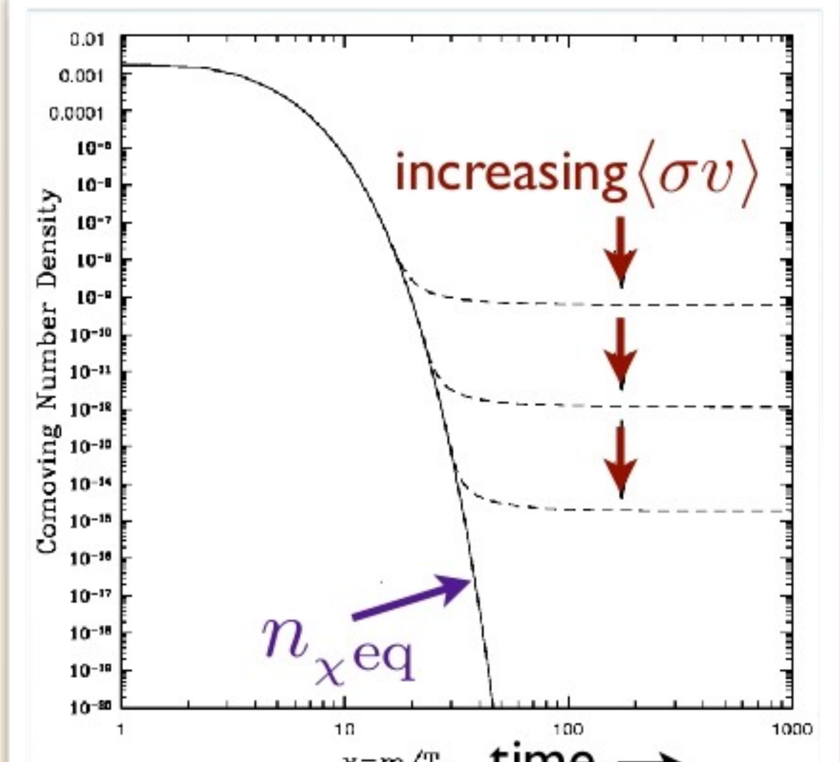

Fig.: Jungman, Kamionkowski & Griest, PR'96

Thermal Relic Density

standard approach

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Critical assumption:

kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\rm eq}$$

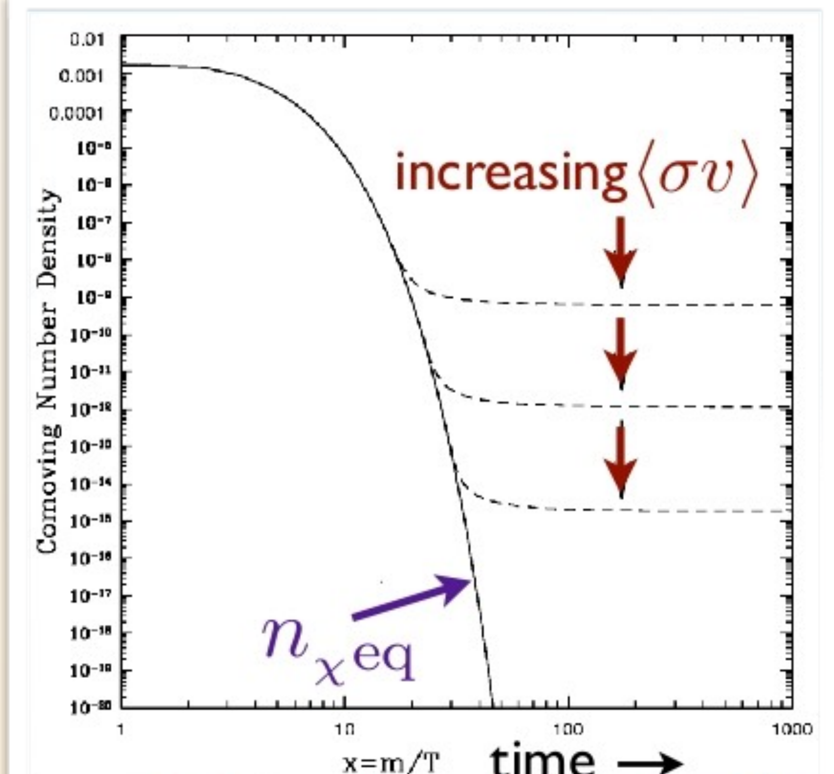
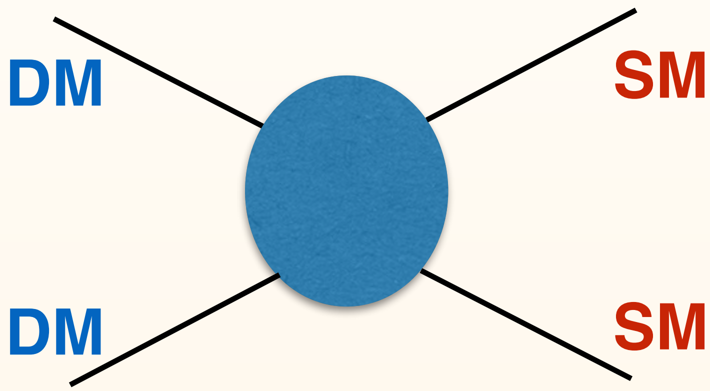


Fig.: Jungman, Kamionkowski & Griest, PR'96

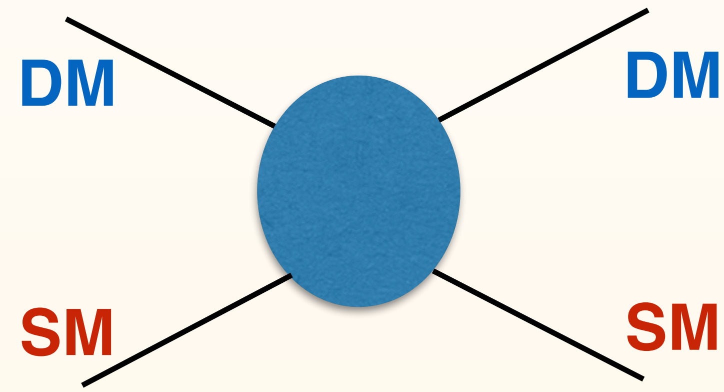
FREEZE-OUT *VS.* DECOUPLING

annihilation



crossing sym.

(elastic) scattering



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Longrightarrow$ scatterings <u>typically</u> more frequent

dark matter frozen-out but <u>typically</u> still kinetically coupled to the plasma

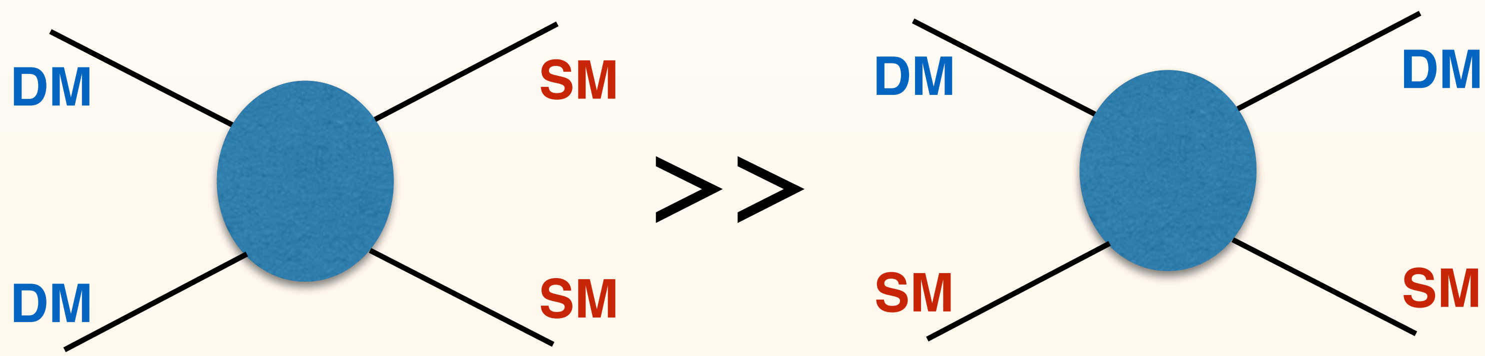
Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Early Kinetic Decoupling?

A necessary and sufficient condition: scatterings weaker than annihilation

i.e. rates around freeze-out: $H \sim \Gamma_{\mathrm{ann}} \gtrsim \Gamma_{\mathrm{el}}$

Possibilities:

A) 

e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

D) Multi-component dark sectors

e.g., additional sources of DM from late decays, …

How to go beyond kinetic equilibrium?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

contains both scatterings and annihilations

Two possible approaches:

fBE

cBE

0-th moment: n_χ
2-nd moment: T_χ
$\dots$

solve numerically for full $f_\chi(p)$

have insight on the distribution
no constraining assumptions

numerically challenging
often an overkill

consider system of equations for moments of $f_\chi(p)$

partially analytic/much easier numerically
manifestly captures all of the relevant physics

finite range of validity
no insight on the distribution

T. Binder, T. Bringmann, M. Gustafsson & A.H. 2103.01944

NEW TOOL!

