EARLY KINETIC DECOUPLING OF DM

WHEN THE STANDARD WAY OF CALCULATING THE THERMAL RELIC DENSITY FAILS

Andrzej Hryczuk

University of Oslo

based on: **T. Binder, T. Bringmann, M. Gustafsson and AH,**
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+ work in progress

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MOTIVATION
THERMAL RELIC DENSITY

Theory:

I. Natural

Comes out automatically from the expansion of the Universe

Naturally leads to cold DM

II. Predictive

No dependence on initial conditions

Fixes coupling(s) $\Rightarrow$ signal in DD, ID & LHC

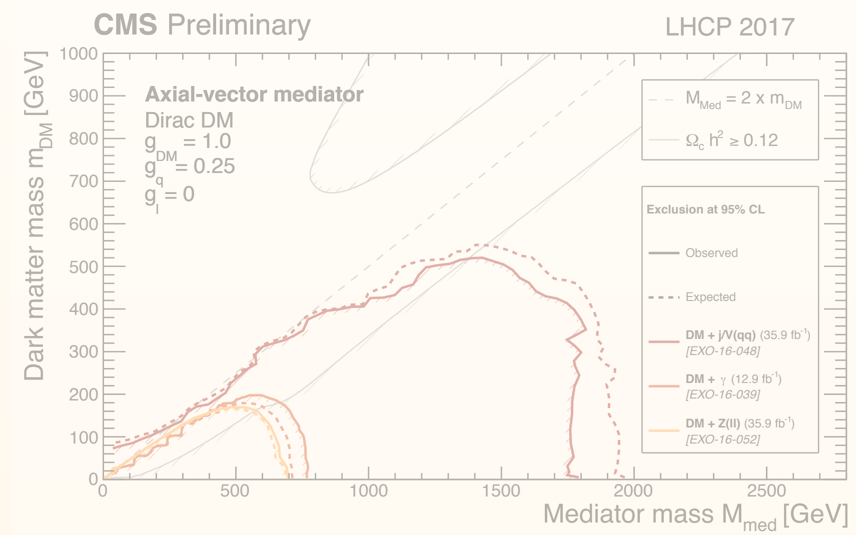
III. It is not optional

Overabundance constraint

To avoid it one needs quite significant deviations from standard cosmology

Experiment:

…as a constraint:



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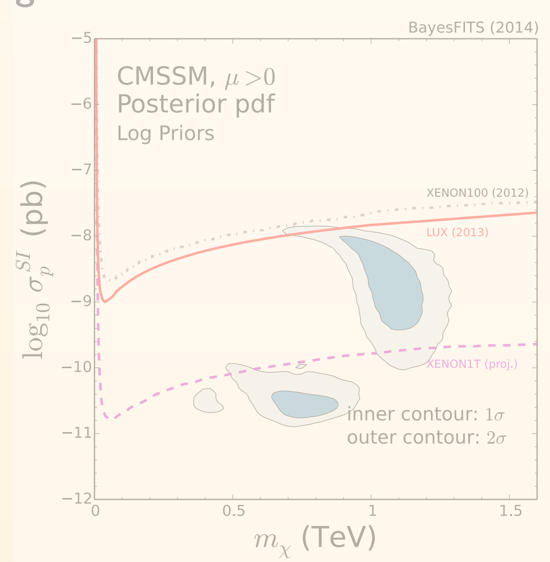

"(…) besides the Higgs boson mass measurement and LHC direct bounds, the constraint showing **by far the strongest impact** on the parameter space of the MSSM is the **relic density**"

Roszkowski et al. '14

…as a pin:

When a dark matter signal is (finally) found: relic abundance can pin-point the particle physics interpretation

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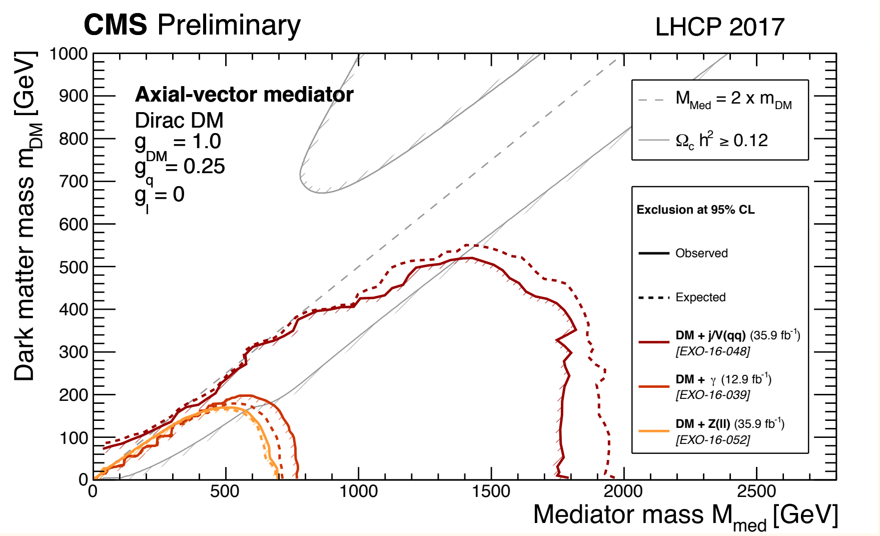

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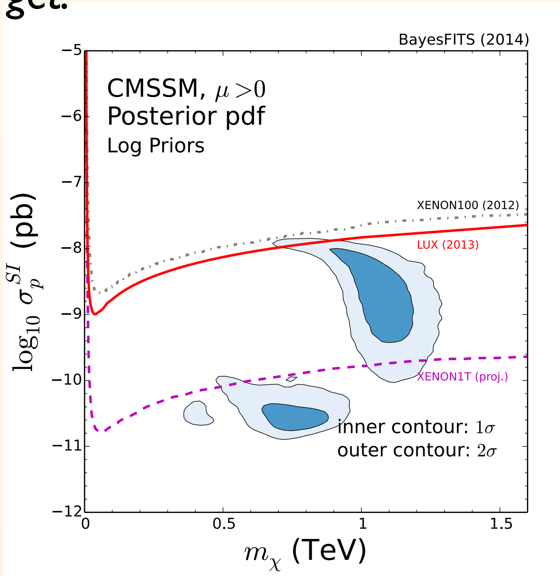

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THERMAL RELIC DENSITY
STANDARD APPROACH

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq:
classical limit, molecular chaos,...

…for derivation from thermal QFT
see e.g., 1409.3049

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$\Downarrow$ integrate over p (i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}} \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\, \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel}\, f_\chi^{\rm eq} f_{\bar{\chi}}^{\rm eq}$$

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$\Downarrow$

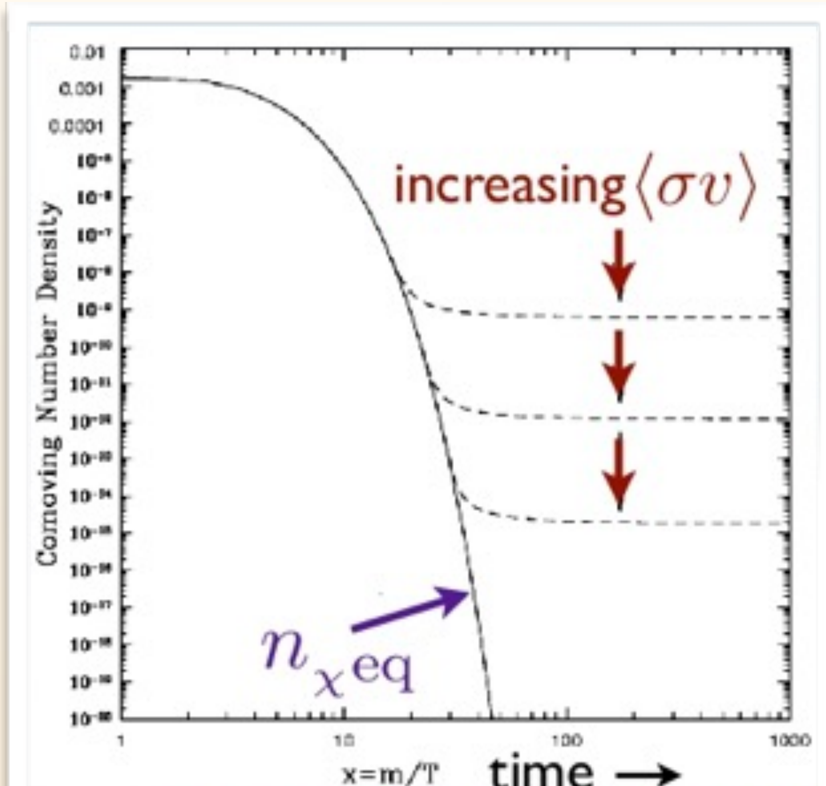

Fig.: Jungman, Kamionkowski & Griest, PR'96

Thermal Relic Density

Standard approach

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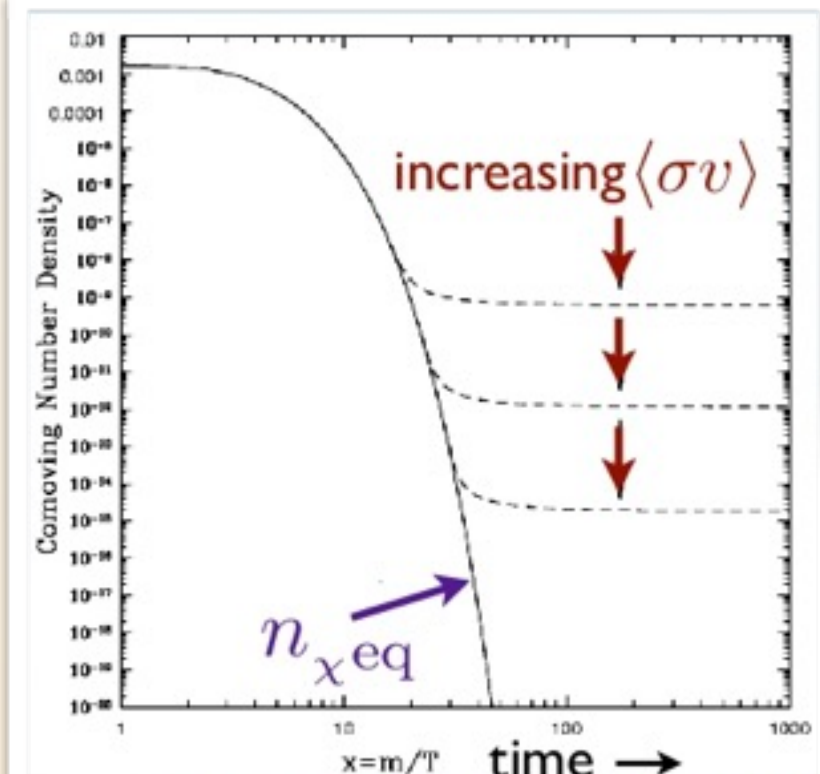

Fig.: Jungman, Kamionkowski & Griest, PR'96

Critical assumption:
kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(\mu) f_\chi^{\rm eq}$$

THERMAL RELIC DENSITY
”EXCEPTIONS”

1. Three ”exceptions”
   Griest, Seckel ’91
2. Non-standard cosmology
   many works… very recent e.g., D’Eramo, Fernandez, Profumo ’17
3. Second era of annihilation
   Feng et al. ’10; Bringmann et al. ’12; …
4. Bound State Formation
   recent e.g., Petraki at al. ’15, ’16; An et al. ’15, ’16; Cirelli et al. ’16; …

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5. $3 \rightarrow 2$ and $4 \rightarrow 2$ annihilation
   e.g., D’Agnolo, Ruderman ’15; Cline at al. ’17; Choi at al. ’17; …
6. Semi-annihilation/Cannibalization
   D’Eramo, Thaler ’10; … e.g., Kuflik et al. ’15; Pappadopulo et al. ’16; …
7. Conversion driven/Co-scattering
   Garny, Heisig, Lulf, Vogl ’17 D’Agnolo, Pappadopulo, Ruderman ’17
8. …

In other words: whenever studying non-minimal scenarios ”exceptions” appear — but most of them come from interplay of new added effects, while do not affect the foundations of modern calculations

WHAT IF NON-MINIMAL SCENARIO?

