

# KINETIC DECOUPLING OF DARK MATTER AND ITS IMPACT ON THE RELIC DENSITY

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&

NCBJ



based on: **T. Binder, T. Bringmann, M. Gustafsson and AH,**  
Phys.Rev. D96 (2017) 115010, [astro-ph.co/1706.07433](https://arxiv.org/abs/1706.07433)

University of Warsaw, 21st December 2017

# OUTLINE

## 1. Introduction

- standard approach to **thermal relic density**
- recent novel models/ideas

## 2. Kinetic decoupling

- **freeze-out vs. decoupling**
- significance for cosmology

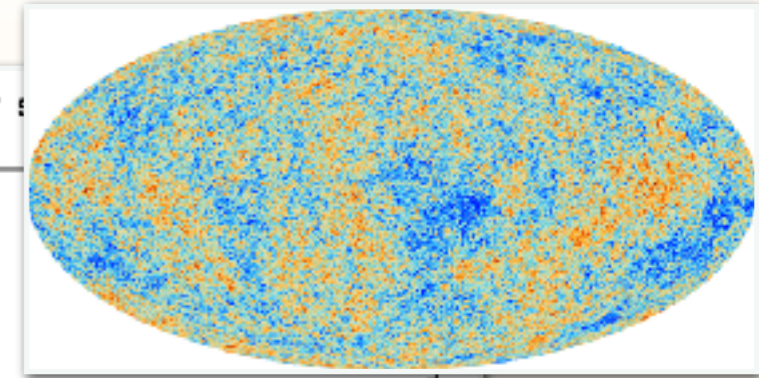
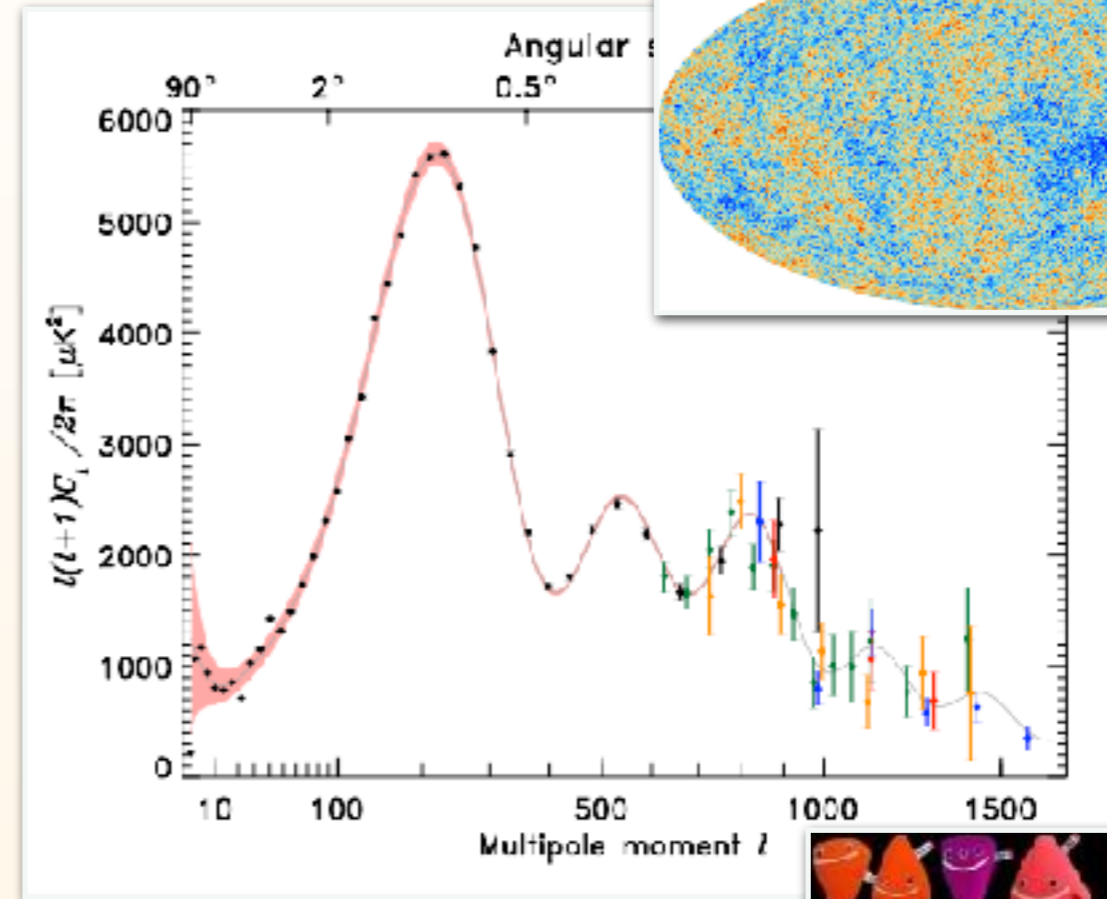
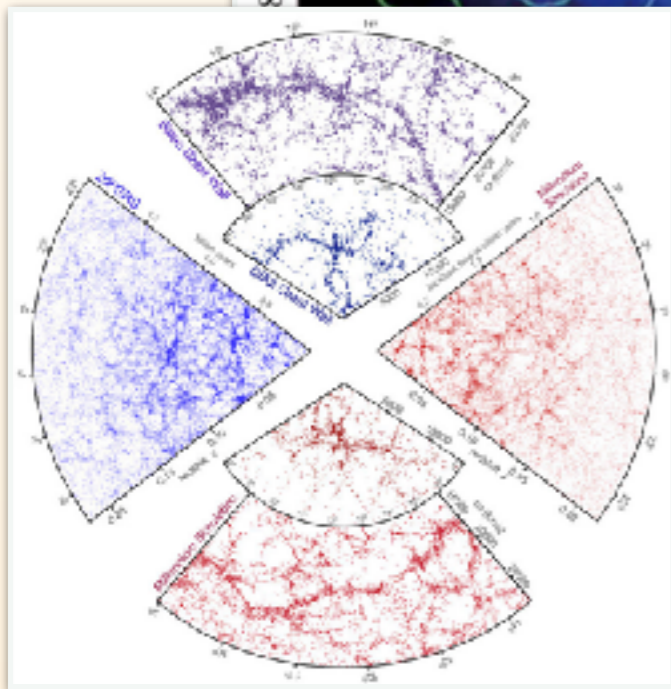
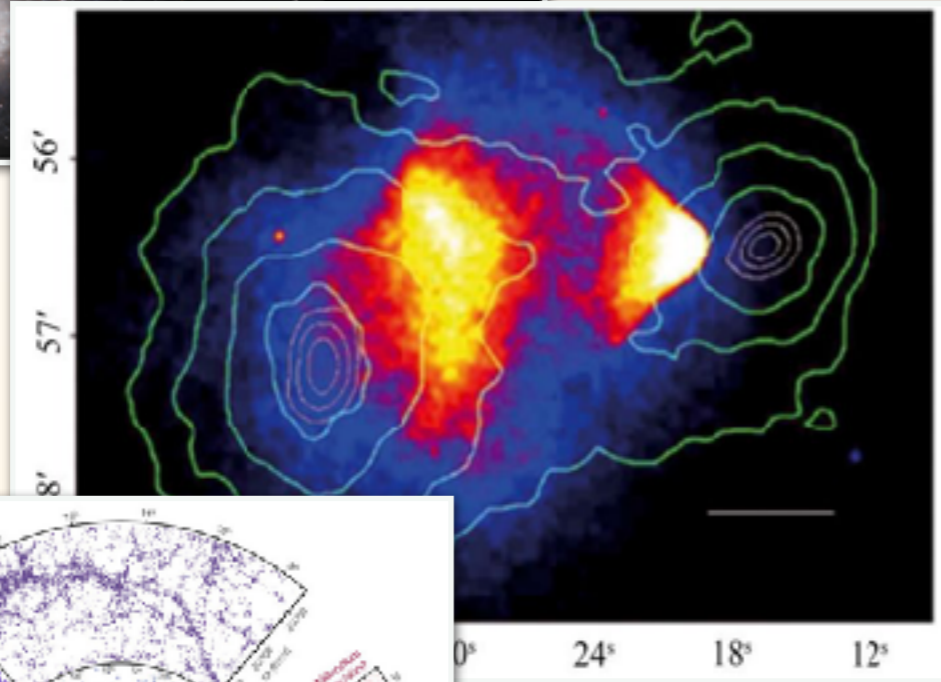
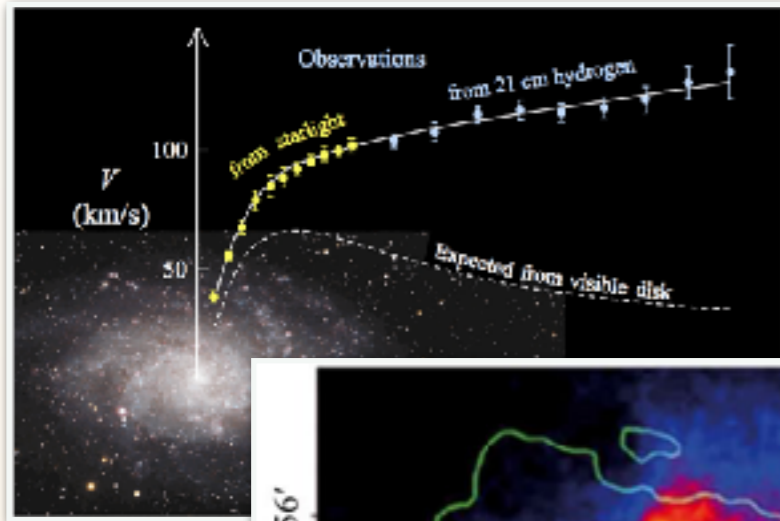
## 3. *n-th* Exception