GOING BEYOND THE STANDARD APPROACH

- Home
- Downloads
- Contact

DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.
DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,**
  Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]

Currently, an user guide can be found in the Appendix A of this reference.
Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for
any (user defined) model*

Interplay between chemical and kinetic decoupling

Prediction for the DM
phase space distribution

Late kinetic decoupling
and impact on cosmology

see e.g., 1202.5456

…

(only) prerequisite:
Wolfram Language (or *Mathematica*)

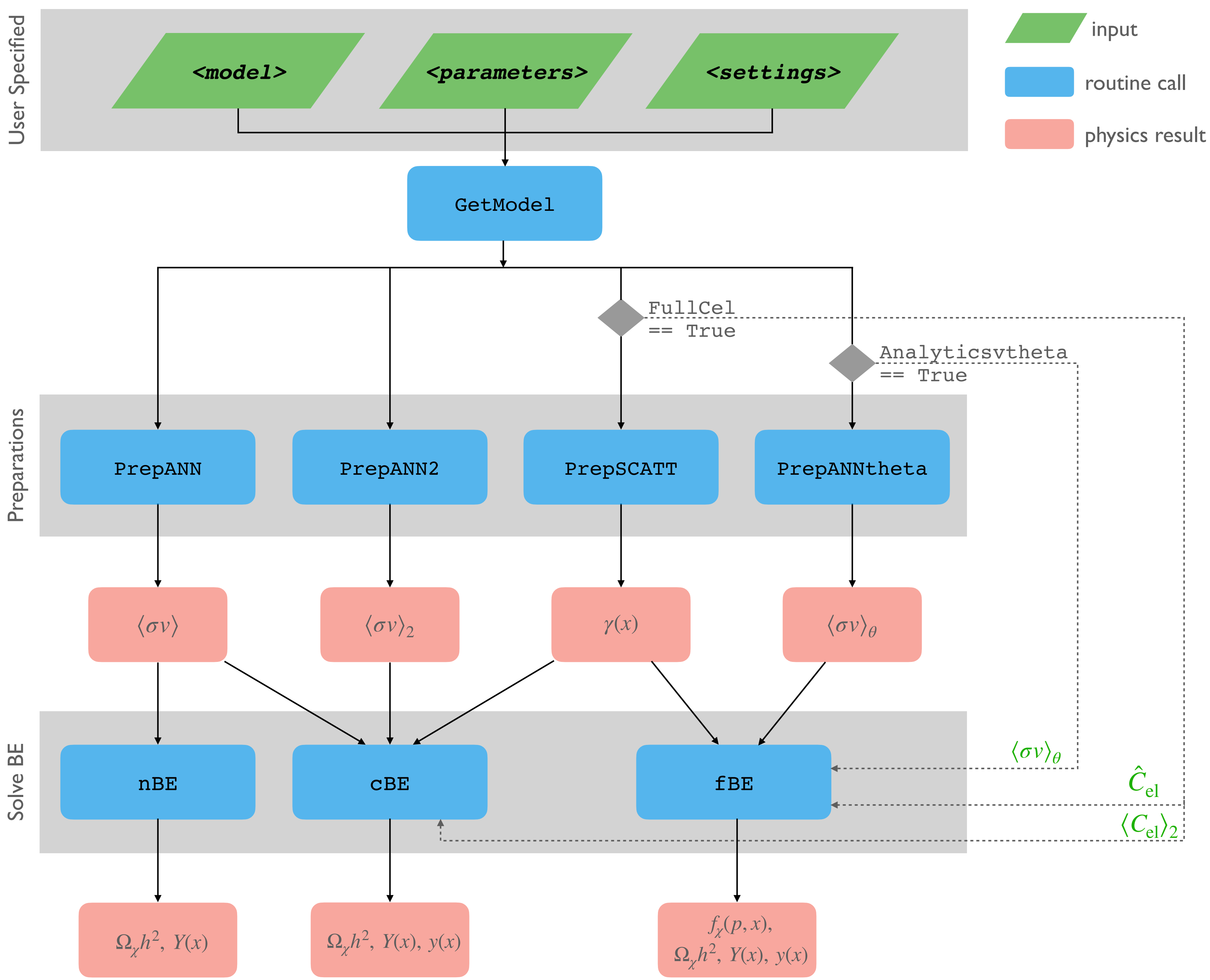
*at the moment for a single DM species and w/o co-annihlations… but stay tuned for extensions!

Dark matter Relic Abundance beyond Kinetic Equilibrium

https://drake.hepforge.org

Few words about the Code

written in *Wolfram Language*, lightweight, modular and simple to use both via script and front end usage



<u>Input</u>: model file (few models implemented, user extendable), parameter values and settings choices

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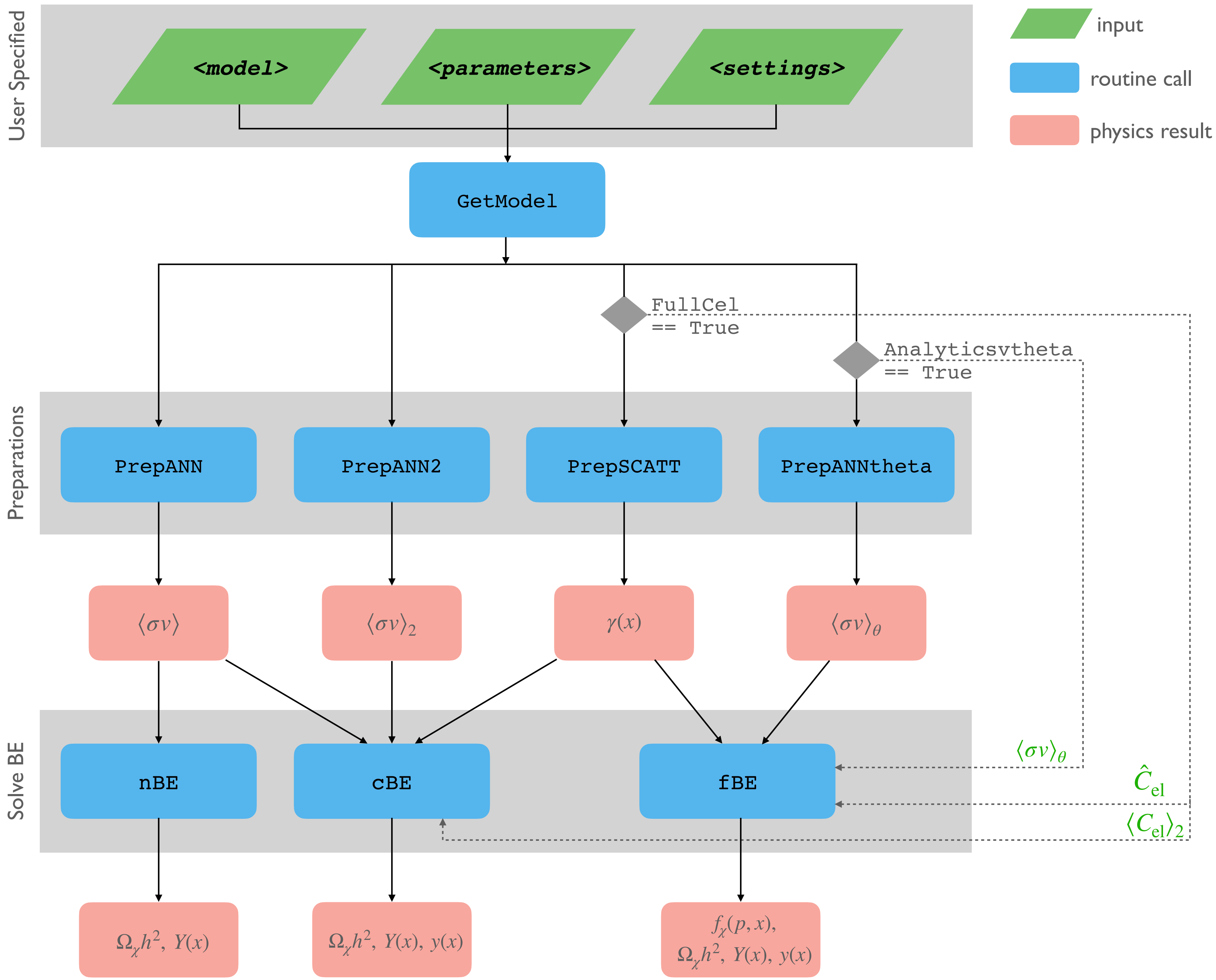