Example: assume two particles in the dark sector: A and B

scenario / process	Co-annihilation	superWIMP	Co-decaying	Conversion-driven/ Co-scattering	Cannbial/Semi-annihilation	Forbidden-like	…
annihilation A A <-> SM SM A B <-> SM SM B B <-> SM SM							
conversion A A <-> B B inelastic scattering A SM <-> B SM							
elastic scattering A SM <-> A SM B SM <-> B SM							
el. self-scattering A A <-> A A B B <-> B B							
decays A <-> B SM A <-> SM SM B <-> SM SM							
semi-ann/3->2 A A A <-> A A A A <-> A B A A A <-> SM A							

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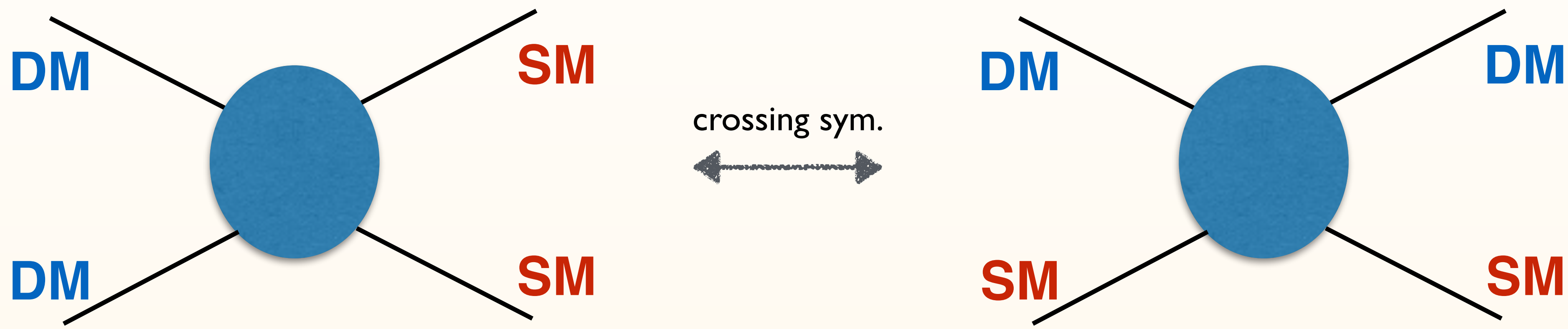
FREEZE-OUT *VS.* DECOUPLING

annihilation | (elastic) scattering



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Rightarrow$ scatterings typically more frequent

dark matter frozen-out but typically
still kinetically coupled to the plasma

$$\tau_r(T_{kd}) \equiv N_{coll}/\Gamma_{el} \sim H^{-1}(T_{kd})$$

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Two consequences:

1. During freeze-out (chemical decoupling) typically: $f_\chi \sim a(\mu) f_\chi^{eq}$
2. If kinetic decoupling much, much later: possible impact on the matter power spectrum

i.e. kinetic decoupling can have observable consequences and affect e.g. missing satellites problem

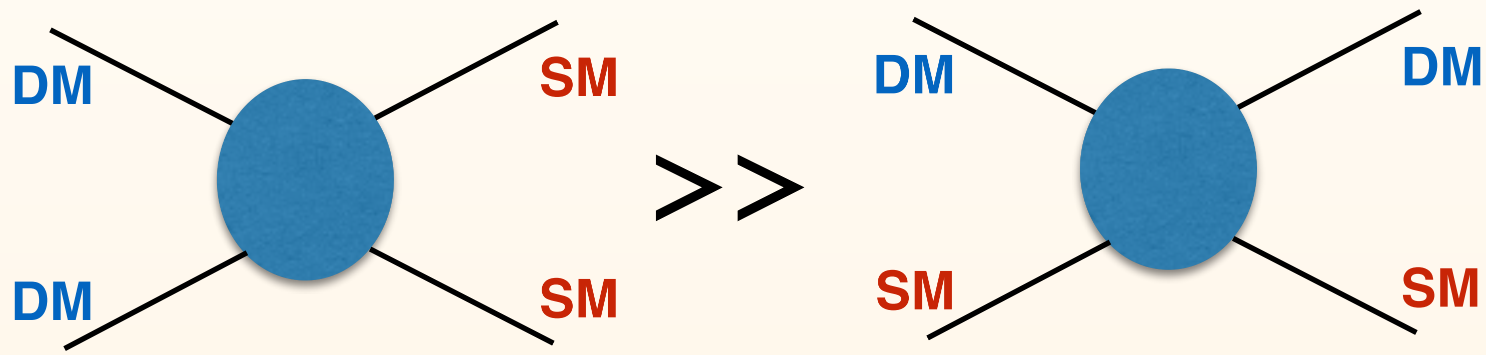
see e.g., Bringmann, Ihle, Karsten, Walia '16

EARLY KINETIC DECOUPLING?

A necessary and sufficient condition: scatterings weaker than annihilation

i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:

A) 

e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

How to describe KD?

All information is in full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

contains both scatterings and annihilation

Two possible approaches:

0-th moment: n_χ
2-nd moment: T_χ
$\ldots$

solve numerically for full $f_\chi(p)$

- have insight on the distribution
- no constraining assumptions
- numerically challenging
- typically overkill

consider system of equations for moments of $f_\chi(p)$

- partially analytic/much easier numerically
- manifestly captures all of the relevant physics
- finite range of validity
- no insight on the distribution

KINETIC DECOUPLING 101

Consider general KD scenario, i.e. coupled temperature and number density evolution:

annihilation and production thermal averages done at different T — feedback of modified y evolution

$$Y = \frac{n_\chi}{s} \qquad y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$$

$$\frac{Y'}{Y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx} sY \left(\langle \sigma v_{\rm rel} \rangle|_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\rm eq}^2}{Y^2} \langle \sigma v_{\rm rel} \rangle|_x \right)$$

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$$+ \frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{3m_\chi} \langle p^4/E^3 \rangle_{x=m_\chi^2/(s^{2/3}y)}$$

These equations still assume the equilibrium shape of $f_\chi(p)$ — but with variant temperature

or more accurately: that the thermal averages computed with true non-equilibrium distributions don't differ much from the above ones

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elastic scatterings term

$$c(T) = \frac{1}{12(2\pi)^3 m_\chi^4 T} \sum_X \int dk\, k^5 \omega^{-1} g^{\pm} \left(1 \mp g^{\pm}\right) \int_{-4k^2}^{0} (-t) \frac{1}{8k^4} |\mathcal{M}_{\rm el}|^2$$

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impact of annihilation

$$\langle \sigma v_{\rm rel} \rangle_2 \equiv \frac{g_\chi^2}{3T m_\chi n_\chi^2} \int \frac{d^3p}{(2\pi)^3} \int \frac{d^3\tilde{p}}{(2\pi)^3}\, p^2 v_{\rm rel} \sigma_{\bar{\chi}\chi \to \bar{X}X} f(E) f(\tilde{E})$$

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NUMERICAL APPROACH

… or one can just solve full phase space Boltzmann eq.

$$
\begin{aligned}
\partial_x f_\chi(x,q) &= \frac{m_\chi^3}{\tilde{H}x^4}\frac{g_{\bar{\chi}}}{2\pi^2}\int d\tilde{q}\,\tilde{q}^2\,\frac{1}{2}\int d\cos\theta\;\; v_{\text{Møl}}\sigma_{\bar{\chi}\chi\to\bar{f}f} \\
&\quad\times\left[f_{\chi,\text{eq}}(q)f_{\chi,\text{eq}}(\tilde{q}) - f_\chi(q)f_\chi(\tilde{q})\right] \\
&+ \frac{2m_\chi c(T)}{2\tilde{H}x}\left[x_q\partial_q^2 + \left(q + \frac{2x_q}{q} + \frac{q}{x_q}\right)\partial_q + 3\right] f_\chi \\
&+ \tilde{g}\frac{q}{x}\partial_q f_\chi,
\end{aligned}
$$

fully general

expanded in NR and small momentum transfer (semi-relativistic!)

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discretization, ~1000 steps

$$
\begin{aligned}
\partial_x f_i &= \\
&\frac{m_\chi^3}{\tilde{H}x^4}\frac{g_{\bar{\chi}}}{2\pi^2}\sum_{j=1}^{N-1}\frac{\Delta\tilde{q}_j}{2}\Big[\tilde{q}_j^2\,\langle v_{\text{Møl}}\sigma_{\bar{\chi}\chi\to\bar{f}f}\rangle_{i,j}^{\theta}\left(f_i^{\text{eq}}f_j^{\text{eq}} - f_i f_j\right) \\
&+ \tilde{q}_{j+1}^2\,\langle v_{\text{Møl}}\sigma_{\bar{\chi}\chi\to\bar{f}f}\rangle_{i,j+1}^{\theta}\left(f_i^{\text{eq}}f_{j+1}^{\text{eq}} - f_i f_{j+1}\right)\Big] \\
&+ \frac{2m_\chi c(T)}{2\tilde{H}x}\left[x_{q,i}\partial_q^2 + \left(q_i + \frac{2x_{q,i}}{q_i} + \frac{q_i}{x_{q,i}}\right)\partial_q + 3\right] f_i \\
&+ \tilde{g}\frac{q_i}{x}\partial_q f_i,
\end{aligned}
$$

Solved numerically with MatLab

Note:

can be extended to e.g. self-scatterings

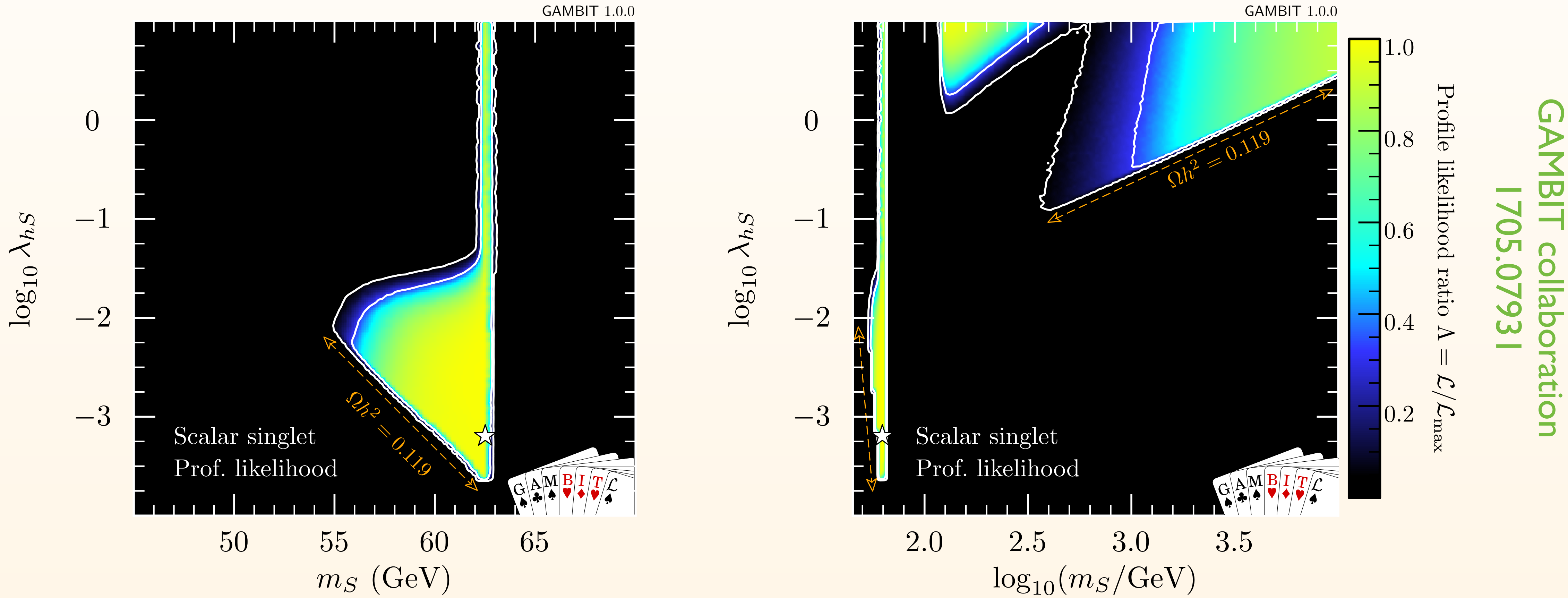
very stiff, care needed with numerics

Example #1:
Scalar Singlet DM

SCALAR SINGLET DM

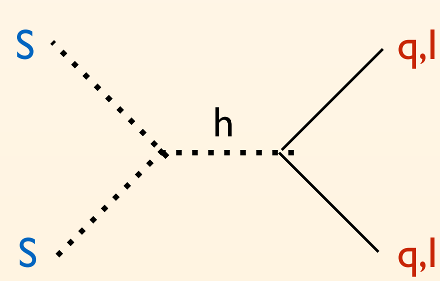
To the SM Lagrangian add one singlet scalar field *S* with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_s S^2 |H|^2 \qquad m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$