- **early kinetic decoupling** with
- **velocity dependent** annihilation

## 4. Summary

# DARK MATTER

## IS EVERYWHERE!



⇒ Evidence on all scales!



# THE ORIGIN OF DARK MATTER

Dark matter could be created in many different ways...

...but every massive particle with not-too-weak interactions with the SM will be produced thermally, with relic abundance:

Lee, Weinberg '77; + others

$$\Omega_\chi h^2 \approx 0.1 \frac{3 \times 10^{-26} \text{cm}^3 \text{s}^{-1}}{\langle \sigma v \rangle}$$

It is very natural to expect that this mechanism is responsible for the origin of **all of dark matter**

...but **even if not, it still is present nevertheless** and it's important to be able to correctly determine thermal population abundance

# HISTORICAL PRELUDE

## THREE EXCEPTIONS Griest & Seckel '91

### 1. Co-annihilations

if more than one state share a conserved quantum number making DM stable

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_i^{\text{eq}} n_j^{\text{eq}}}{n_{\text{eq}}^2}$$

with:  $\sigma_{ij} = \sum_X \sigma(\chi_i \chi_j \rightarrow X)$   
e.g., SUSY

### 2. Annihilation to forbidden channels

if DM is slightly below mass threshold for annihilation  $\Rightarrow$  „forbidden” channel can still be accessible in thermal bath

recent e.g., [1505.07107](#)

### 3. Annihilation near poles

expansion in velocity  
(s-wave, p-wave, etc.) not safe

(more historical issue:  
these days most people  
use numerical codes)

# THERMAL RELIC DENSITY

## MODERN "EXCEPTIONS"

### 1. Non-standard cosmology

many works... very recent e.g., D'Eramo, Fernandez, Profumo '17

### 2. Bound State Formation

recent e.g., Petraki et al. '15, '16; An et al. '15, '16; Cirelli et al. '16; ...

### 3. $3 \rightarrow 2$ and $4 \rightarrow 2$ annihilation

e.g., D'Agnolo, Ruderman '15; Cline et al. '17; Choi et al. '17; ...

### 4. Second era of annihilation

Feng et al. '10; Bringmann et al. '12; ...

### 5. Semi-annihilation

D'Eramo, Thaler '10; ...

### 6. Cannibalization

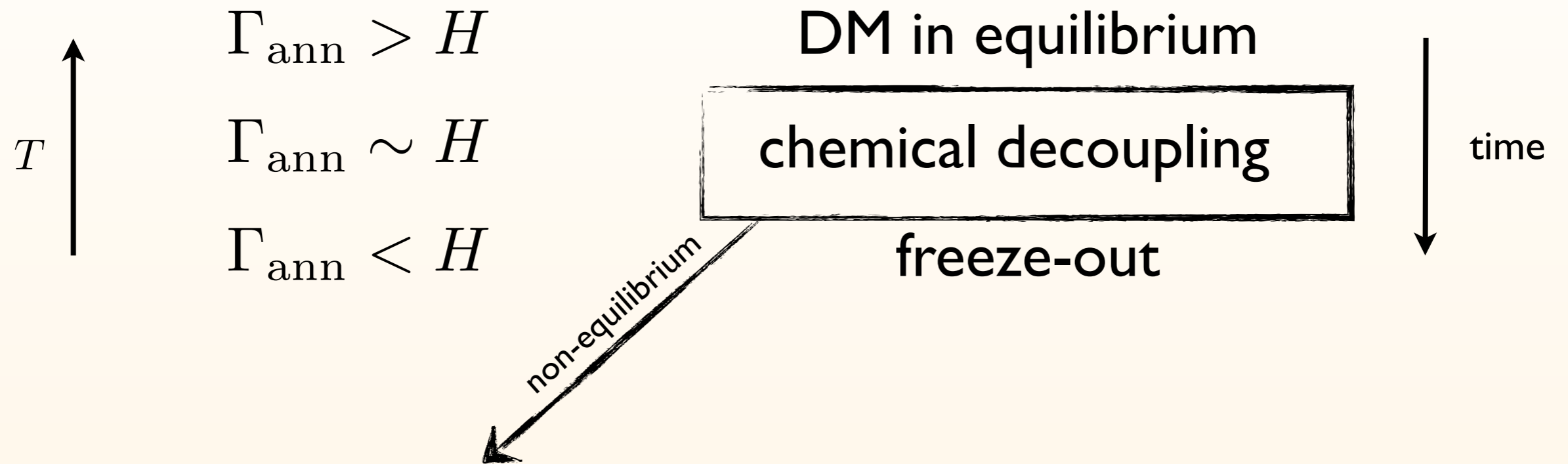
e.g., Kuflik et al. '15; Pappadopulo et al. '16; ...