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[slowest part (~ few sec - some min), can be bypassed by user own routines/results…]

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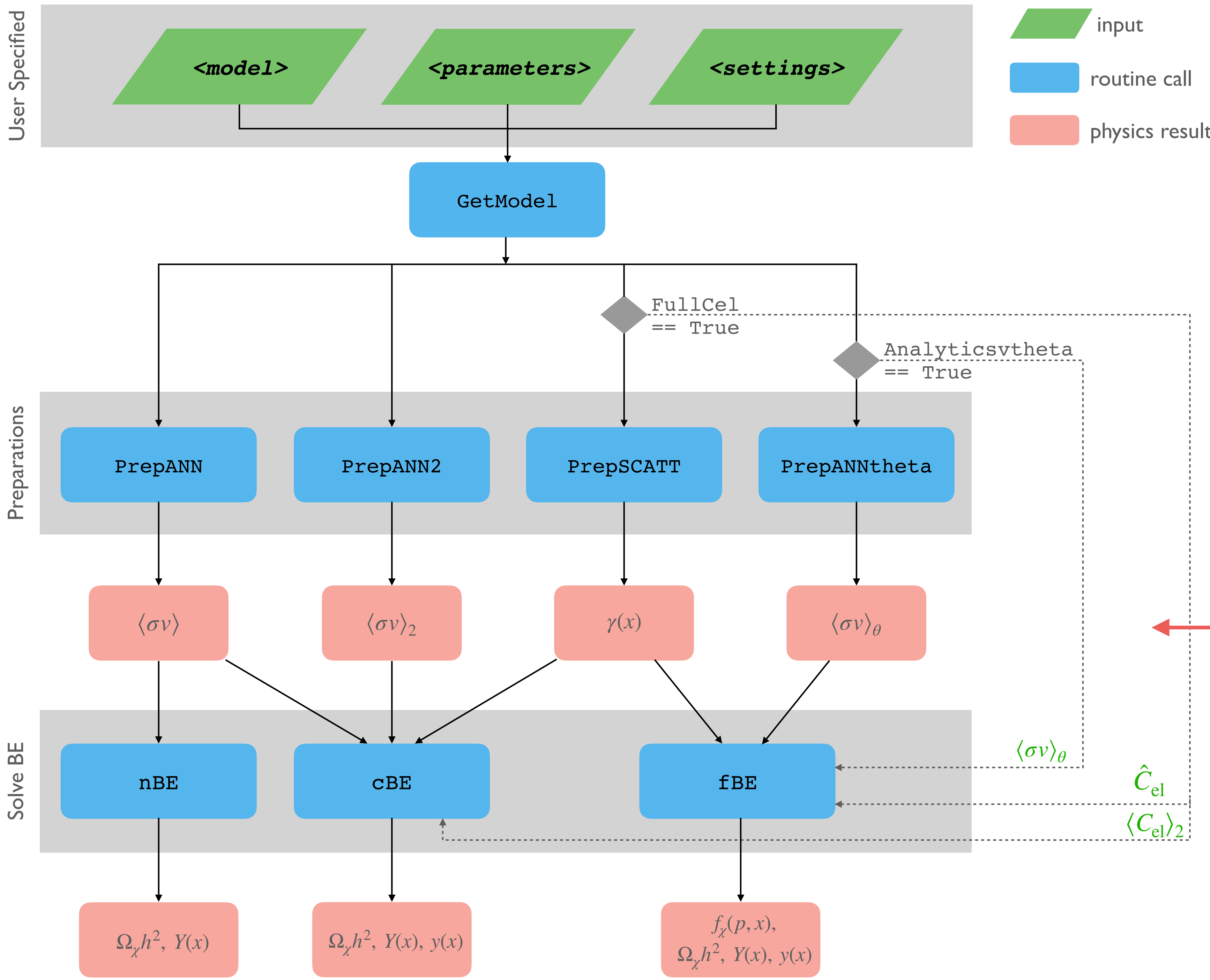

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(intermediate) results for rates

DRAKE

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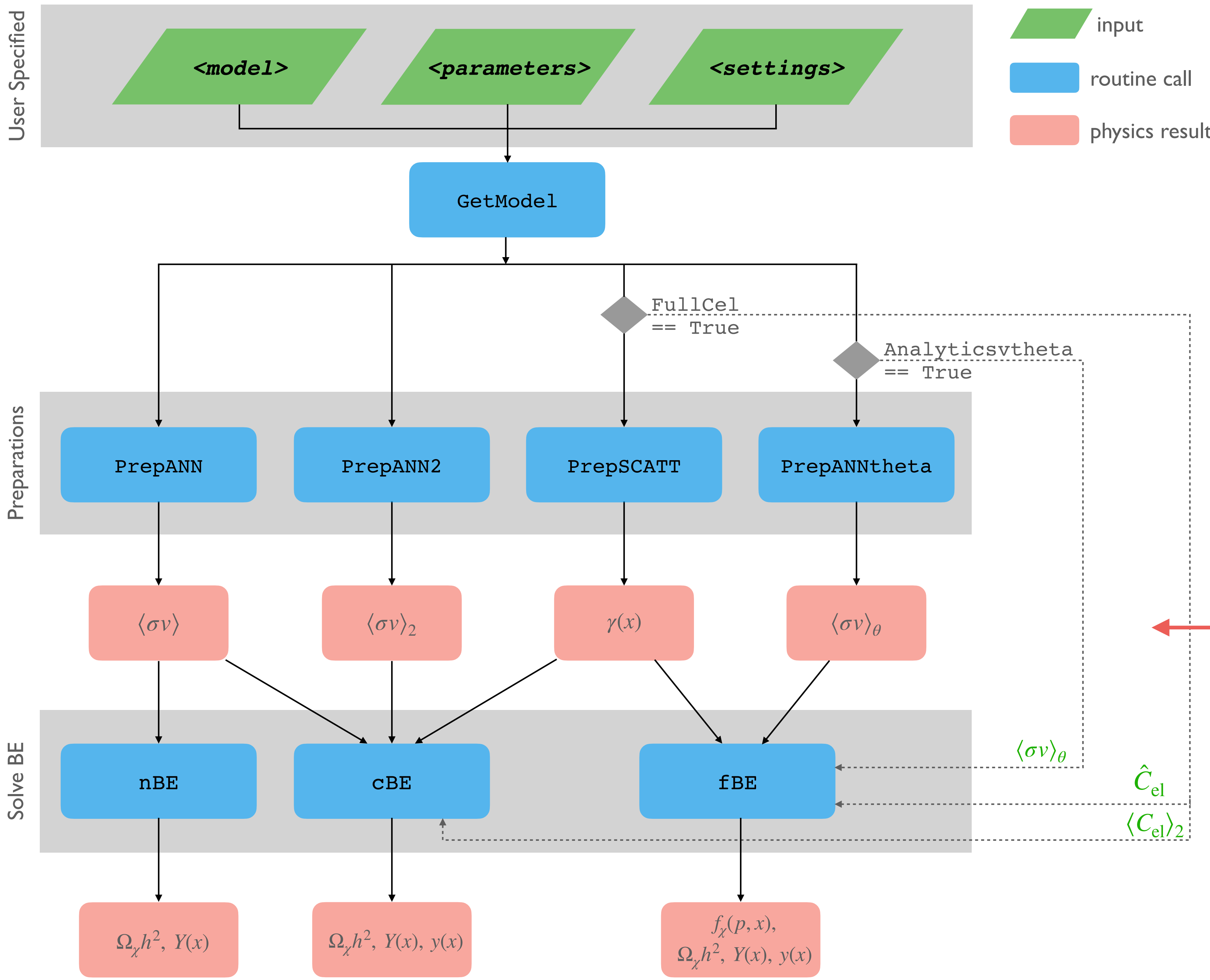

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(intermediate) results for rates

<u>Final phase</u>: tailored solvers for Boltzmann eq.