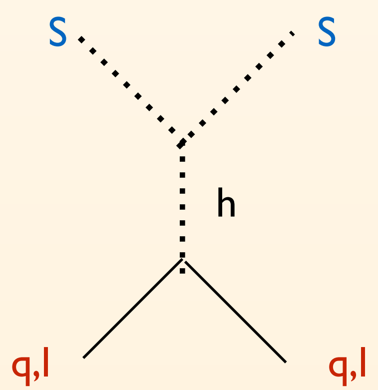
Annihilation processes:

resonant



El. scattering processes:

non-resonant



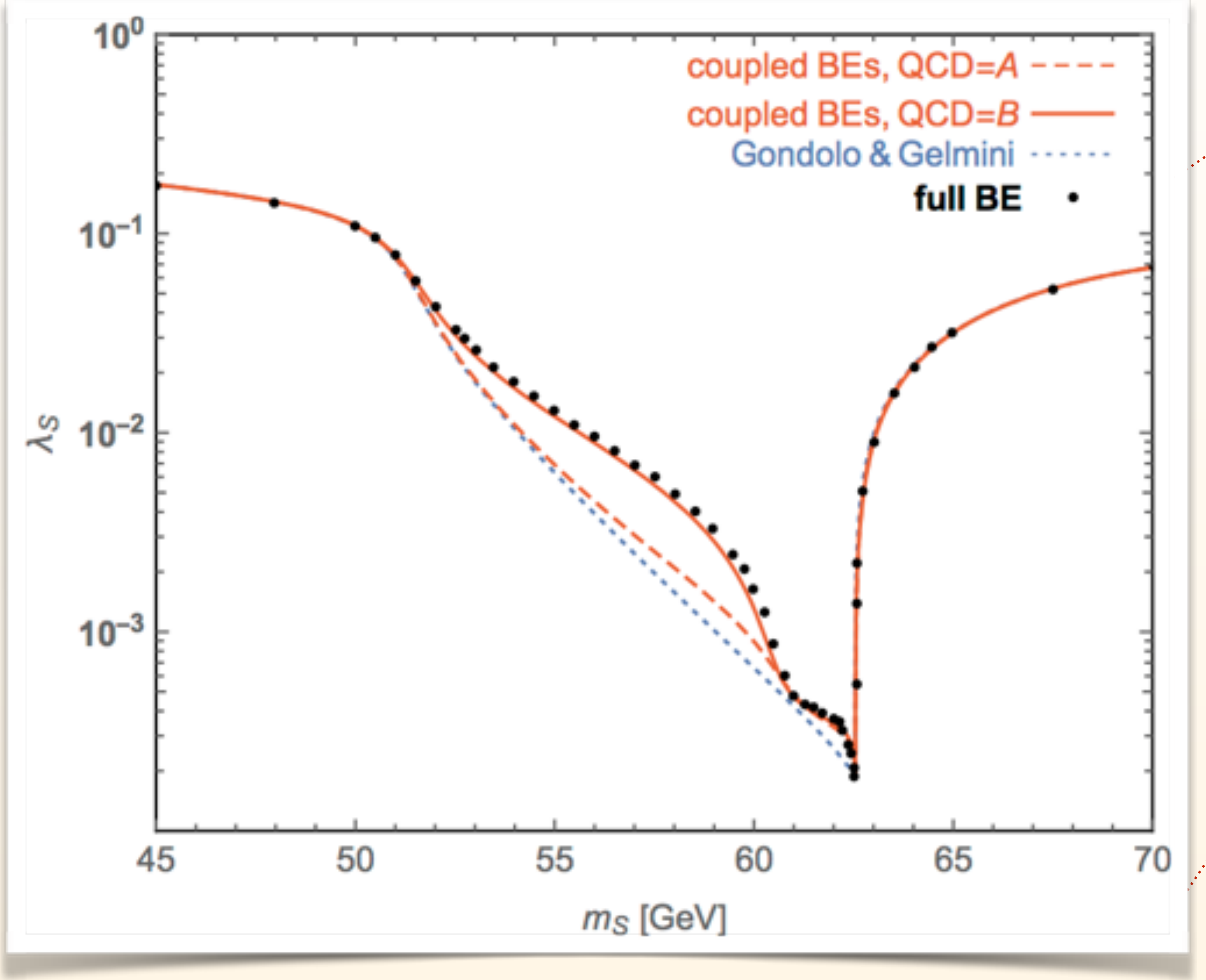
Hierarchical Yukawa couplings: strongest coupling to more Boltzmann suppressed quarks/leptons

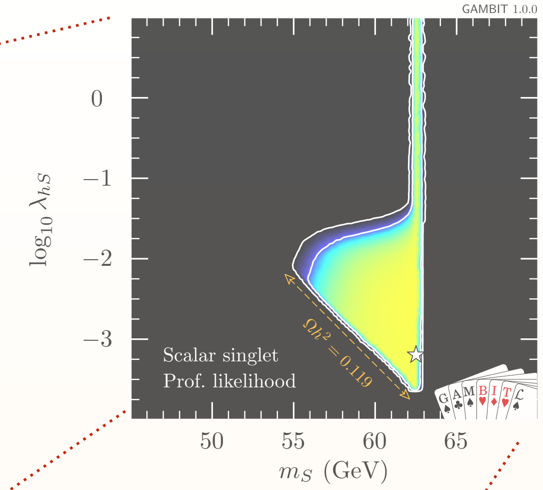
RESULTS

RD CONTOURS

QCD = A - all quarks are free and present down to T_c =154 MeV

QCD = B - only light quarks in the plasma and only down to $4T_c$





essentially the only region left for this model

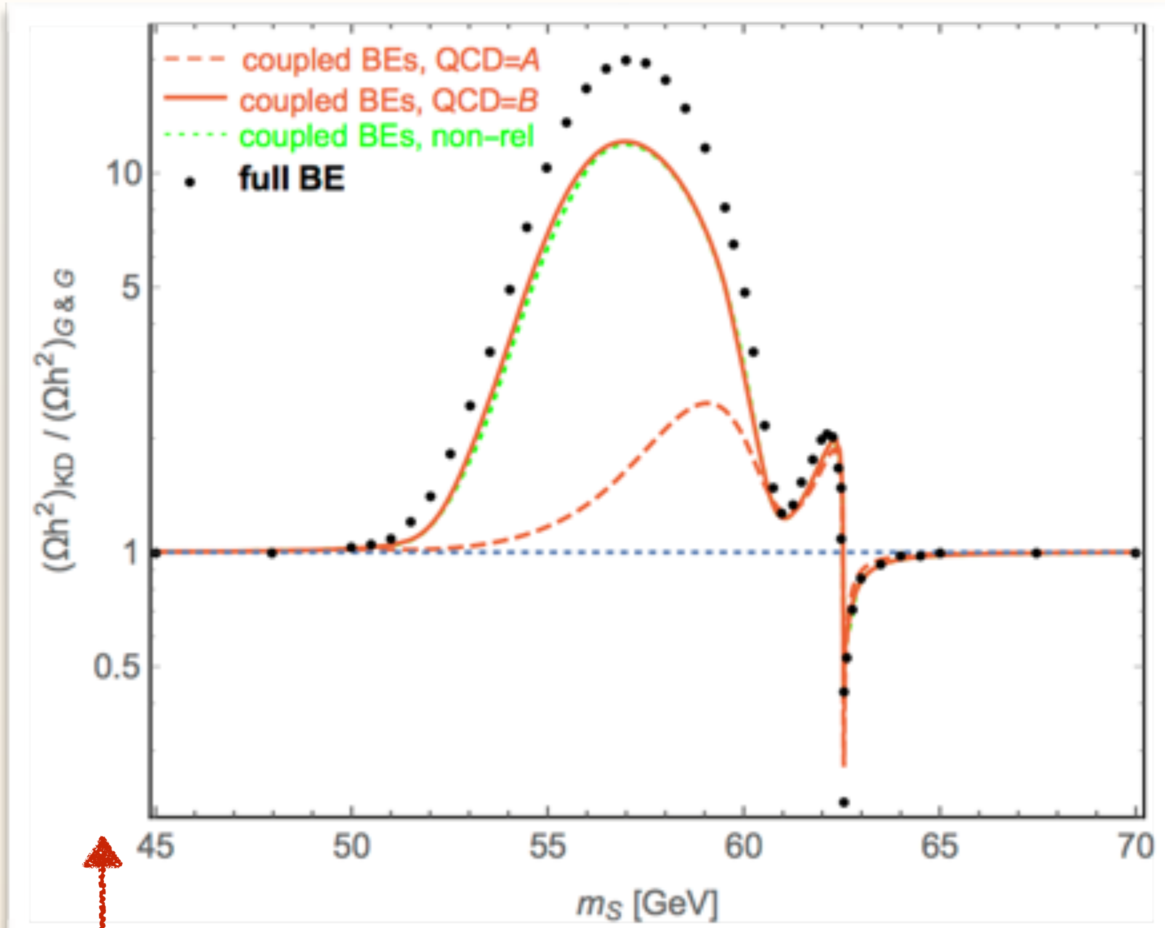
Significant <u>modification</u> of the observed relic density contour in the Scalar Singlet DM model

→ larger coupling needed → better chance for closing the last window

RESULTS

EFFECT

effect on relic density:



effect on relic density:
up to O(~10)

Why such non-trivial shape of the effect of early kinetic decoupling?