### 7. ...

In other words: whenever studying non-minimal scenarios "exceptions" appear — but most of them come from interplay of **new added effects**, while do **not affect the foundations** of modern calculations

# THERMAL RELIC DENSITY

## STANDARD APPROACH



time evolution of  $f_\chi(p)$  in kinetic theory:

$$E (\partial_t - H \vec{p} \cdot \nabla_{\vec{p}}) f_\chi = \mathcal{C}[f_\chi] \implies \frac{dn_\chi}{dt} + 3Hn_\chi = C$$

Liouville operator in FRW background

the collision term

integrated

assumptions for using Boltzmann eq: classical limit, molecular chaos,...

...for derivation from thermal QFT see e.g., M. Beneke, F. Dighera, AH; JHEP 1410 (2014) 45 7

# THERMAL RELIC DENSITY

## THE COLLISION TERM

for  $2 \leftrightarrow 2$  CP invariant process:

$$C_{\text{LO}} = -h_{\chi}^2 \int \frac{d^3 \vec{p}_{\chi}}{(2\pi)^3} \frac{d^3 \vec{p}_{\bar{\chi}}}{(2\pi)^3} \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} [f_{\chi} f_{\bar{\chi}} (1 \pm f_i)(1 \pm f_j) - f_i f_j (1 \pm f_{\chi})(1 \pm f_{\bar{\chi}})]$$

assuming kinetic equilibrium at chemical decoupling:  $f_{\chi} \sim a(\mu) f_{\chi}^{\text{eq}}$

$$C_{\text{LO}} = -\langle \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} \rangle^{\text{eq}} (n_{\chi} n_{\bar{\chi}} - n_{\chi}^{\text{eq}} n_{\bar{\chi}}^{\text{eq}})$$

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} \rangle^{\text{eq}} = -\frac{h_{\chi}^2}{n_{\chi}^{\text{eq}} n_{\bar{\chi}}^{\text{eq}}} \int \frac{d^3 \vec{p}_{\chi}}{(2\pi)^3} \frac{d^3 \vec{p}_{\bar{\chi}}}{(2\pi)^3} \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} f_{\chi}^{\text{eq}} f_{\bar{\chi}}^{\text{eq}}$$



# THERMAL RELIC DENSITY

## BOLTZMANN EQ.

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}\rightarrow ij}\sigma_{\text{rel}}\rangle^{\text{eq}} (n_\chi n_{\bar{\chi}} - n_\chi^{\text{eq}} n_{\bar{\chi}}^{\text{eq}})$$

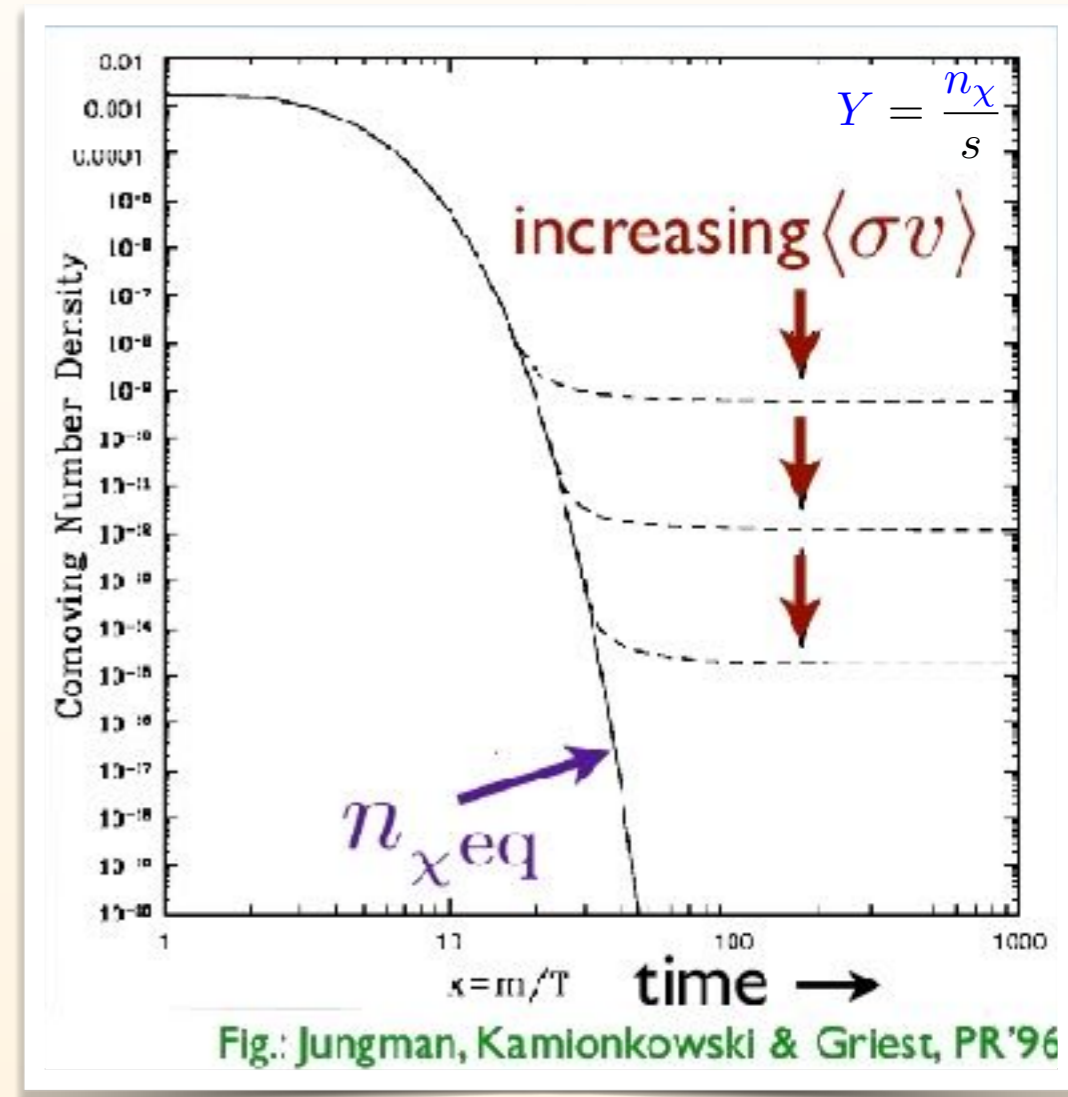
Re-written for the comoving number density:

$$\frac{dY}{dx} = \sqrt{\frac{g_*\pi m_\chi^2}{45G}} \frac{\langle\sigma_{\chi\bar{\chi}\rightarrow ij}\sigma_{\text{rel}}\rangle^{\text{eq}}}{x^2} (Y^2 - Y_{\text{eq}}^2)$$

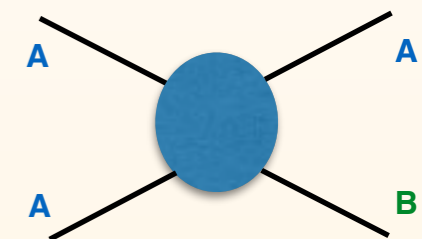
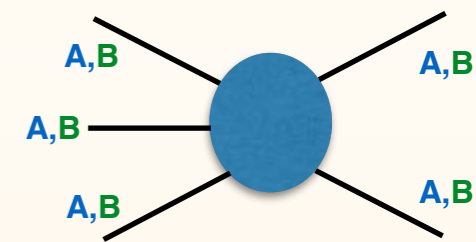
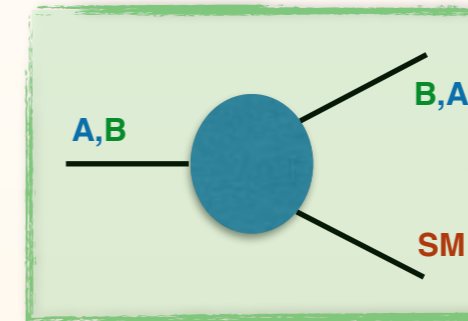
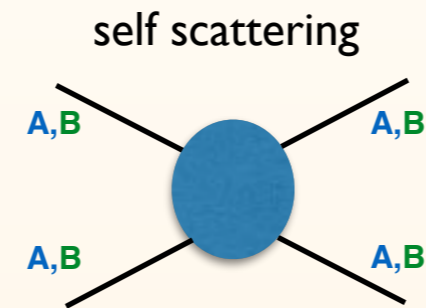
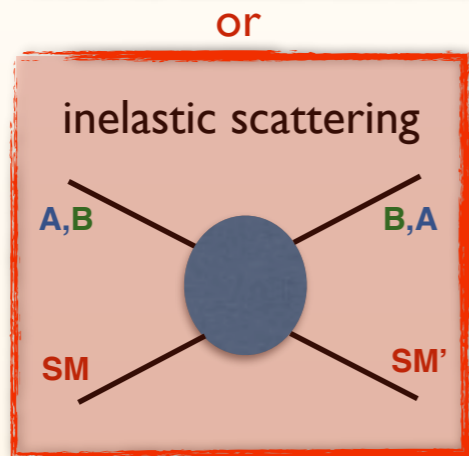
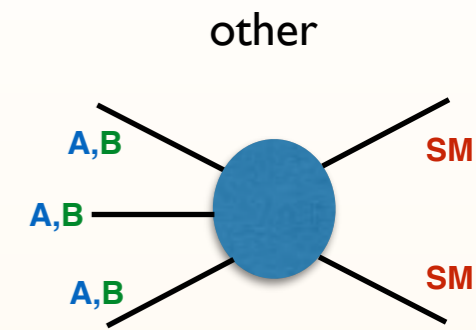
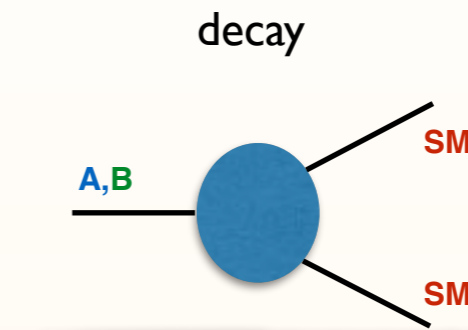
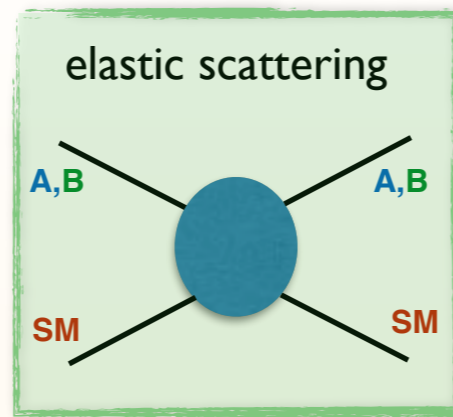
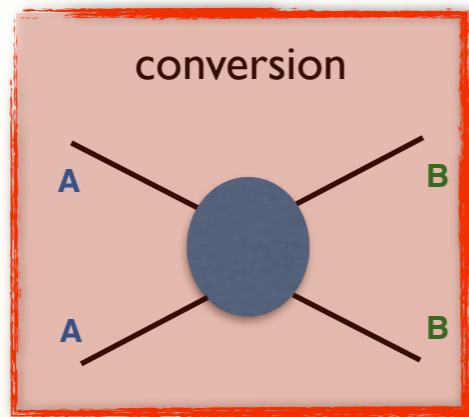
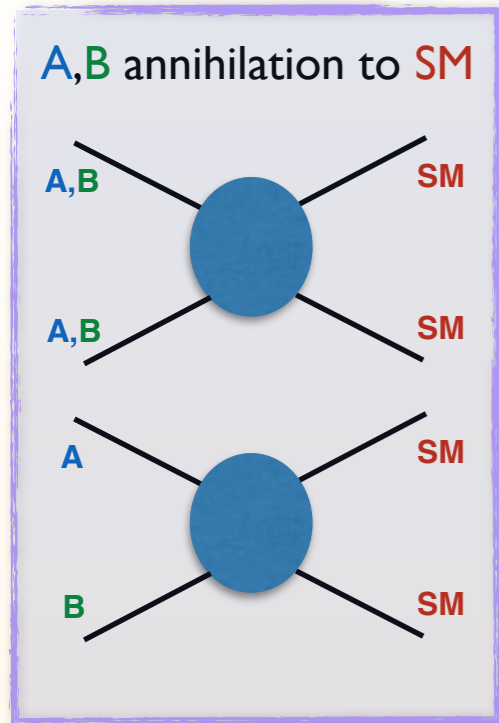
$$\lim_{x\rightarrow 0} Y = Y_{\text{eq}} \quad \lim_{x\rightarrow\infty} Y = \text{const}$$

**Recipe:**

compute annihilation **cross-section**,  
 take a **thermal bath average**,  
 throw it into **BE**... and voilà



# WHAT IF NON-MINIMAL SCENARIO?



+.....