[~ few seconds]

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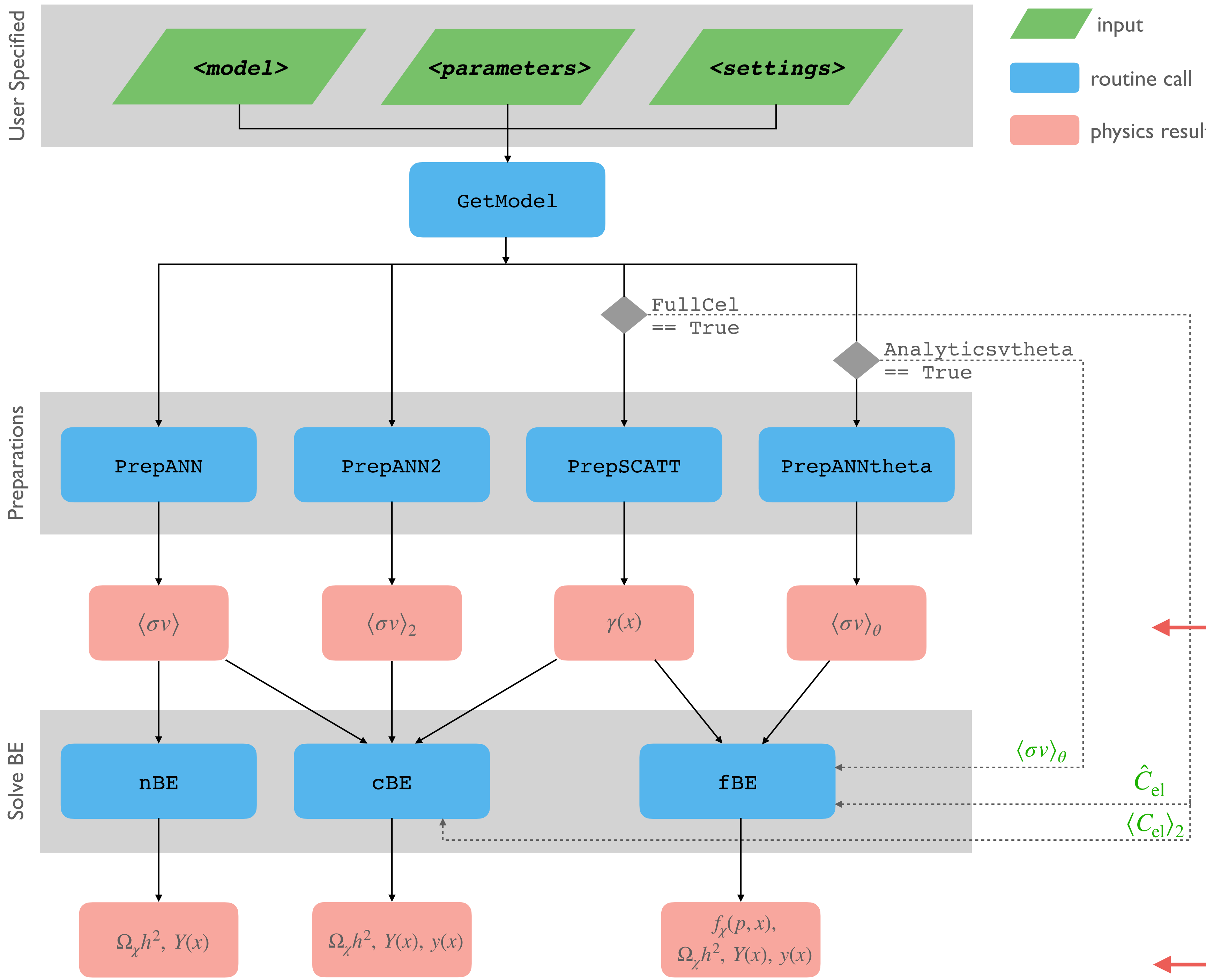

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[~ few seconds]

(final) results for $\Omega h^2, f_{DM}(p)$

Example A:
Scalar Singlet DM

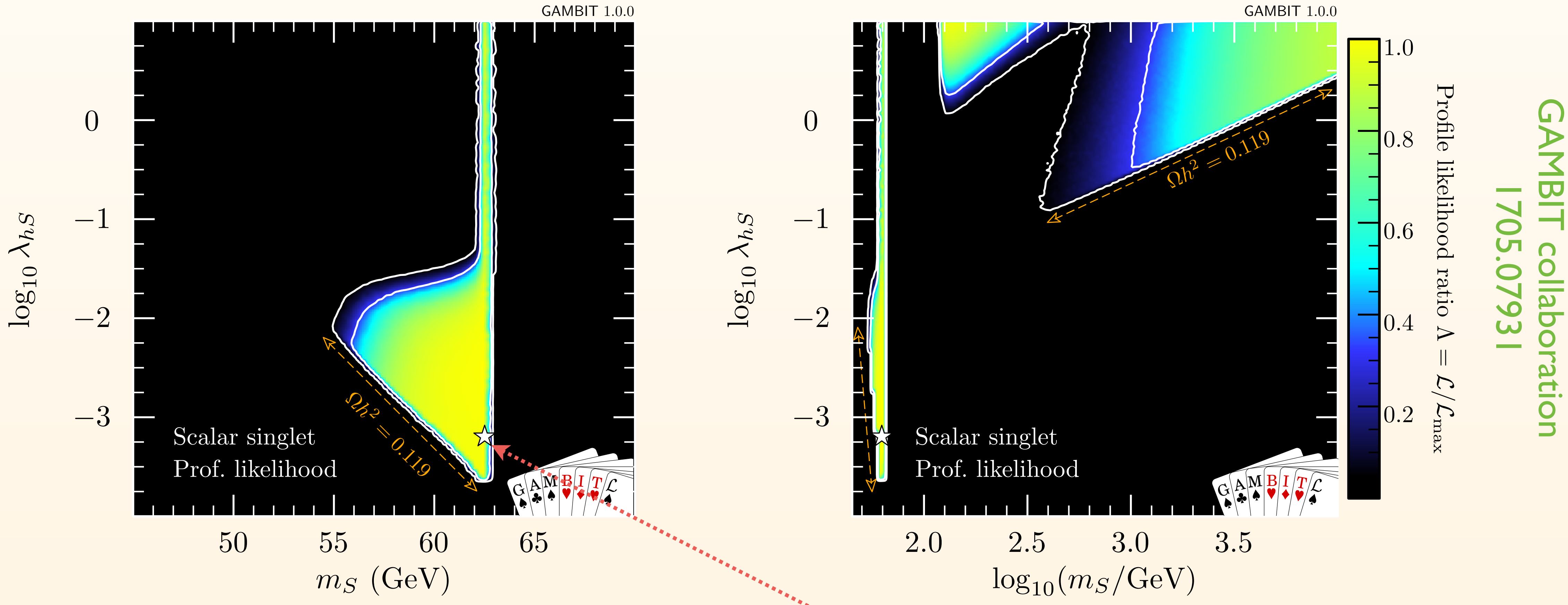
A)

EXAMPLE A
SCALAR SINGLET DM

To the SM Lagrangian add one singlet scalar field *S* with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_s S^2 |H|^2$$

$$m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$



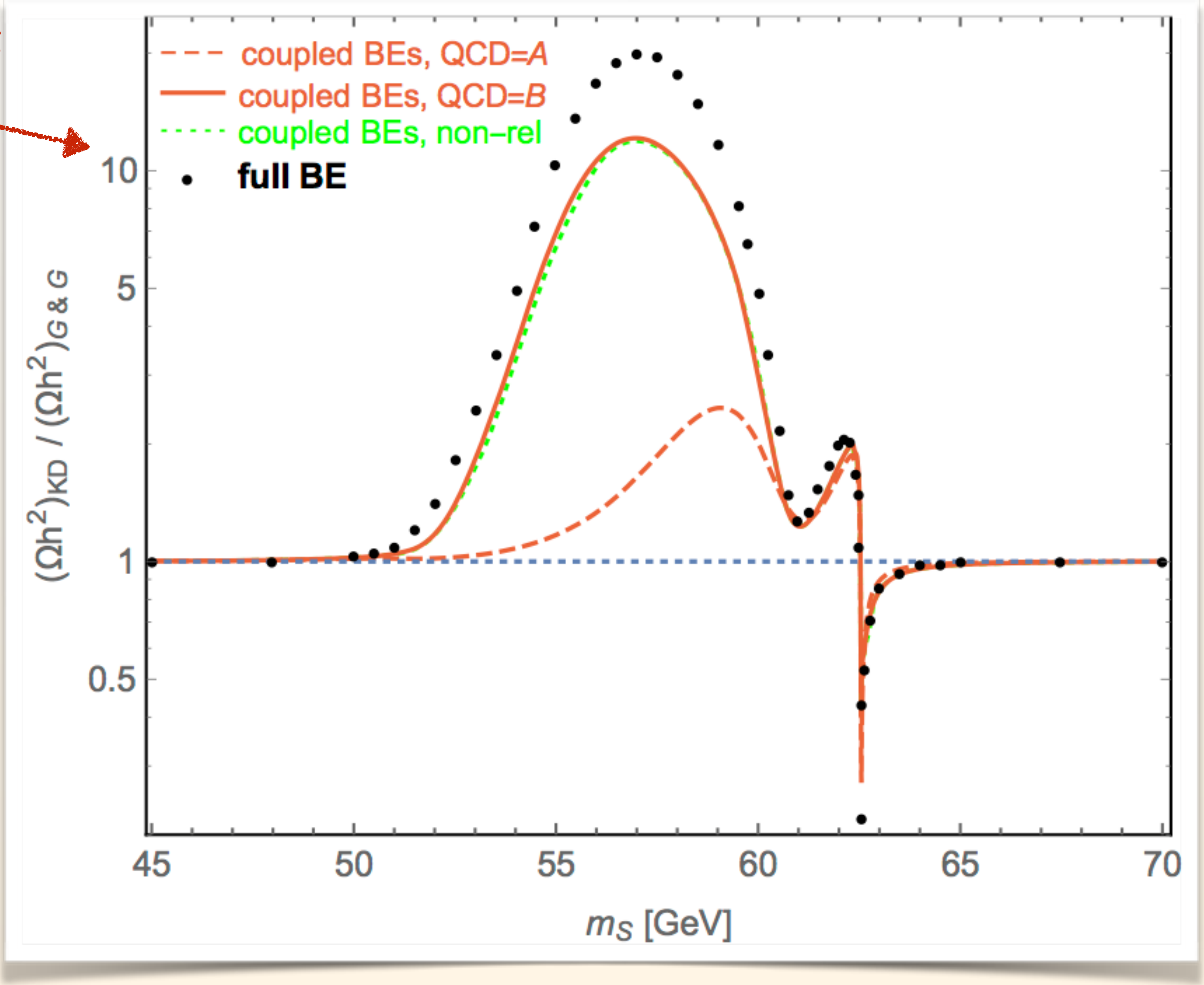
Most of the parameter space excluded, but… even such a simple model is hard to kill