→ we'll inspect the *y* and *Y* evolution…

RESULTS

EFFECT

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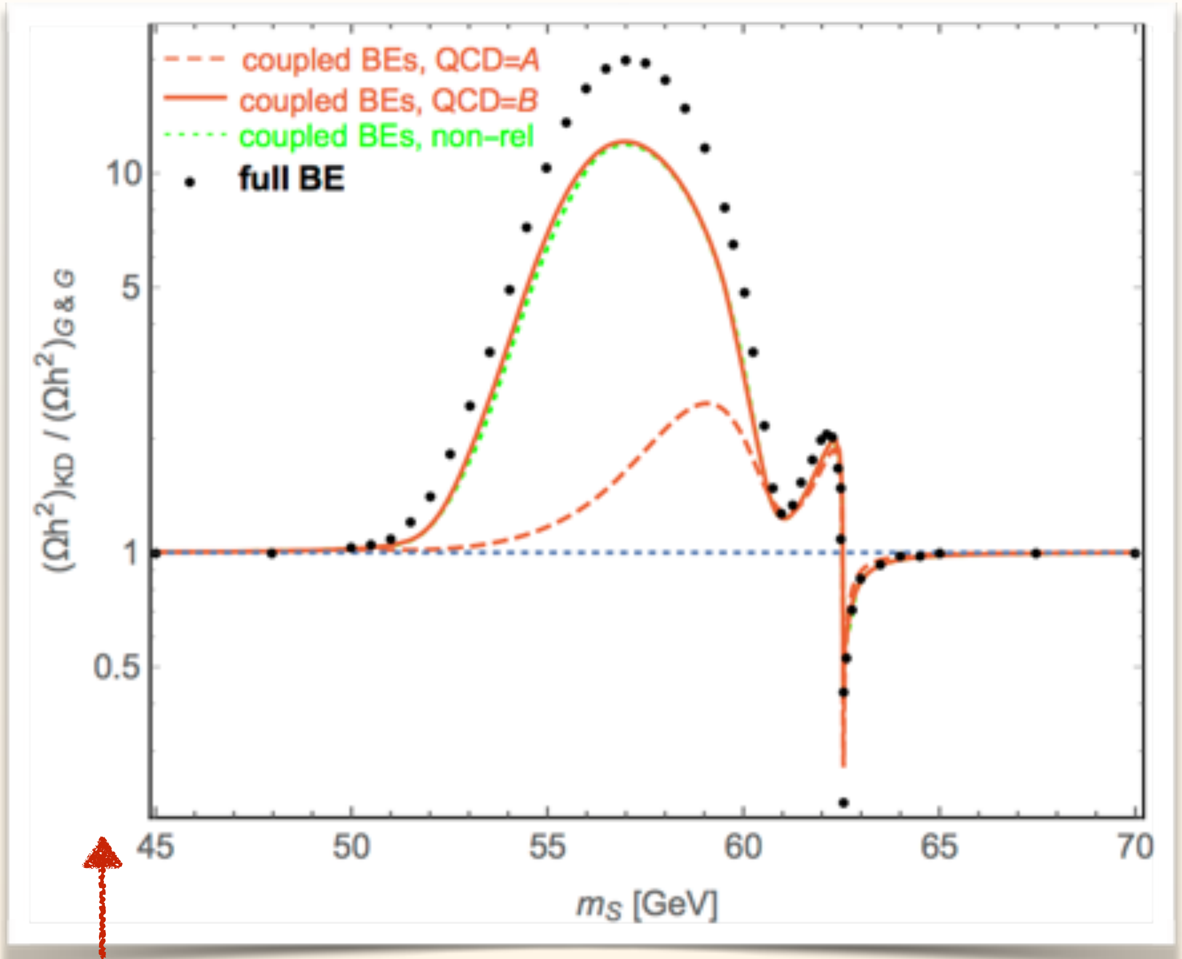
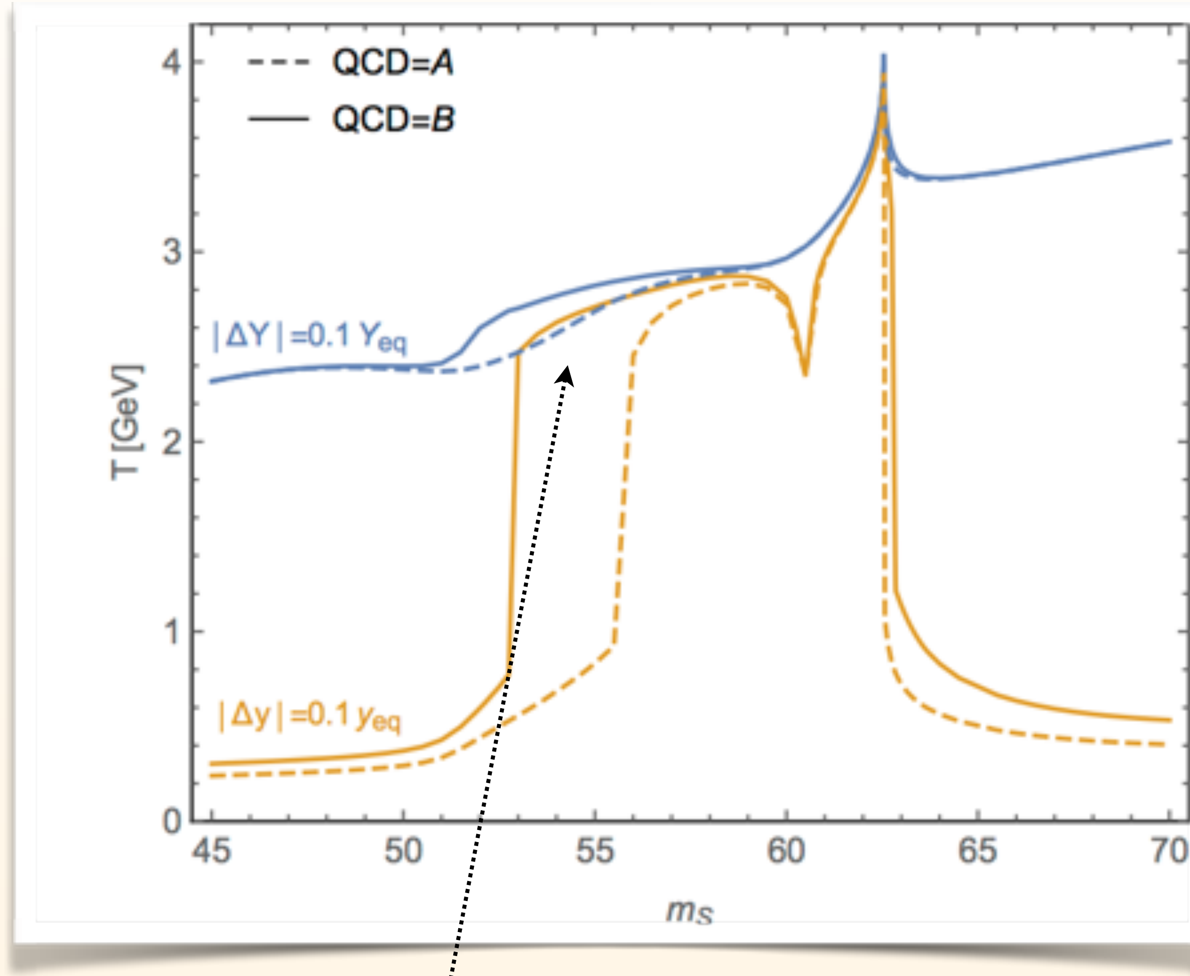


kinetic and chemical decoupling:



effect on relic density:
up to O(~10)

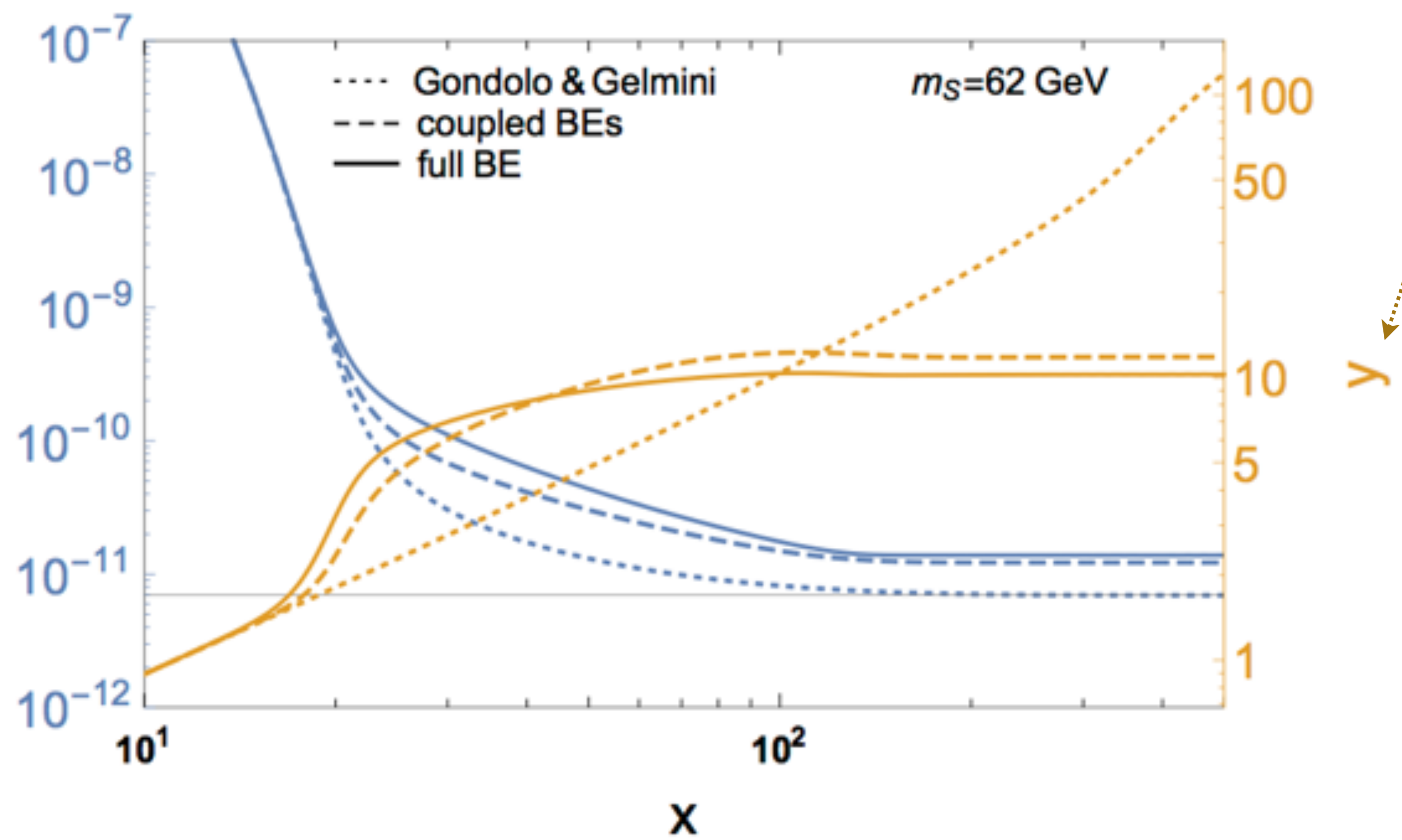
ratio approaches 1,
but does not reach it!

Why such non-trivial shape of the effect of early kinetic decoupling?

→ we'll inspect the y and Y evolution…

DENSITY AND T_{DM} EVOLUTION

for m_{DM} = 62 GeV, i.e. just below the resonance:



co-moving number density

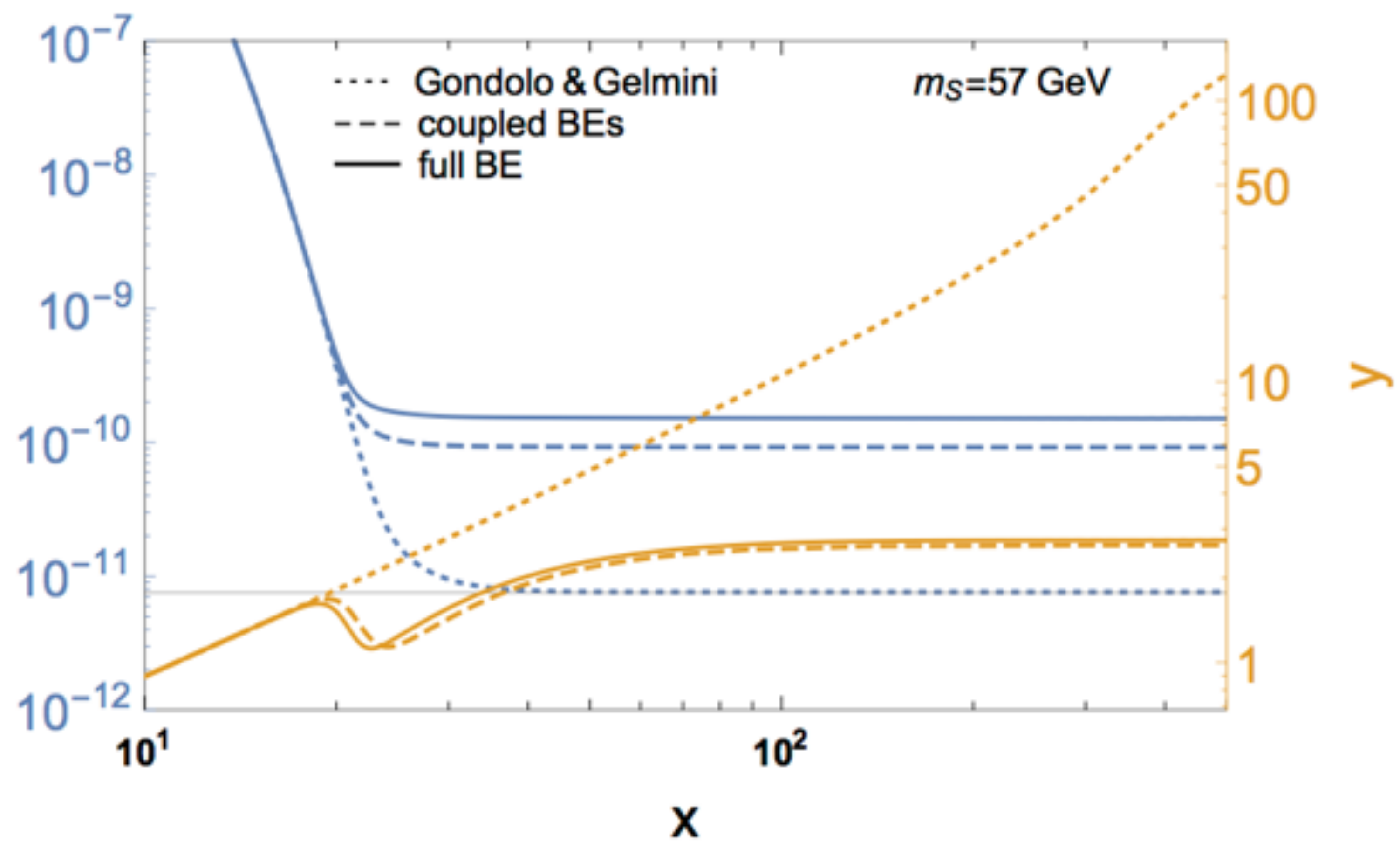
DM temperature

Resonant annihilation **most effective** for **low momenta**

⟶ DM fluid goes through "heating" phase before leaves kinetic equilibrium

DENSITY AND T_{DM} EVOLUTION

for m_{DM} = 57 GeV,   i.e. further away from the resonance:



Resonant annihilation **most effective** for **high momenta**

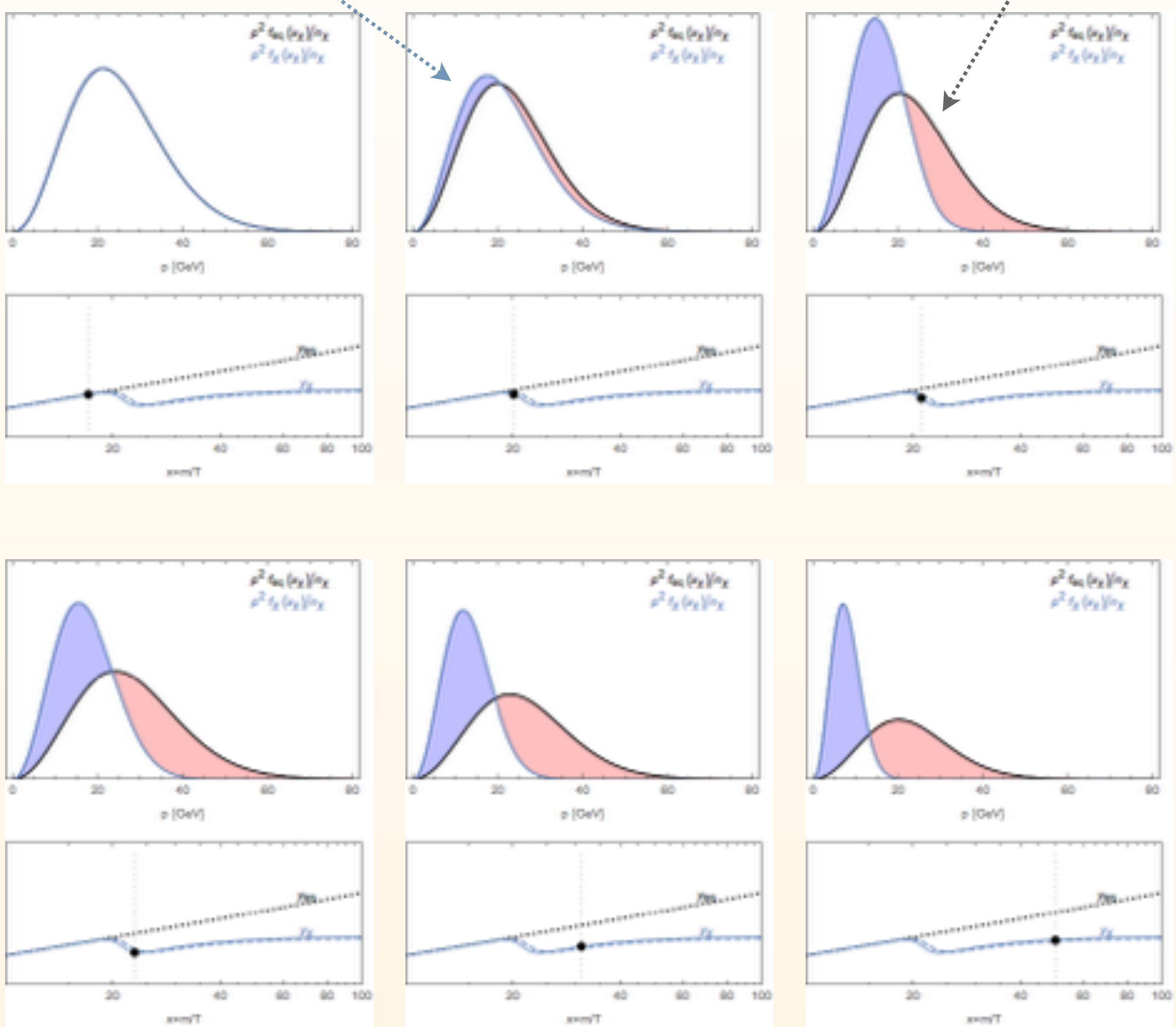
⟶ DM fluid goes through fast "cooling" phase
after that when T_{DM} drops to much annihilation not effective anymore

Full phase-space Evolution

blue - full solution for f_{DM} at T_{DM}

m_{DM} = 58 GeV

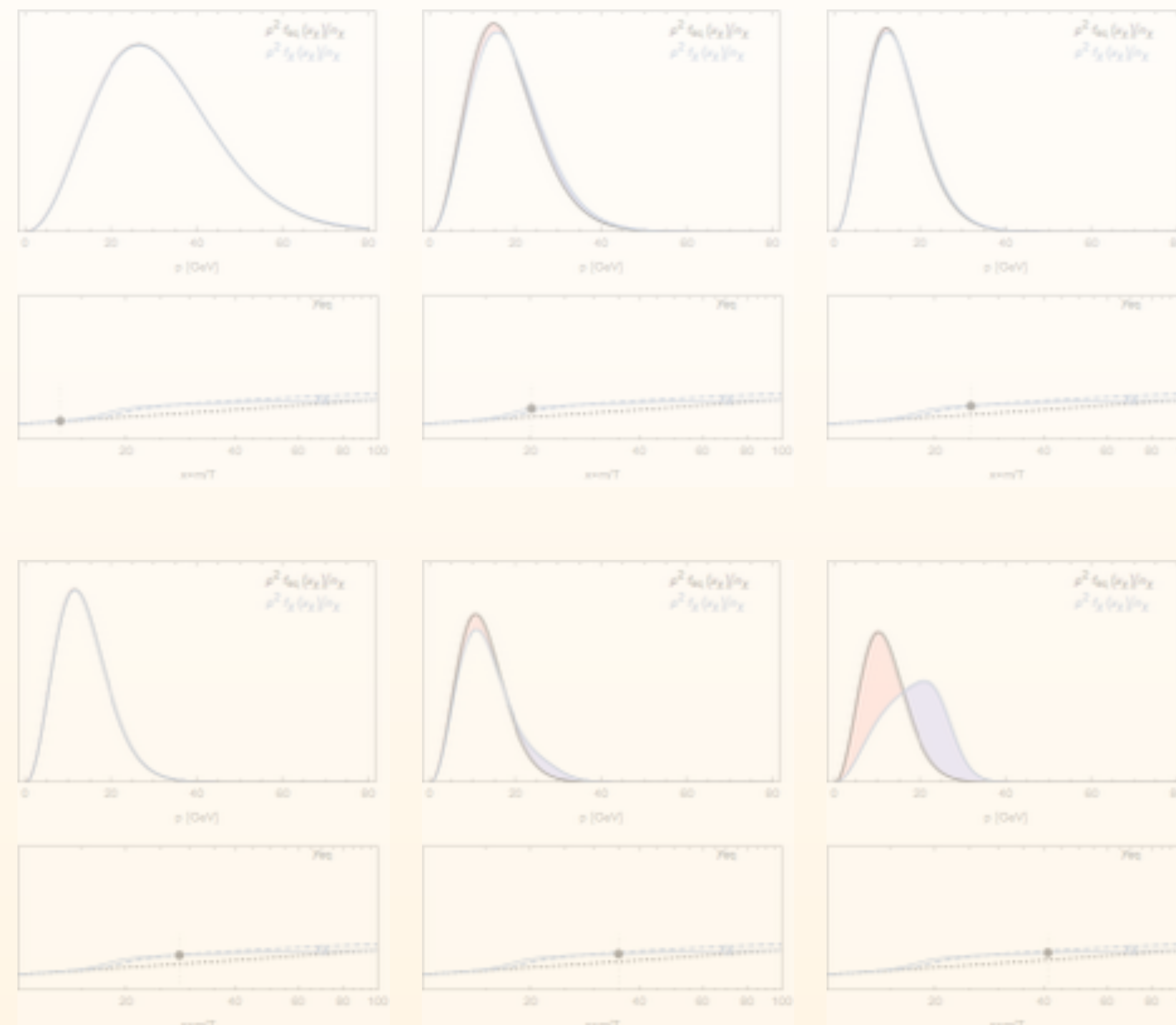
black - equilibrium at T_{DM}



significant deviation from equilibrium shape already around freeze-out

⟶ effect on relic density largest, both from different T and f_{DM}

m_{DM} = 62.5 GeV



large deviations only at later times, around freeze-out not far from eq. shape

⟶ effect on relic density ~only from different T

Full phase-space Evolution

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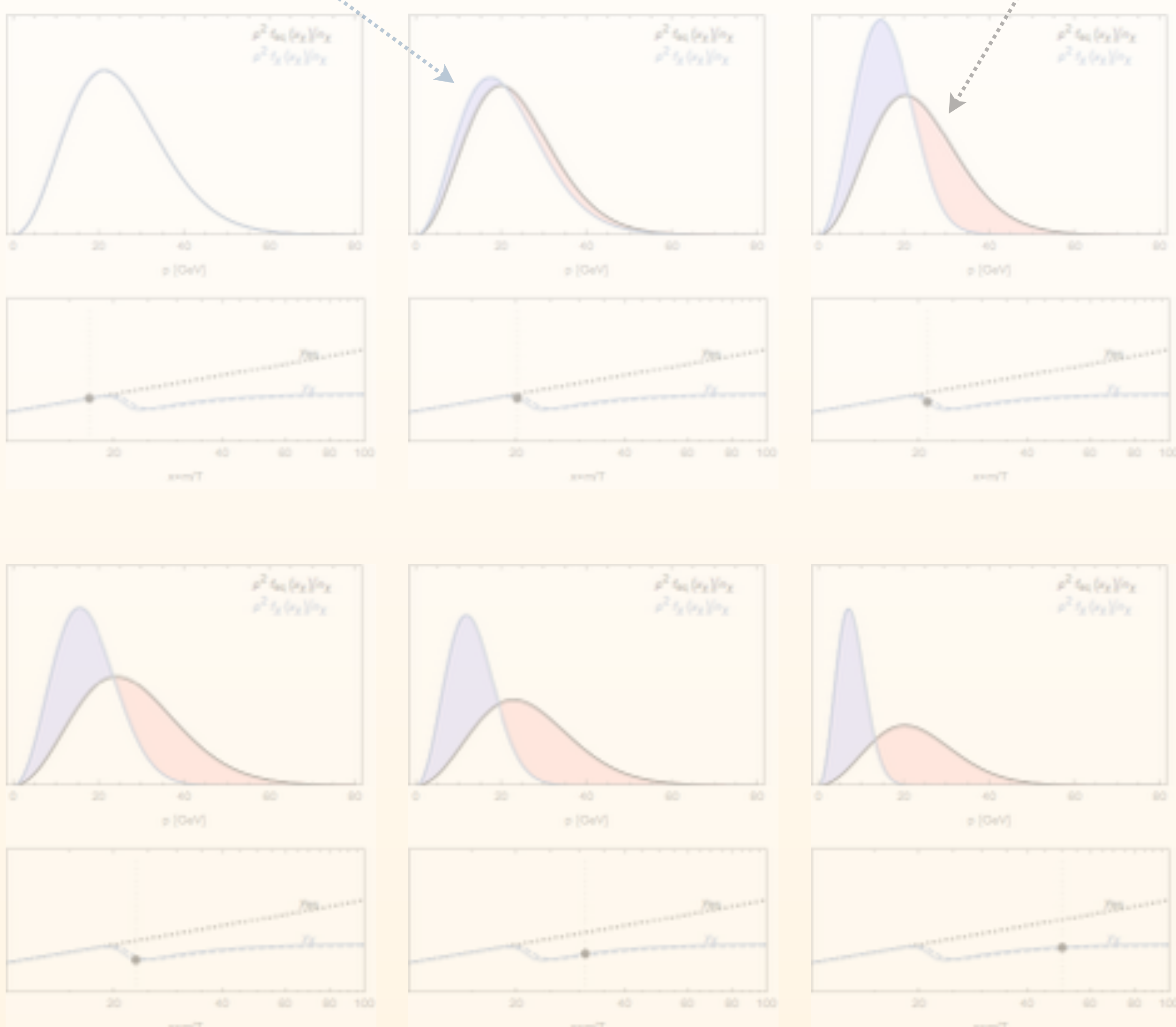