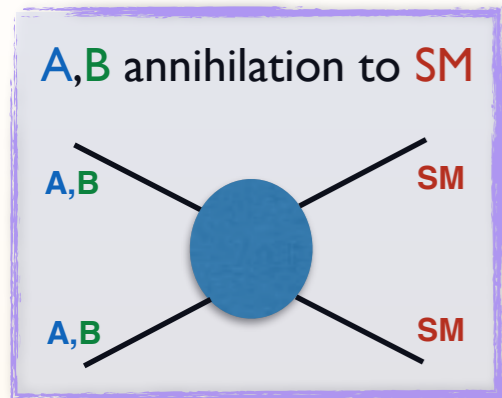
↳ typically forbidden by symmetry

Co-annihilation → Griest, Seckel '91

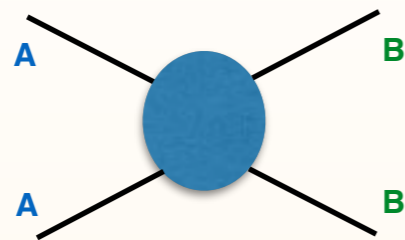
due to **efficient conversion processes** one can trace only number density of sum of the states with shared conserved quantum number using **weighted annihilation cross section**

- needed to be efficient for mechanism to work
- setting the relic density
- assumed in computation

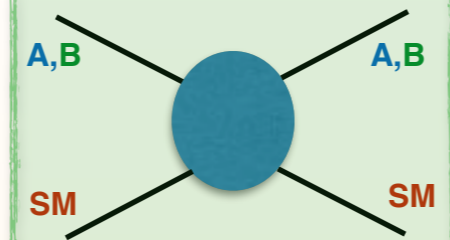
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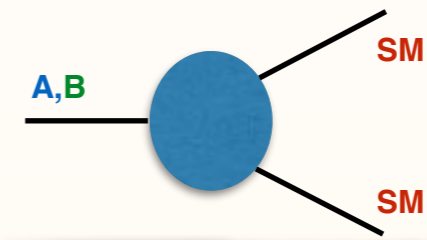
conversion



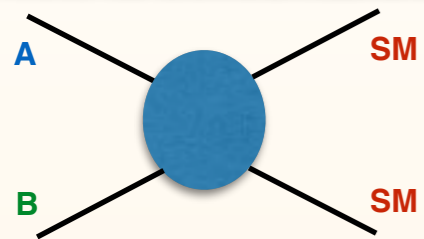
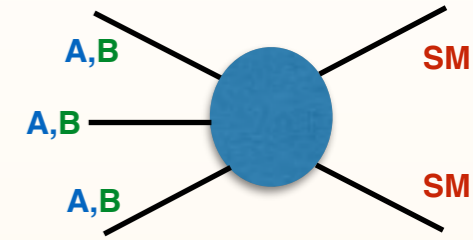
elastic scattering



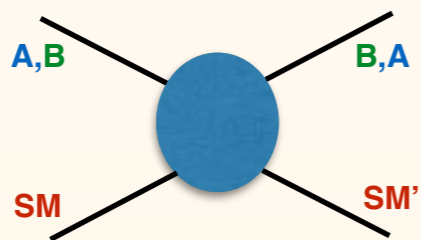
decay



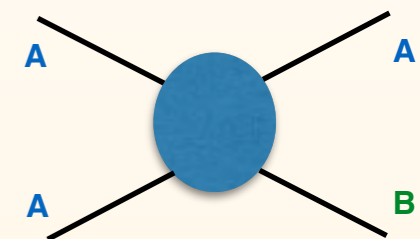
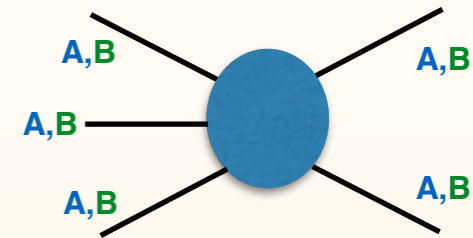
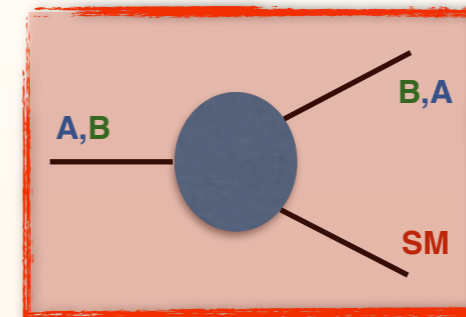
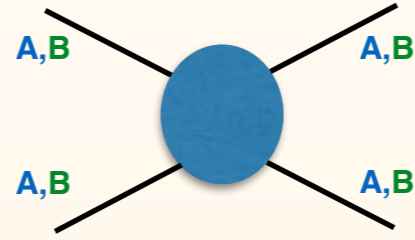
other



inelastic scattering



self scattering



+.....

↳ typically forbidden by symmetry

SuperWIMP



Feng, Rajaraman, Takayama '03

DM abundance is an effect of complete decay of heavier state which freezes-out as standard WIMP