best fit point hides in the resonance region!

RESULTS

EFFECT ON THE Ωh^2

effect on relic density:
up to O(~10)



[… Freeze-out at few GeV ⟶ what is the abundance of heavy quarks in QCD plasma?

two scenarios: QCD = A - all quarks are free and present in the plasma down to T_c =154 MeV
QCD = B - only light quarks contribute to scattering and only down to $4T_c$ …]

cBE vs. fBE

which is more accurate?!

They correspond to the opposite limits of self-interaction strengths:

very efficient - **cBE**

inefficient - f**BE**

cBE vs. fBE

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They correspond to the opposite limits of self-interaction strengths:

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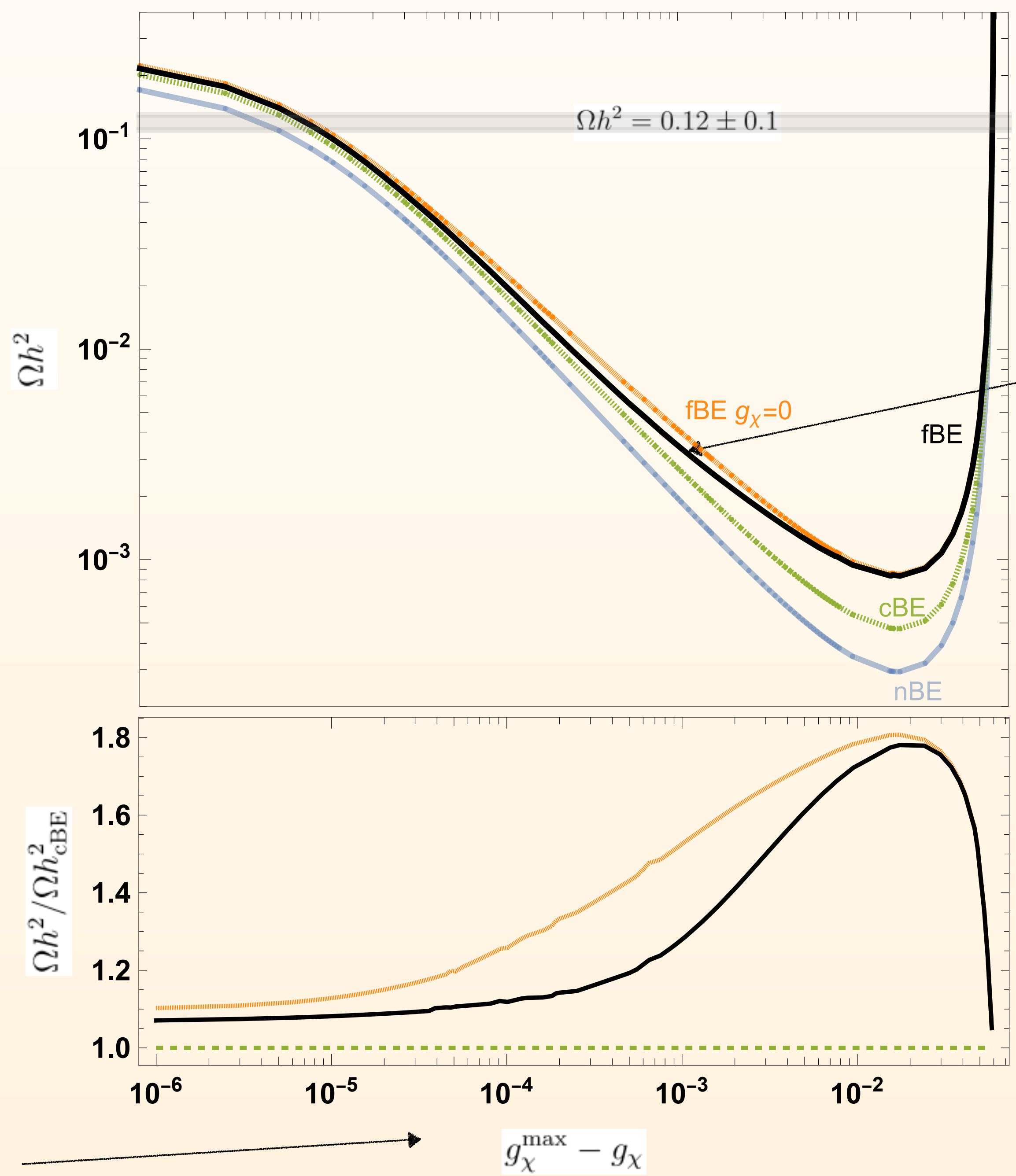
inefficient - **fBE**

Which limit is closer to reality depends on the model, but it seems that fBE is typically more accurate, unless self-scattering is tuned up, e.g:



black line gives the result including self-scattering processes! (being between pure fBE and cBE)

coupling to the mediator; governs self-scatterings

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CBE VS. FBE
WHICH IS MORE ACCURATE?!

They correspond to the opposite limits of self-interaction strengths:

very efficient - **cBE**

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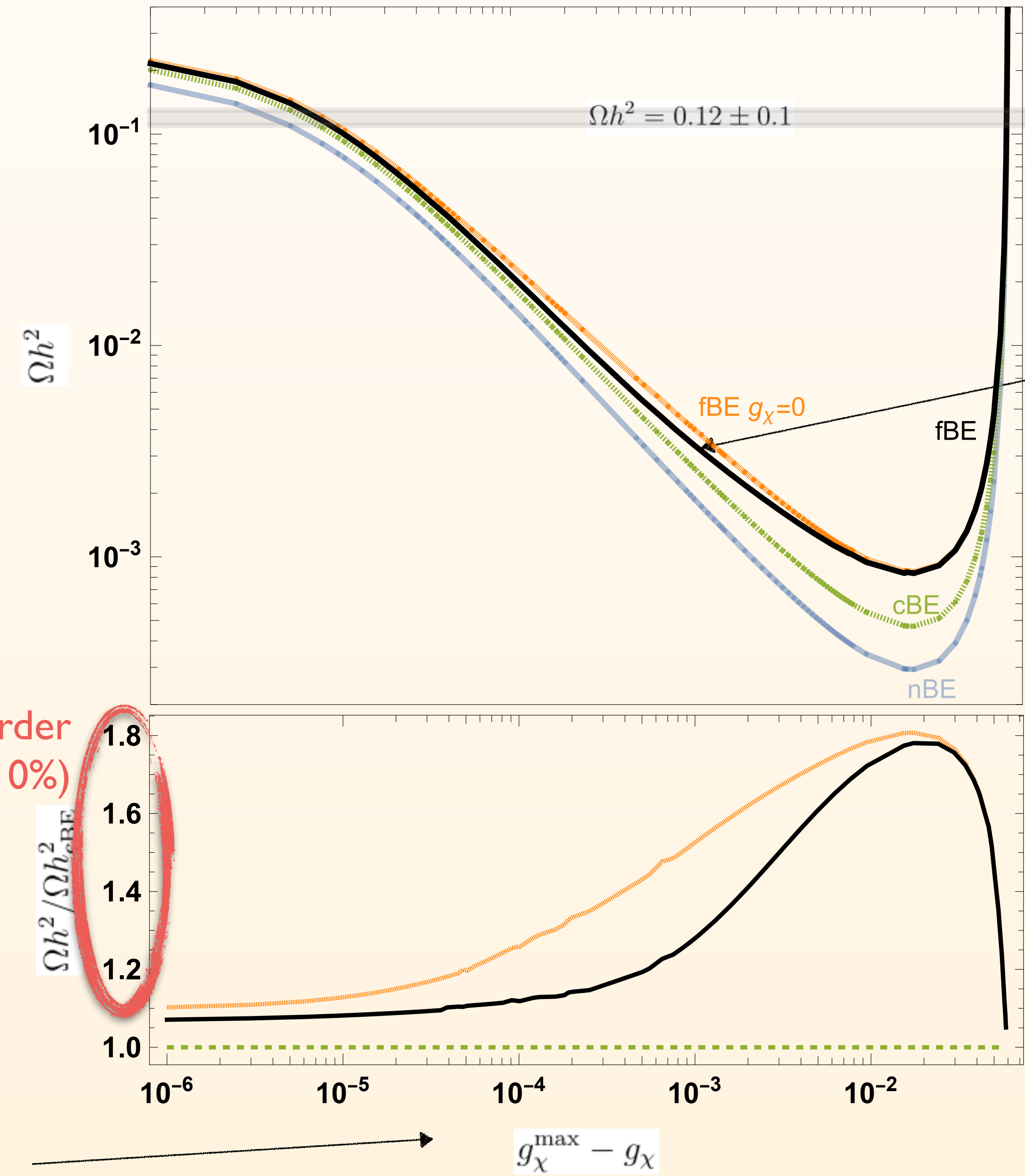