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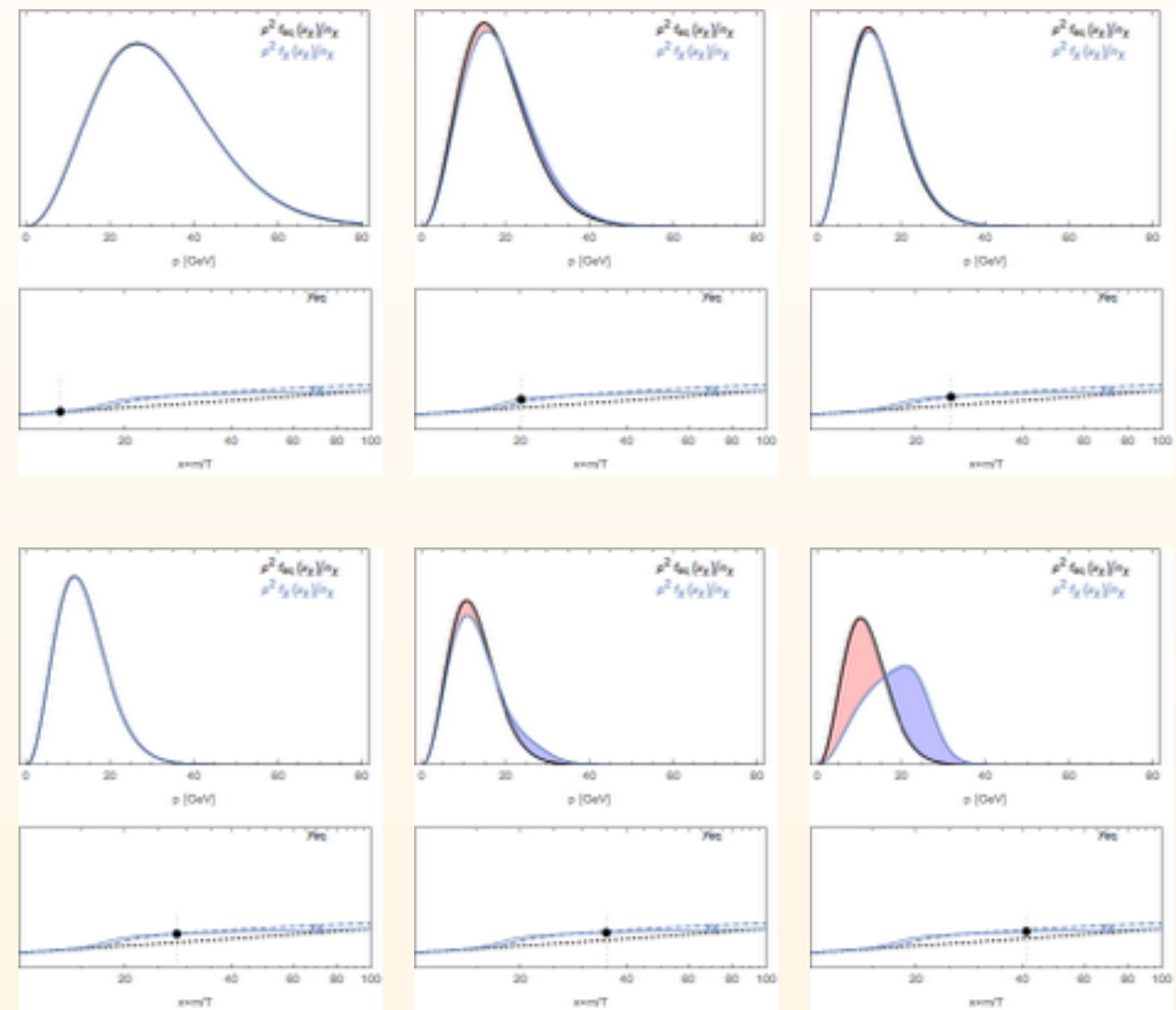

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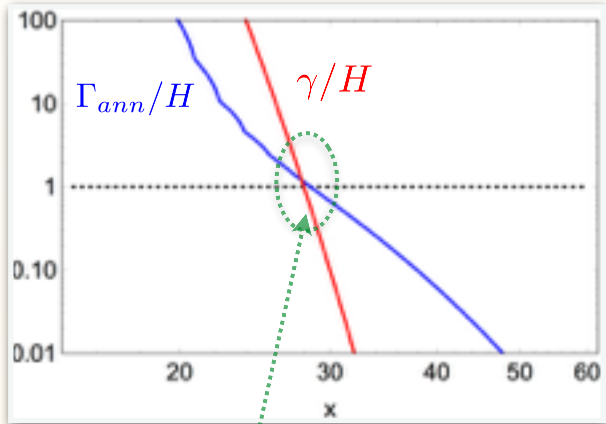
More Examples:
Forbidden DM & Semi-annihilation

FORBIDDEN DM

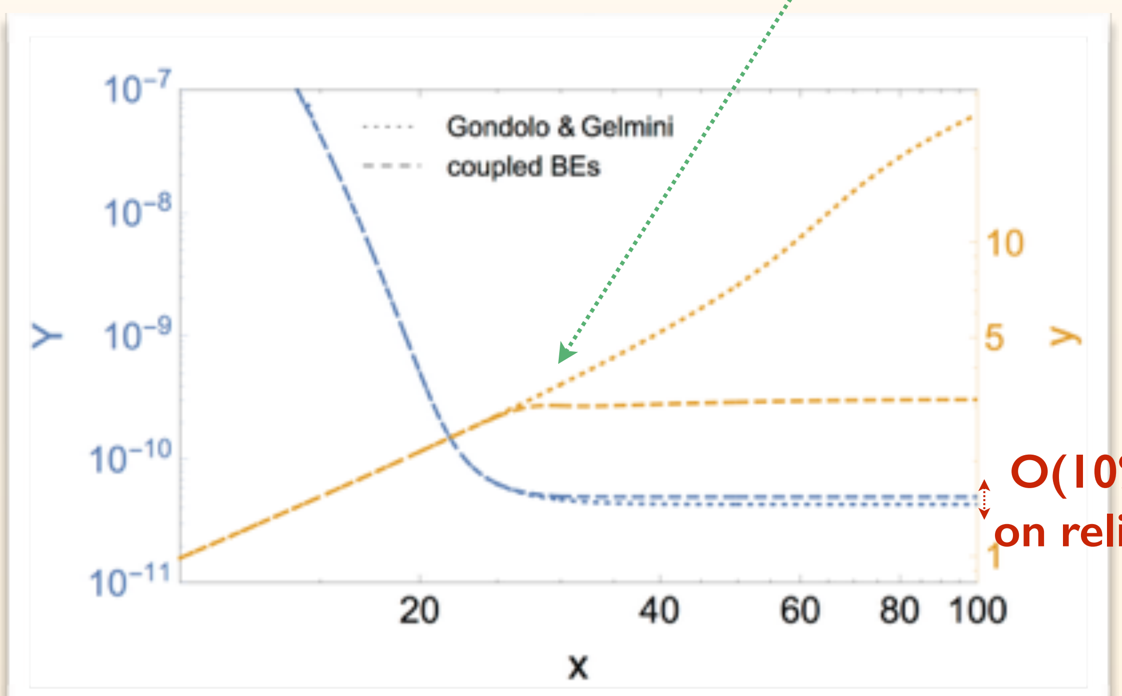
m_{DM} = 10 GeV, m_{SM} = 11 GeV; $|M|^2$ = *const.*

Annihilation threshold → velocity dependence

"heavy" SM particle → scattering rate low



kinetic and chemical decoupling close



O(10%) effect on relic density

*Caveats: toy example, only tree level, only cBE, non-negligible momentum transfer in el. scatt. (Fokker-Planck approx. problematic)

SEMI-ANNIHILATION

see also Cai, Spray 1807.00832

Z_3 complex scalar singlet

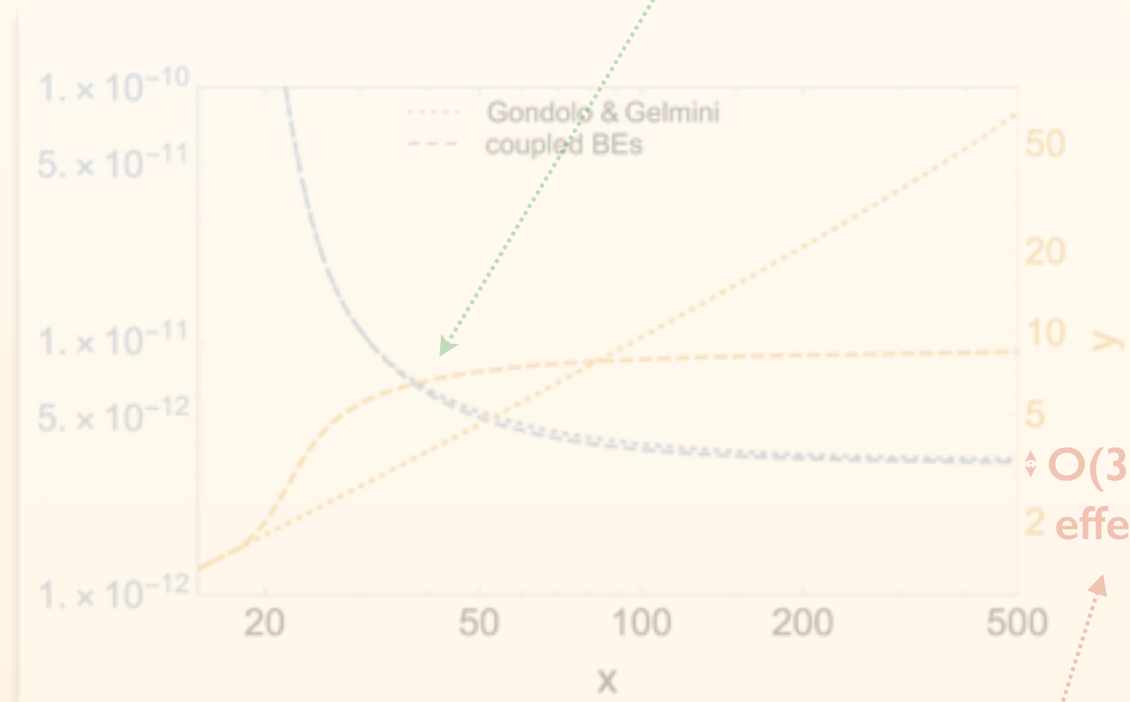
just above the Higgs threshold semi-annihilation dominant!

Belanger, Kannike, Pukhov, Raidal '13

very weak elastic scatterings → semi-annihilation by itself does not equilibrate DM

↓

but rather leads to self-heating!