needed to be efficient for mechanism to work

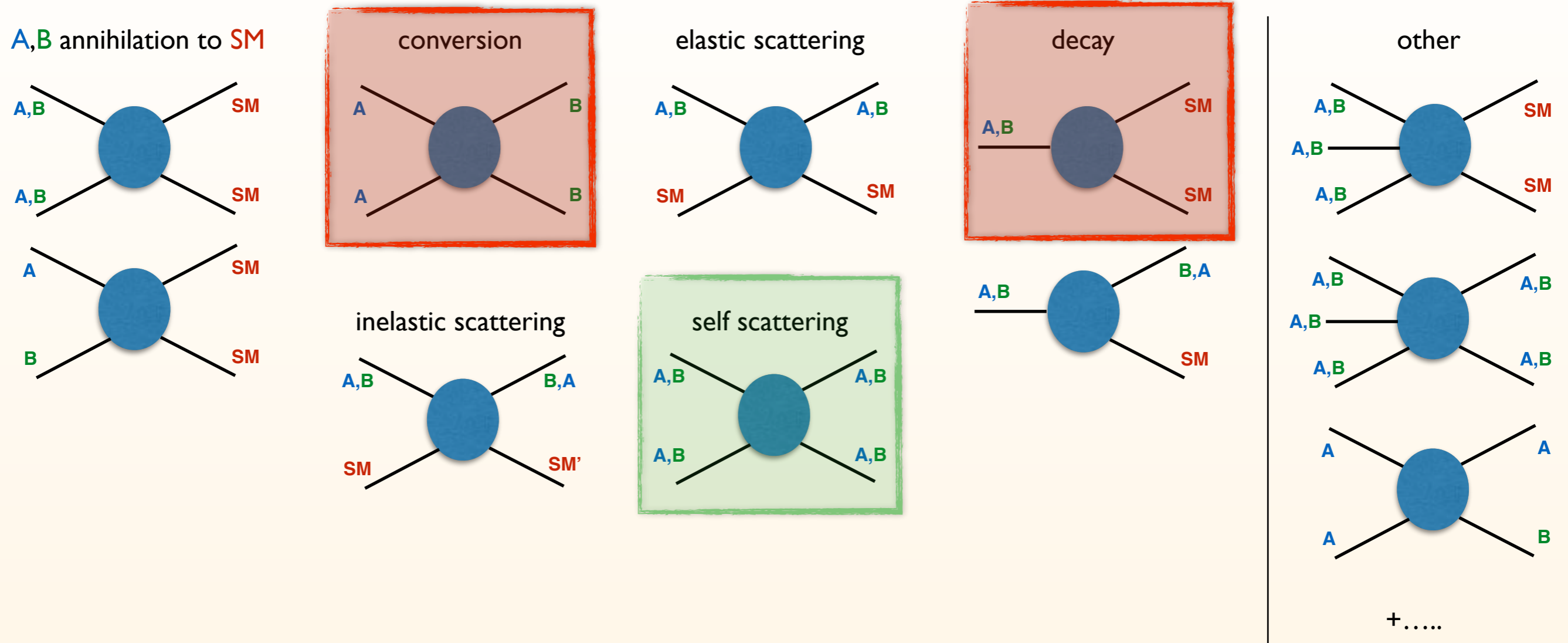


setting the relic density



assumed in computation

# WHAT IF NON-MINIMAL SCENARIO?



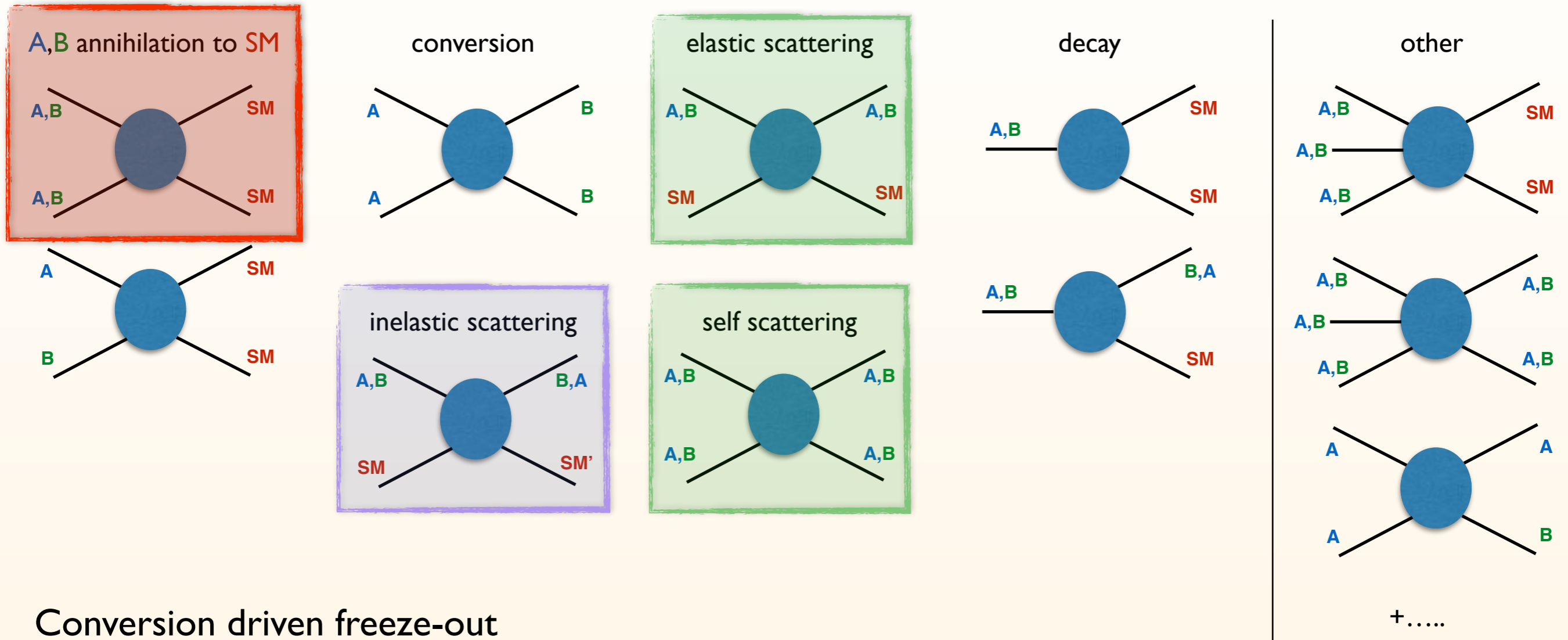
Co-decaying  
Dror, Kuflik, Ng '16

DM decouples when relativistic but then one of the dark sector states **decays** and this effect important as long as **conversions** are

+.....  
↳ typically forbidden by symmetry

- needed to be efficient for mechanism to work
- setting the relic density
- assumed in computation

# WHAT IF NON-MINIMAL SCENARIO?



Conversion driven freeze-out

Garny, Heisig, Lulf, Vogl '17

Co-scattering




D'Agnolo, Pappadopulo, Ruderman '17



only one of the dark sector states annihilates efficiently, but also conversions stop being efficient which blocks co-annihilation

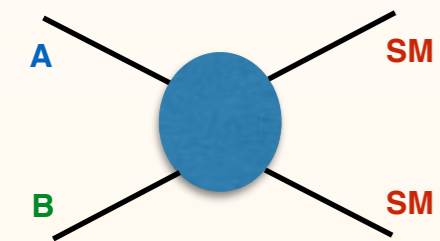
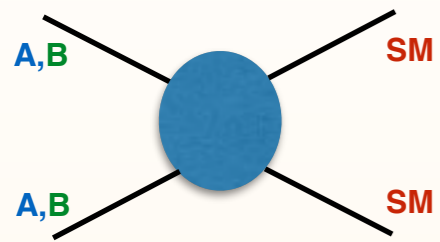
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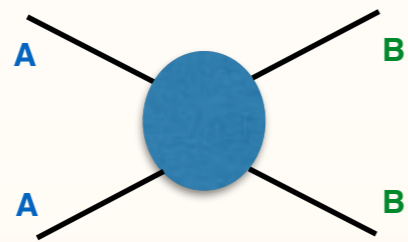
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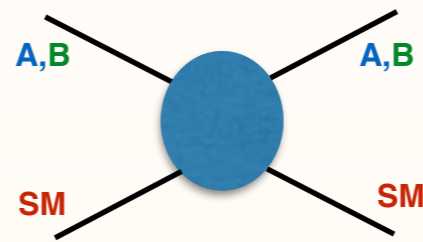
A,B annihilation to SM