black line gives the result including self-scattering processes! (being between pure fBE and cBE)

difference of order O(10%)

coupling to the mediator; governs self-scatterings

EXAMPLE D:
WHEN ADDITIONAL INFLUX OF DM ARRIVES

D) Multi-component dark sectors

Sudden injection of more DM particles distorts $f_\chi(p)$
(e.g. from a decay or annihilation of other states)

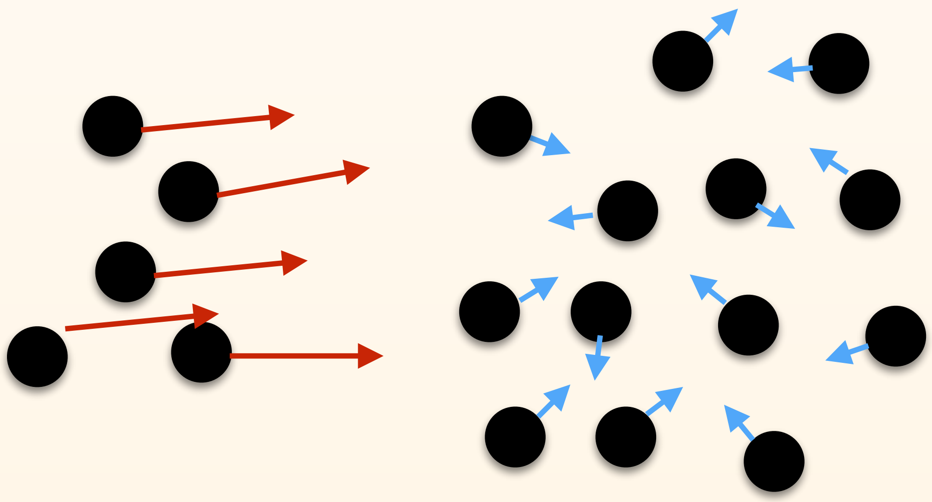
- this can modify the annihilation rate (if still active)
- how does the thermalization due to elastic scatterings happen?

1) DM produced via:

- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

2) DM annihilation has a threshold

e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$



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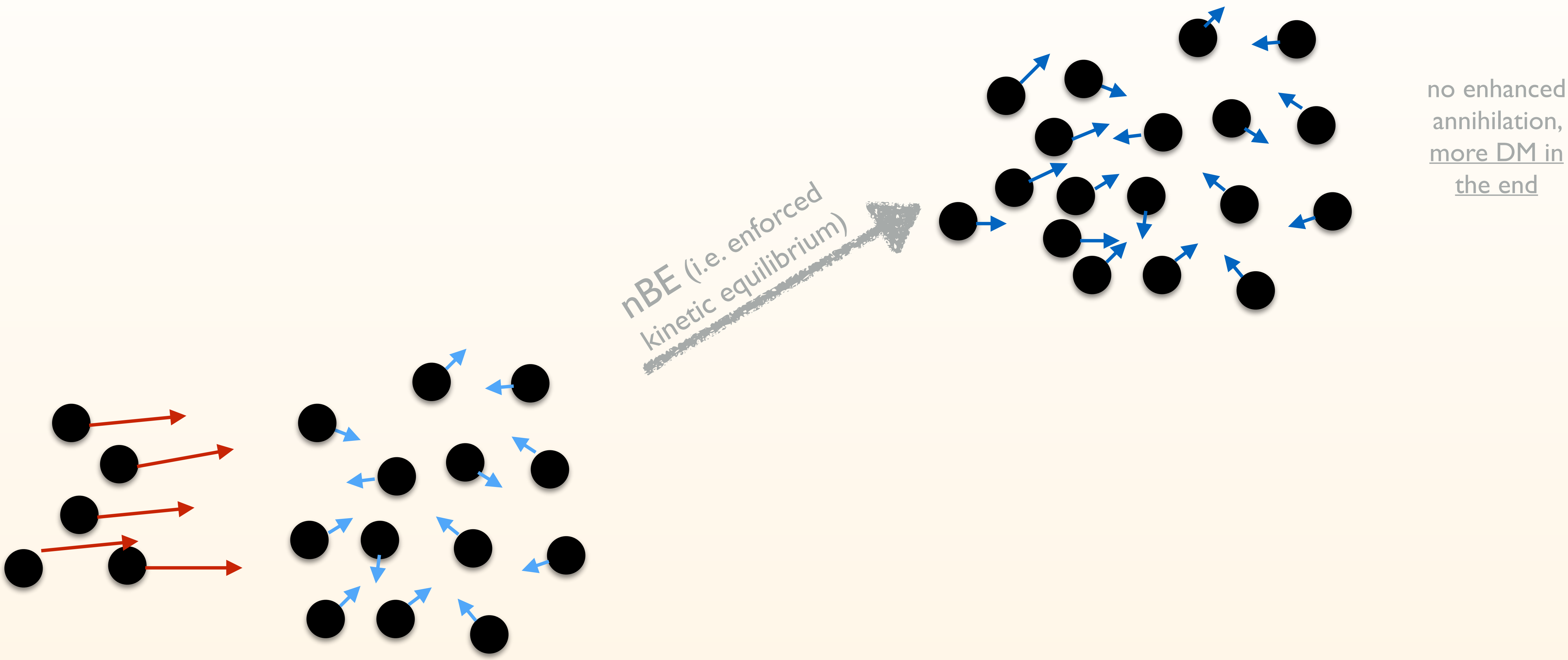

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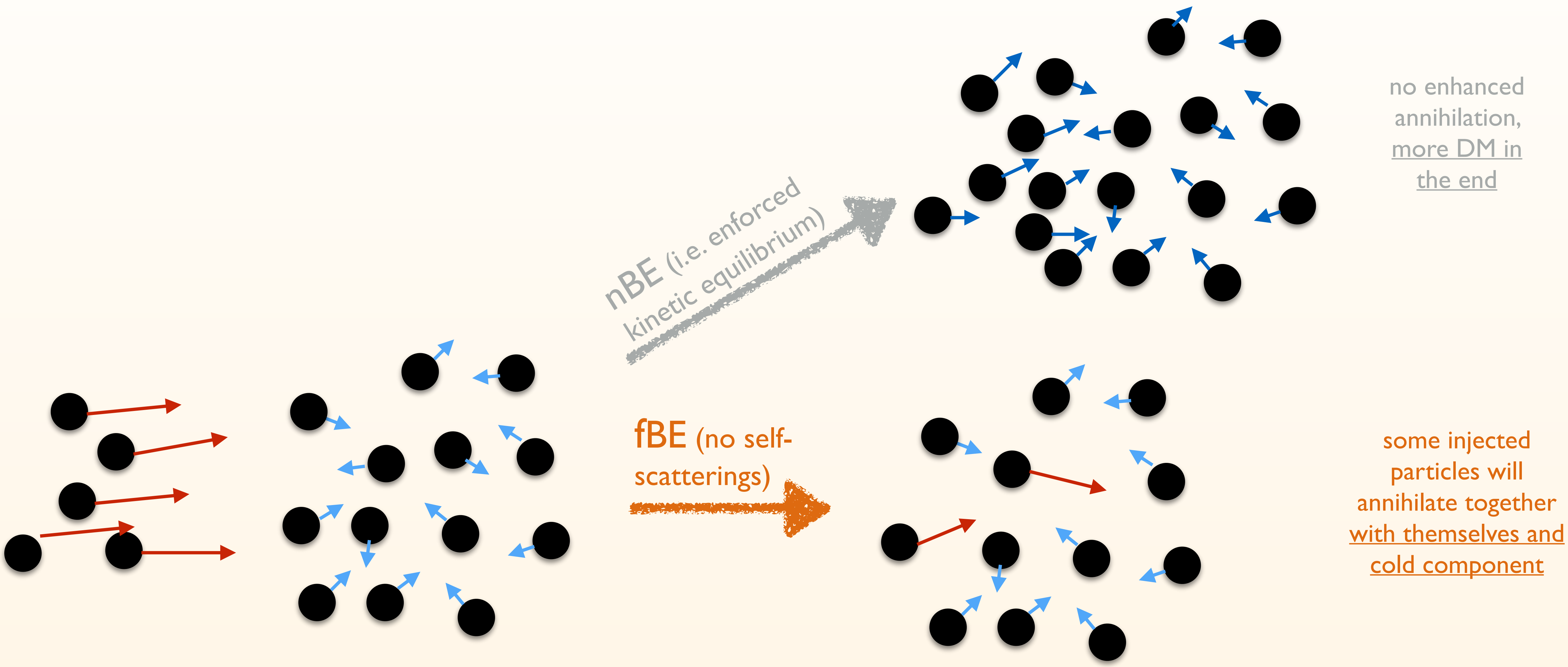