O(3%) effect

will be much larger in case with stronger v-dependnece

*Caveats: only cBE, numerical accuracy challenging

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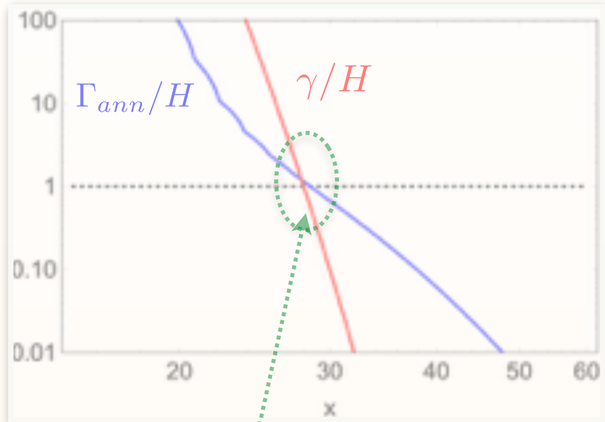

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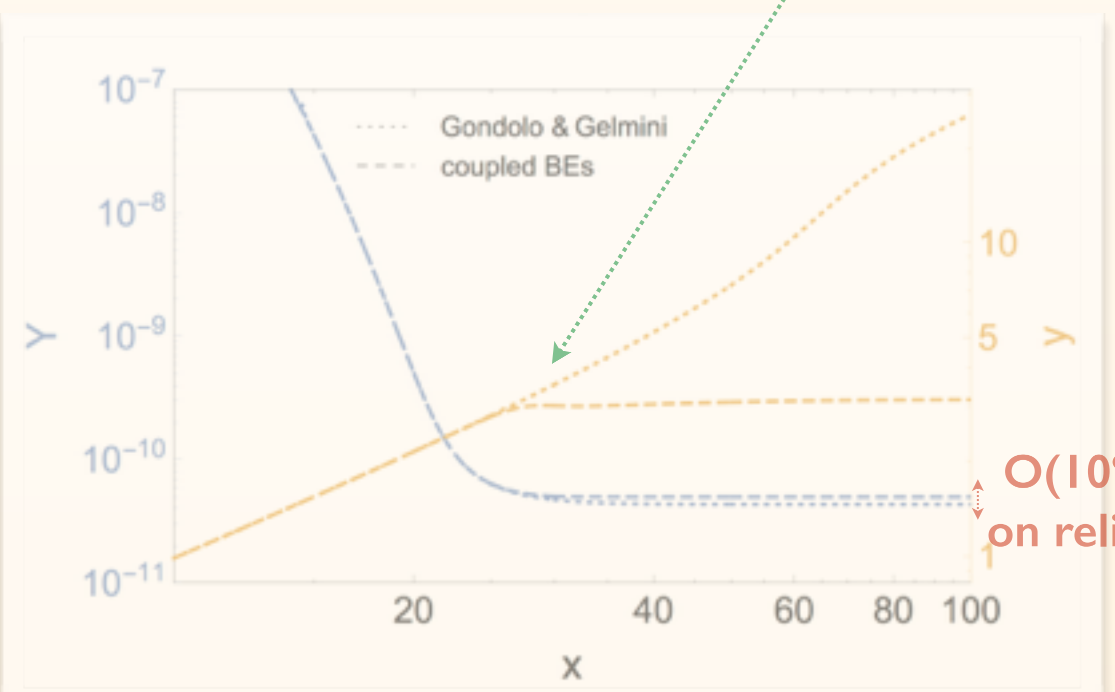

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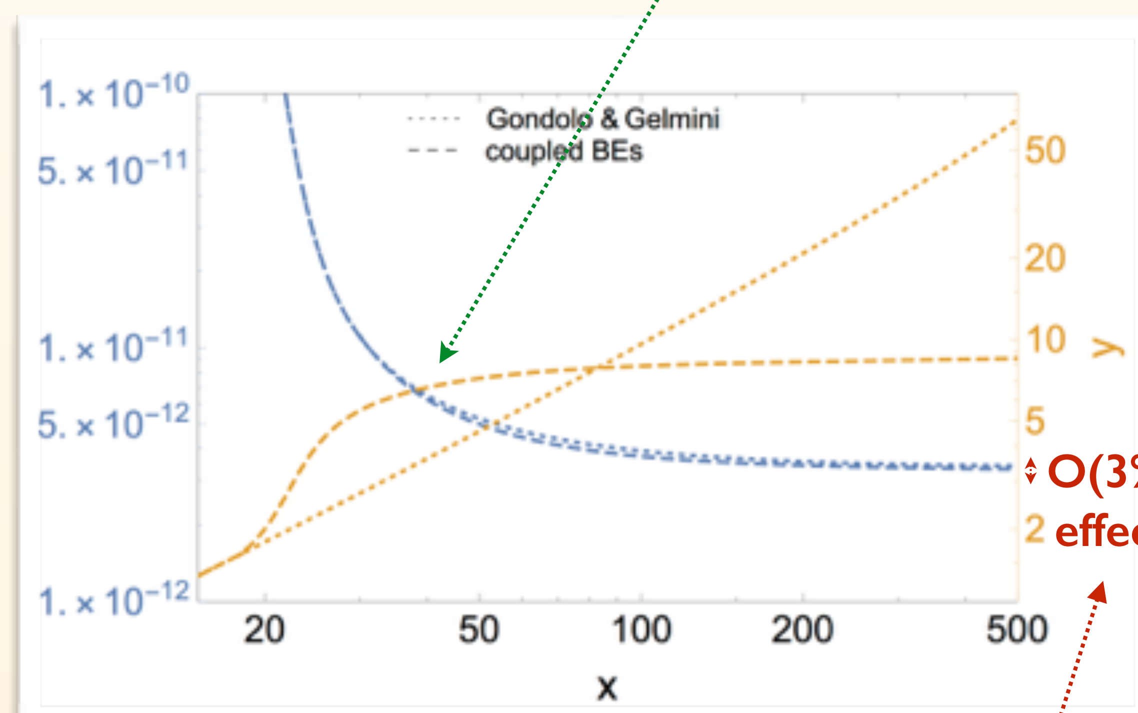

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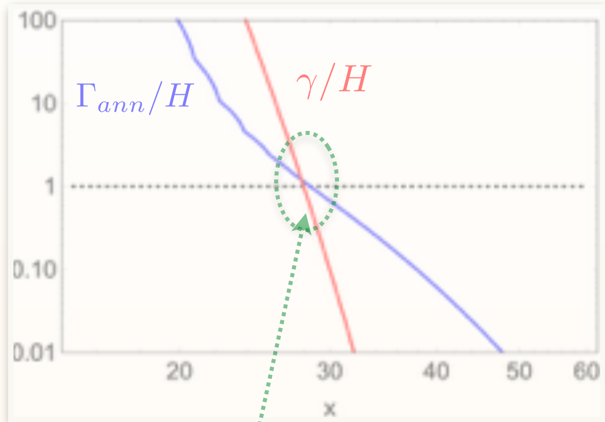

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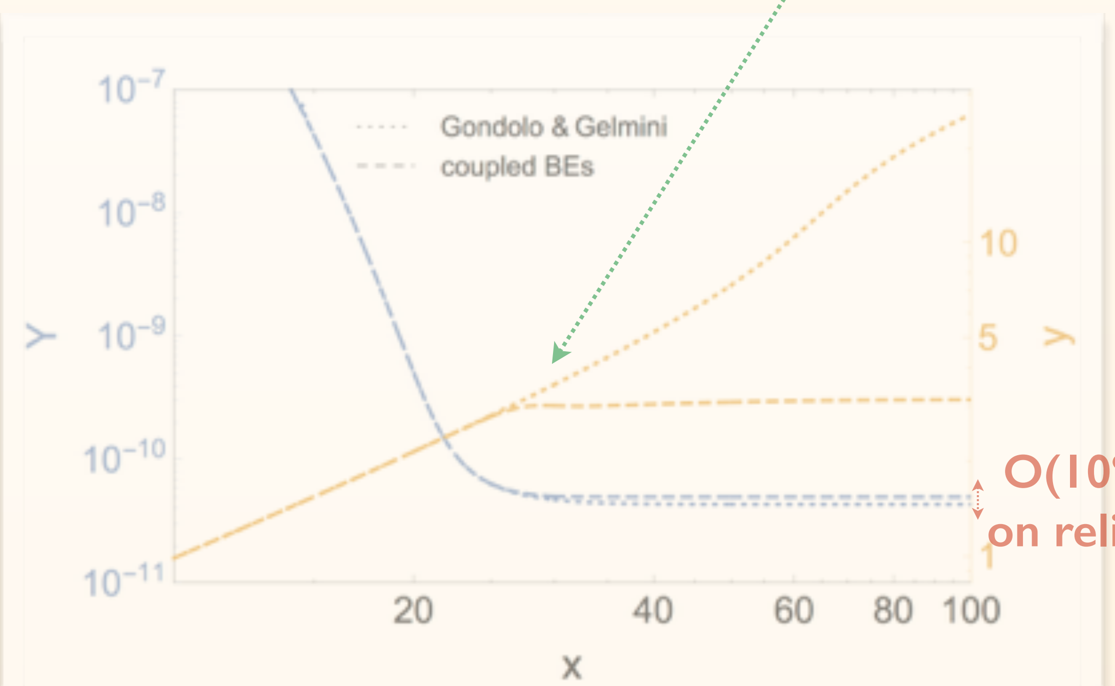

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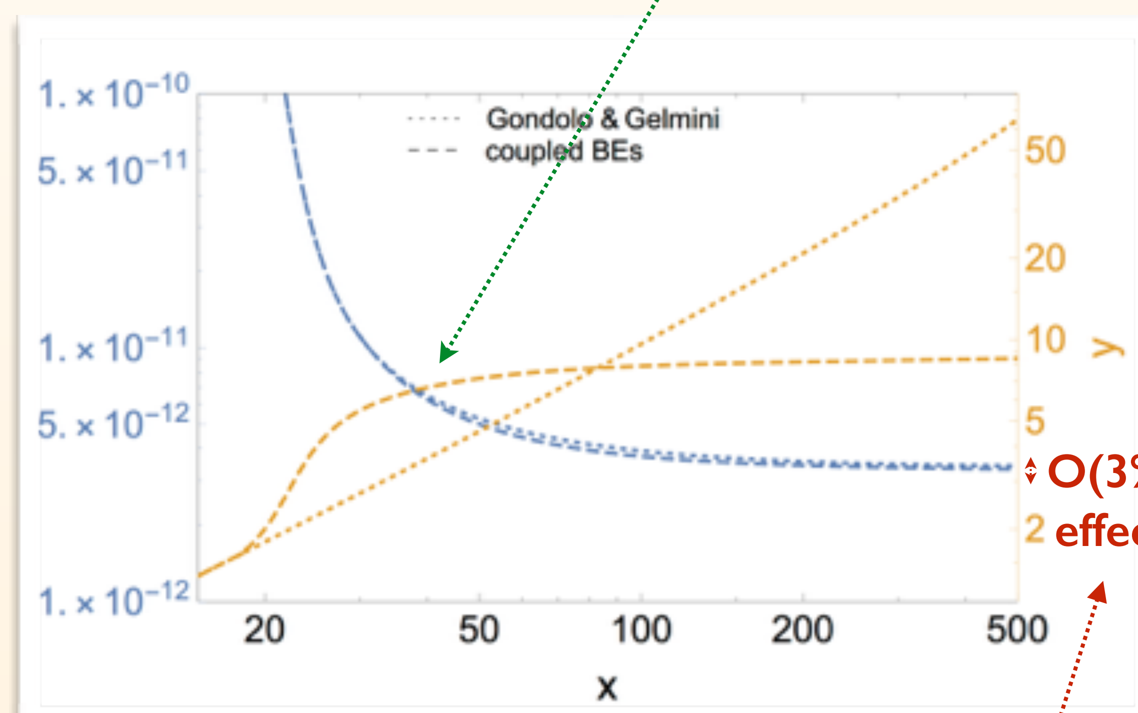
implications for ID



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CONCLUSIONS

1. One needs to remember that kinetic equilibrium is a necessary assumption for standard relic density calculations

2. Coupled system of Boltzmann equations for 0th and 2nd moments allow for a very accurate treatment of the kinetic decoupling and its effect on relic density

3. In special cases the full phase space Boltzmann equation can be necessary — especially if one wants to trace DM temperature as well

4. A public release of the full phase space Boltzmann code coming soon

Stay tuned for this!