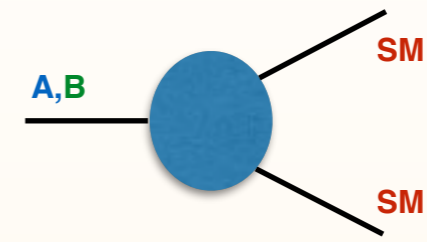
conversion



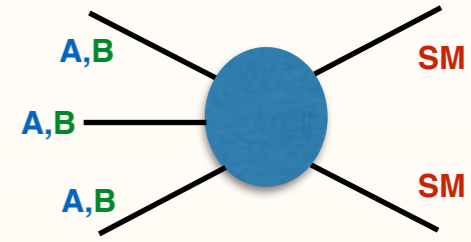
elastic scattering



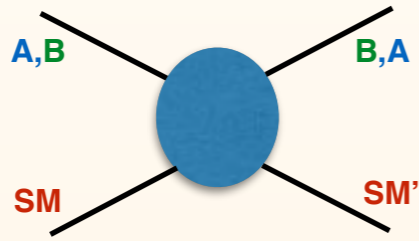
decay



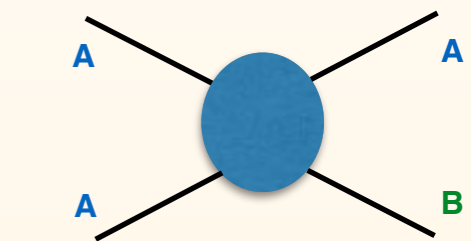
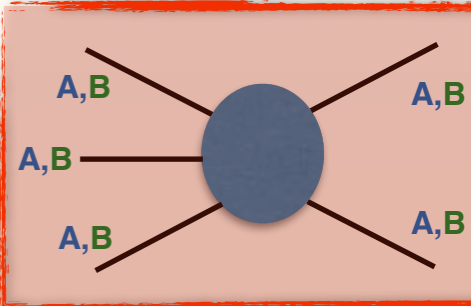
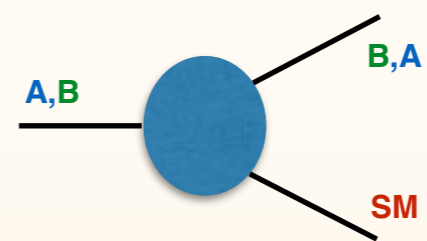
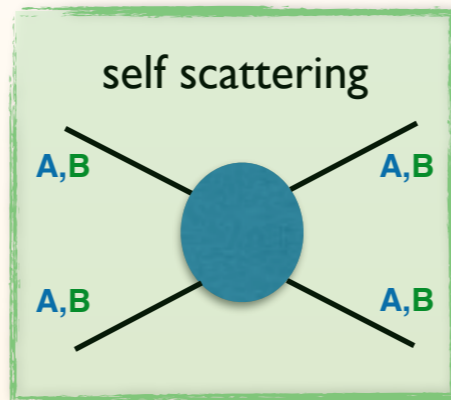
other



inelastic scattering



self scattering




Cannibalization →


Kuflik et al. '15


secluded dark sector, with efficient 3-2 annihilation leading to **DM zero chemical potential** and keeping DM at **much higher temperature** than SM plasma

+.....

↳ typically forbidden by symmetry

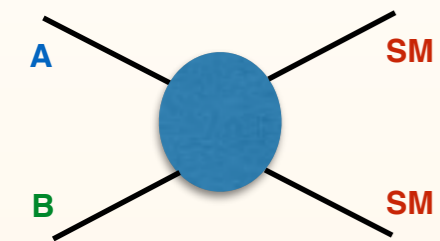
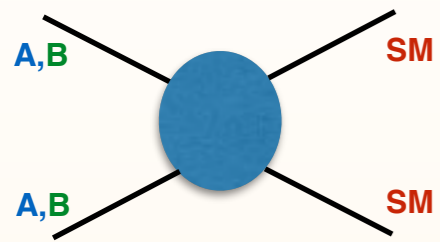
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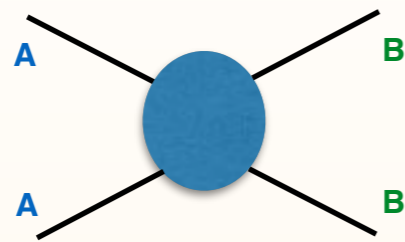
 assumed in computation

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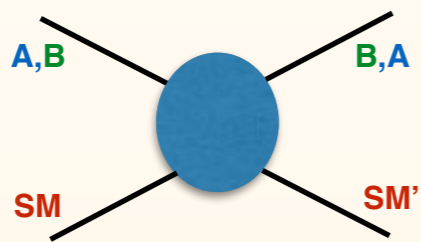
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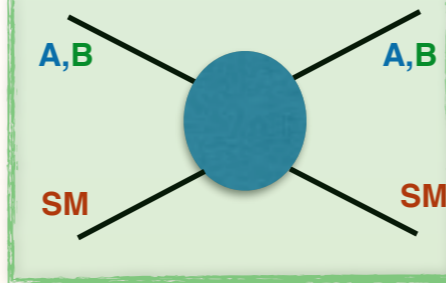
conversion