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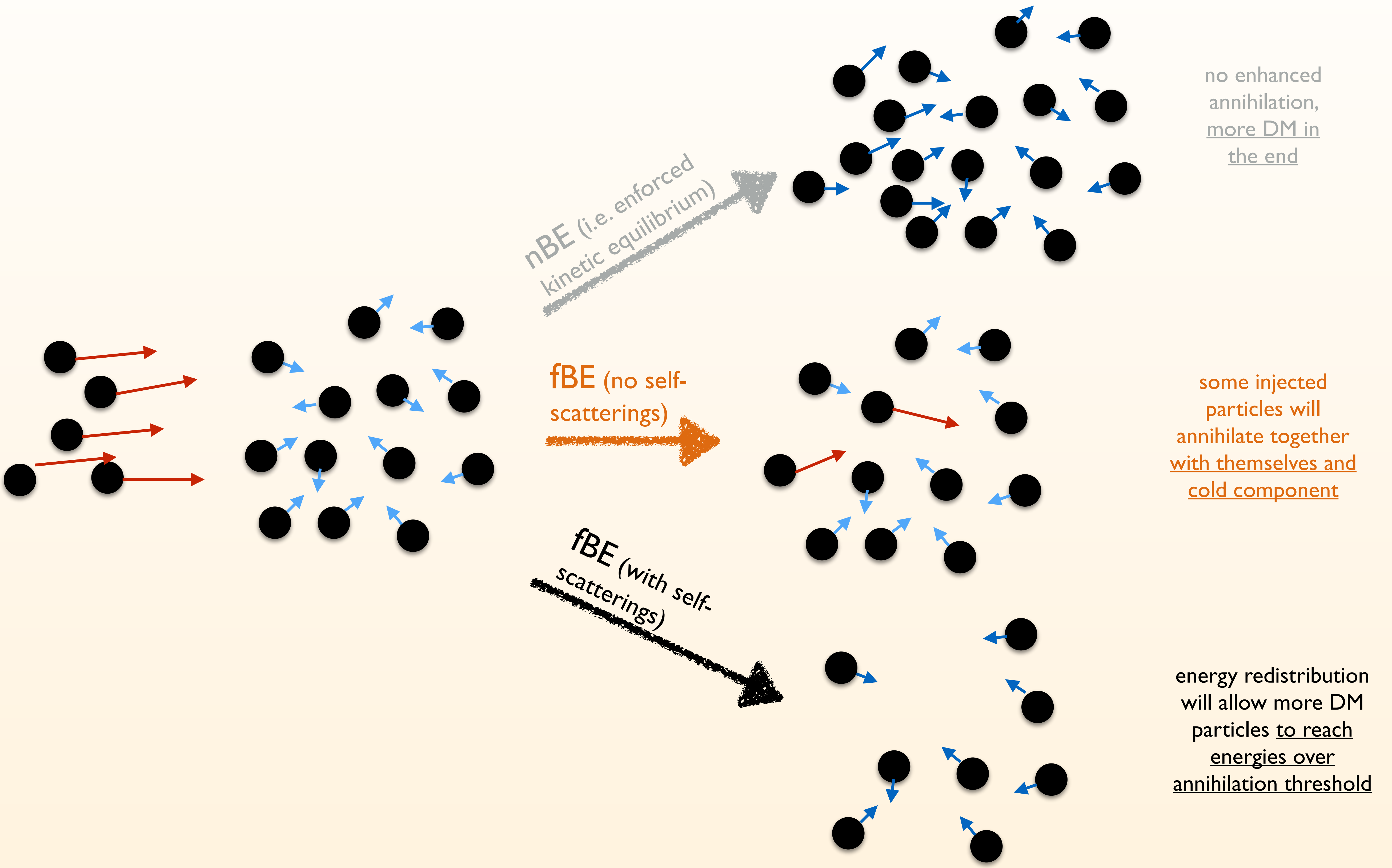

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EXAMPLE EVOLUTION

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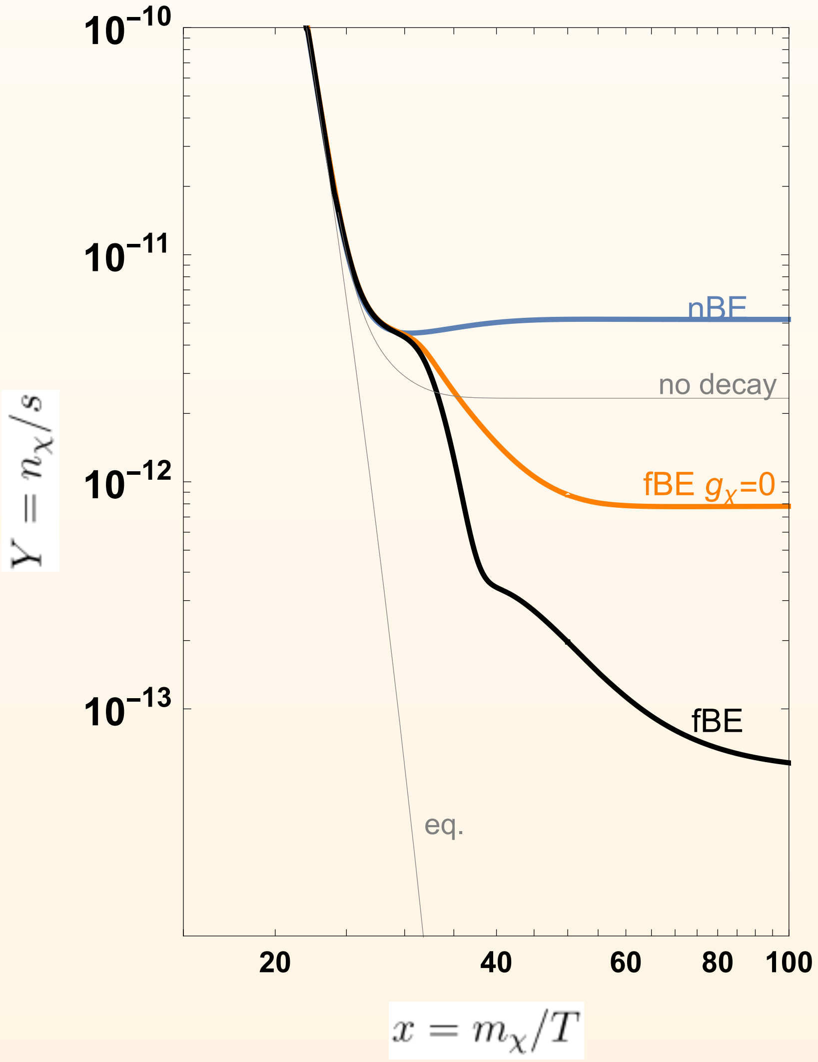
$Y \sim$ number density