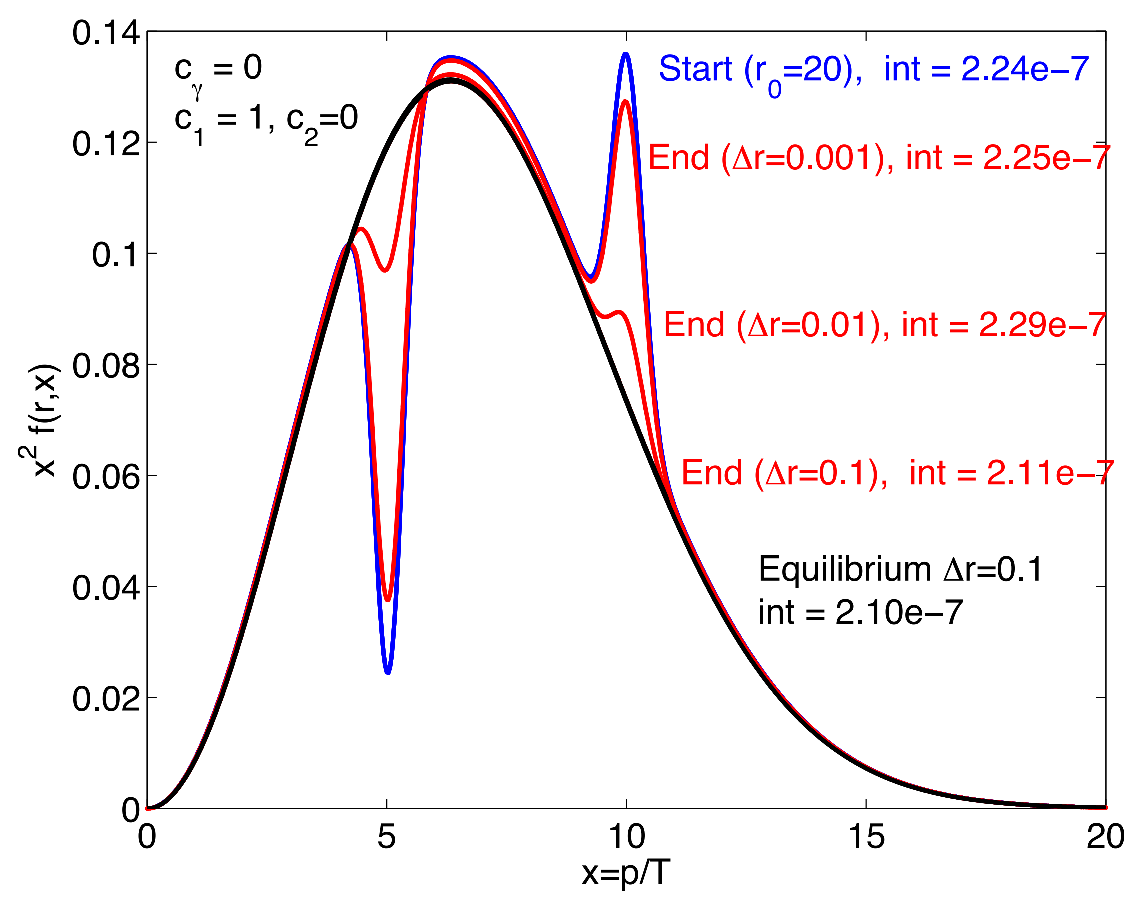
BACKUP

KD BEFORE CD?

Obvious issue:
How to define exactly the kinetic and chemical decouplings and what is the significance of such definitions?

⇓

Improved question:
Can kinetic decoupling happen much earlier than chemical?



we have already seen that even if scatterings were very inefficient compared to annihilation, departure from equilibrium for both Y and y happened around the same time…

← turn off scatterings and take s-wave annihilation; look at local disturbance

annihilation/production precesses drive to restore kinetic equilibrium!

SCATTERING

The elastic scattering collision term:

$$C_{\rm el} = \frac{1}{2g_\chi}\int\frac{d^3k}{(2\pi)^3 2\omega}\int\frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}}\int\frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}}$$
$$\times(2\pi)^4\delta^{(4)}(\tilde{p}+\tilde{k}-p-k)|\mathcal{M}|^2_{\chi f\leftrightarrow\chi f}$$
$$\times\left[(1\mp g^\pm)(\omega)\,g^\pm(\tilde{\omega})f_\chi(\tilde{\mathbf{p}})-(\omega\leftrightarrow\tilde{\omega},\mathbf{p}\leftrightarrow\tilde{\mathbf{p}})\right]$$

→ equilibrium functions for SM particles

Expanding in **NR** and small **momentum transfer**: Bringmann, Hofmann '06

$$C_{\rm el} \simeq \frac{m_\chi}{2}\gamma(T)\left[Tm_\chi\partial_p^2+\left(p+2T\frac{m_\chi}{p}\right)\partial_p+3\right]f_\chi$$

More generally, Fokker-Planck scattering operator (relativistic, but still small **momentum transfer**): Binder et al. '16

physical interpretation: scattering rate

$$C_{\rm el} \simeq \frac{E}{2}\nabla_{\mathbf{p}}\cdot\left[\gamma(T,\mathbf{p})\left(ET\nabla_{\mathbf{p}}+\mathbf{p}\right)f_\chi\right]$$

<u>Semi-relativistic</u>: assume that scattering $\gamma(T,\mathbf{p})$ is momentum independent

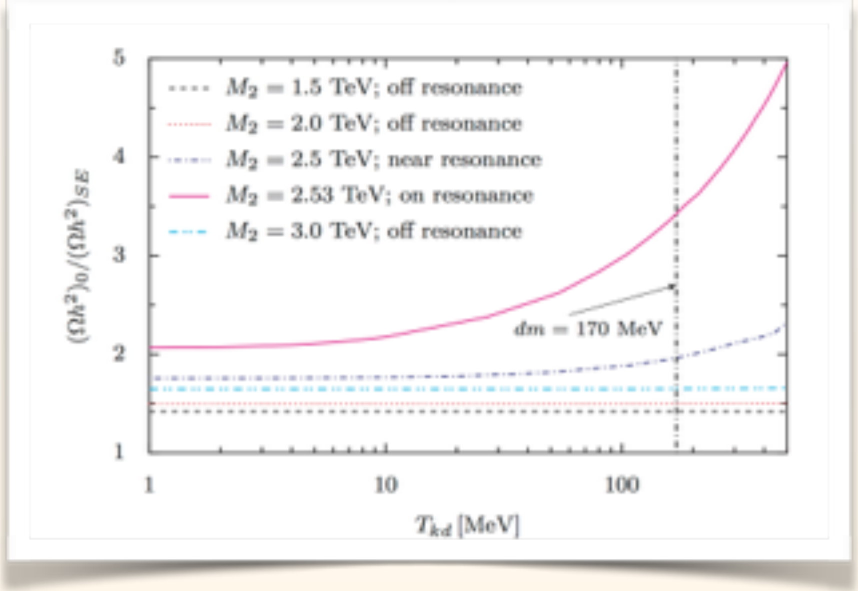
EARLY KD AND RESONANCE

our work wasn't the first to realize that resonant annihilation can lead to early kinetic decoupling…

Feng, Kaplinghat, Yu '10 — noted that for Sommerfeld-type resonances KD can happen early

Dent, Dutta, Scherrer '10 — looked at potential effect of KD on thermal relic density

Since then people were aware of this effect and sometimes tried to estimate it assuming instantaneous KD, e.g., in the case of Sommerfeld effect in the MSSM:



AH, Iengo, Ullio '11

but no systematic studies of decoupling process were performed, until…

…models with very late KD become popular, in part to solve „missing satellites" problem

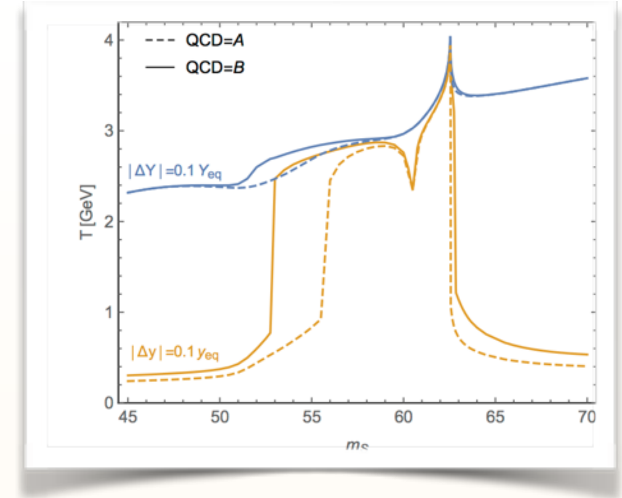
van den Aarssen et al '12; Bringmann et al '16, x2; Binder et al '16

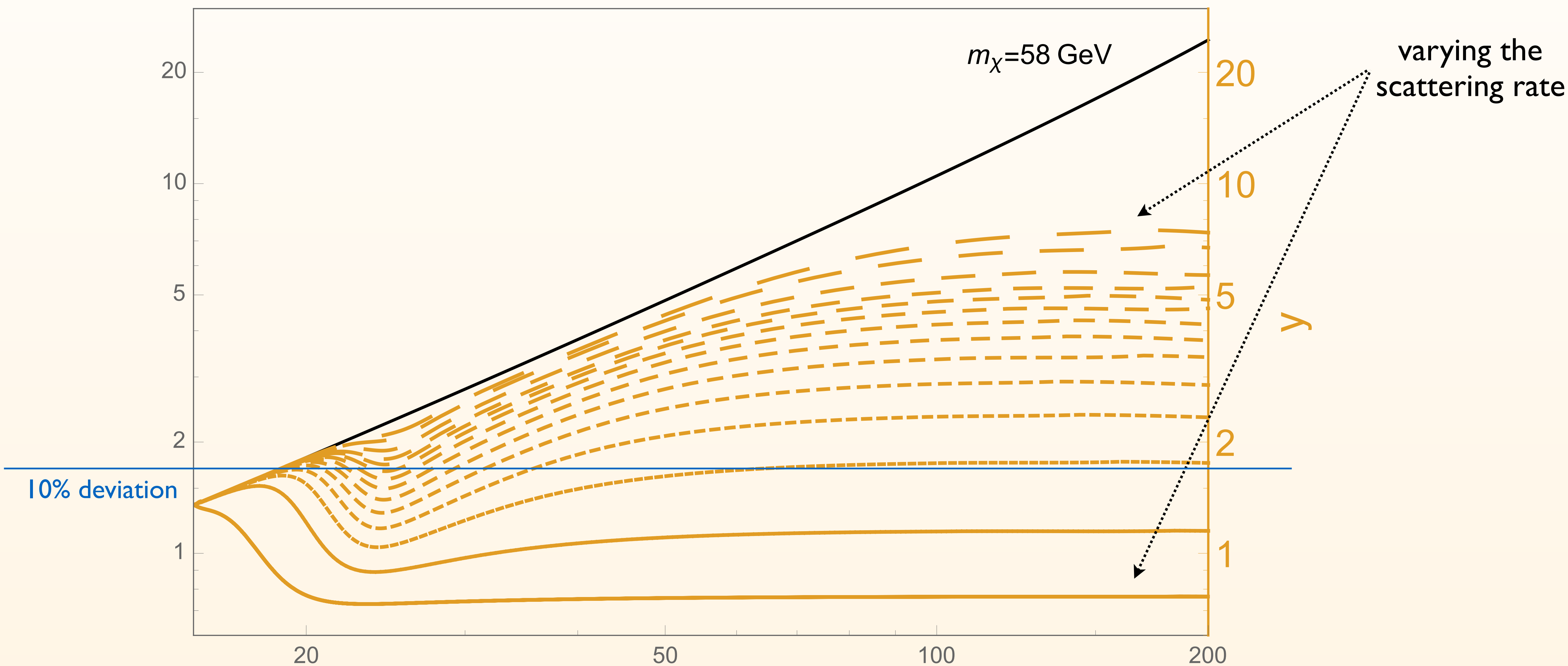
this progress allowed for better approach to early KD scenarios as well and was applied to the resonant annihilation case in

Duch, Grządkowski '17

… but we developed a dedicated accurate method/code to deal with this and other similar situations

WHY SPIKES IN T_{KD}?





Effect resembling first order „phase transition” —
artificial as dependent on a particular choice of T_{KD} definition