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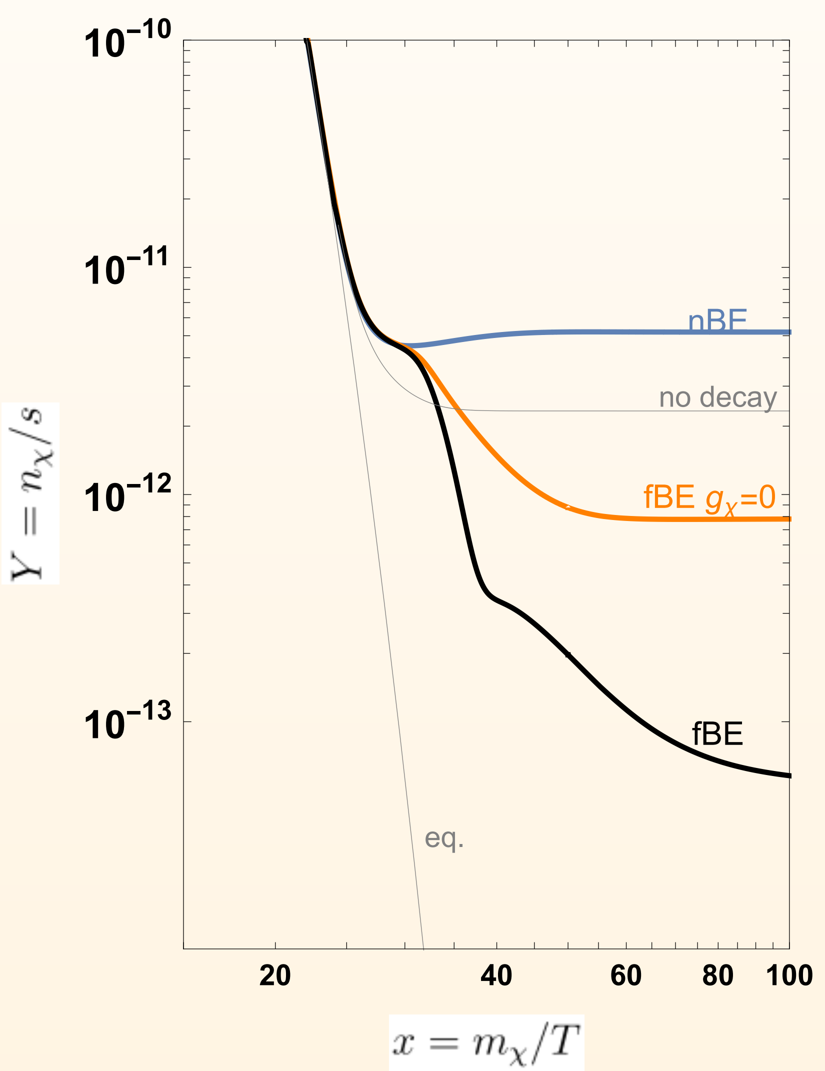
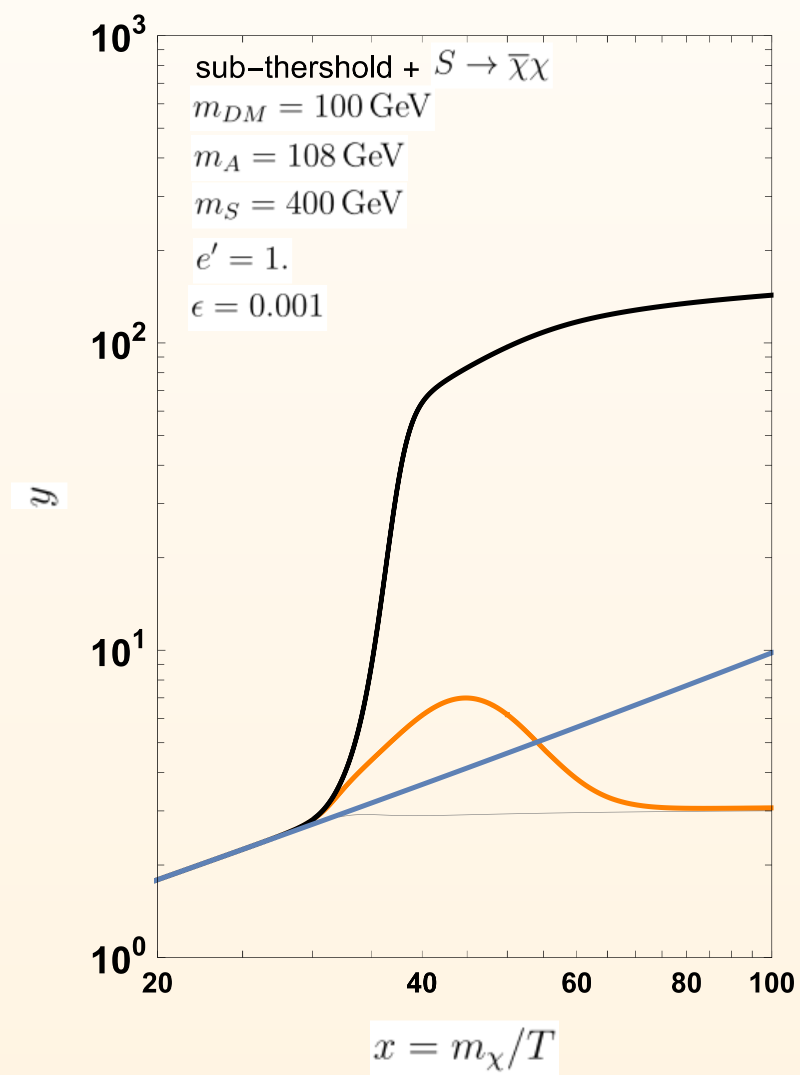


$y \sim$ temperature



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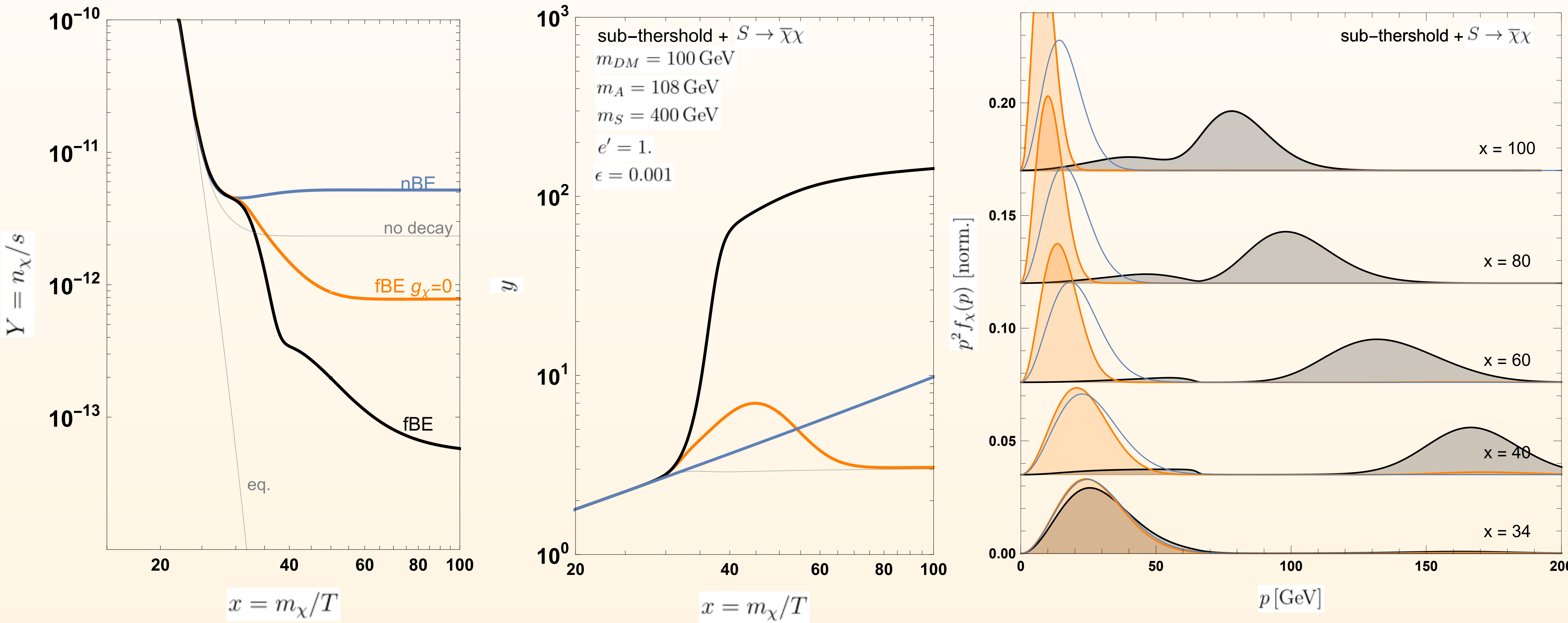
$Y \sim$ number density

$y \sim$ temperature

$p^2 f(p) \sim$ momentum distribution



SUMMARY

1. Kinetic equilibrium is a necessary (often implicit) assumption for standard relic density calculations in all the numerical tools…
…while it is not always warranted!

2. Much more accurate treatment comes from solving the full phase space Boltzmann equation (fBE) to obtain result for $f_{\rm DM}(p)$ where one can study also self-thermalization from self-scatterings

3. Introduced DRAKE : a new tool to extend the current capabilities to the regimes beyond kinetic equilibrium

4. Multi-component sectors, when studied at the fBE level, can reveal quite unexpected behavior