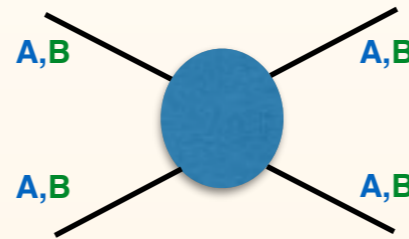
inelastic scattering



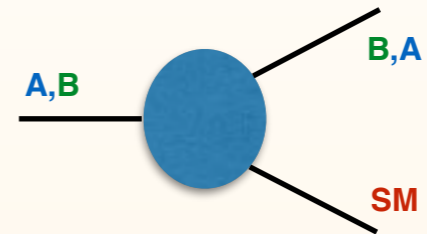
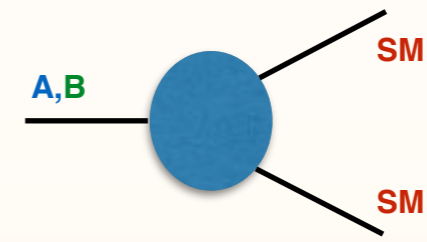
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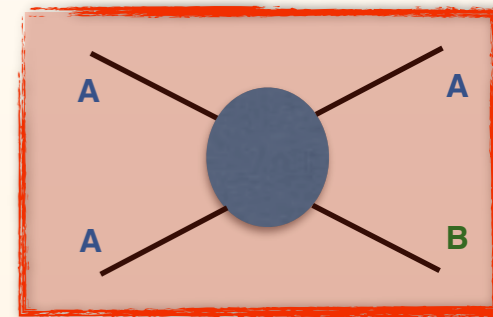
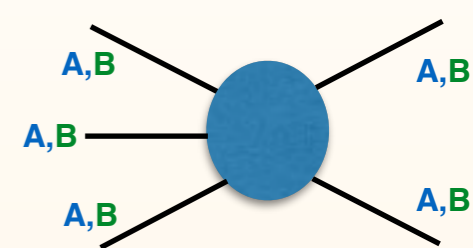
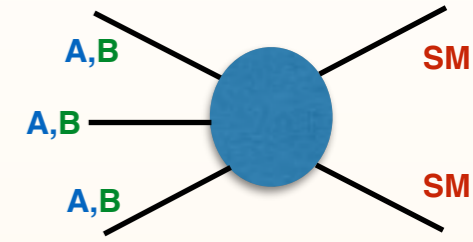
self scattering



decay



other



+.....

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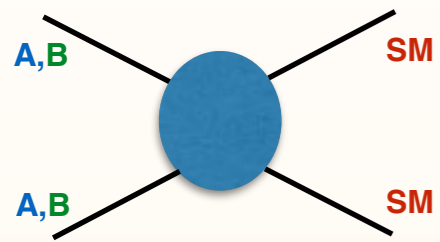
Semi-annihilation  $\longrightarrow$  D'Eramo, Thaler '10

new **type of annihilation** process that can dominate the freeze-out dynamics; occurs when new „flavour” or „baryon” structure in dark sector

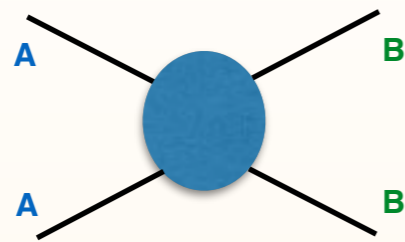
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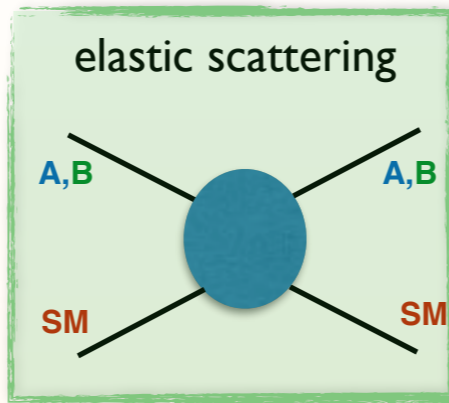
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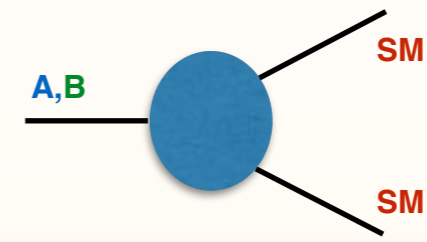
conversion



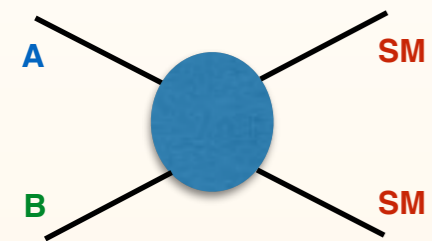
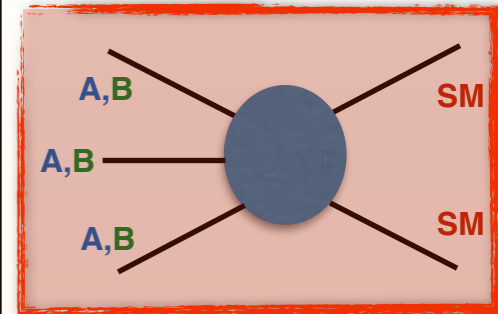
elastic scattering



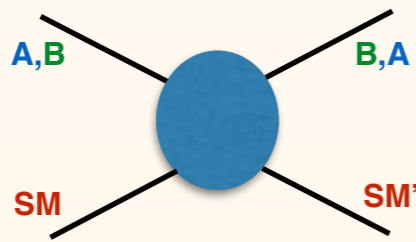
decay



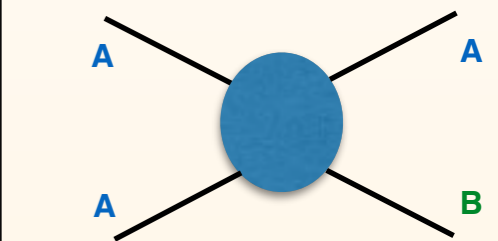
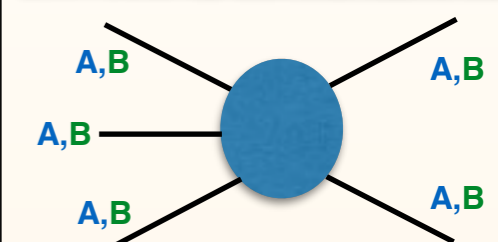
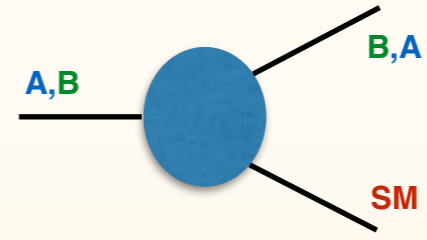
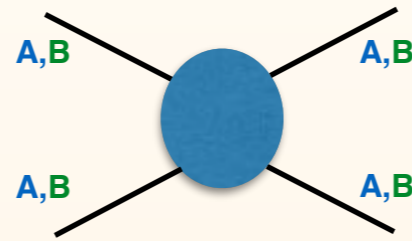
other



inelastic scattering



self scattering



Forbidden-like DM  $\longrightarrow$

e.g., D'Agnolo, Ruderman '15;  
Cline et al. '17; Choi et al. '17;

annihilation to a state heavier than DM,  
possible from tail of the Boltzmann  
distribution or when 3-2 processes are  
non-negligible

+.....

$\longleftarrow$  typically  
forbidden by  
symmetry



needed to be efficient for mechanism to work



setting the relic density

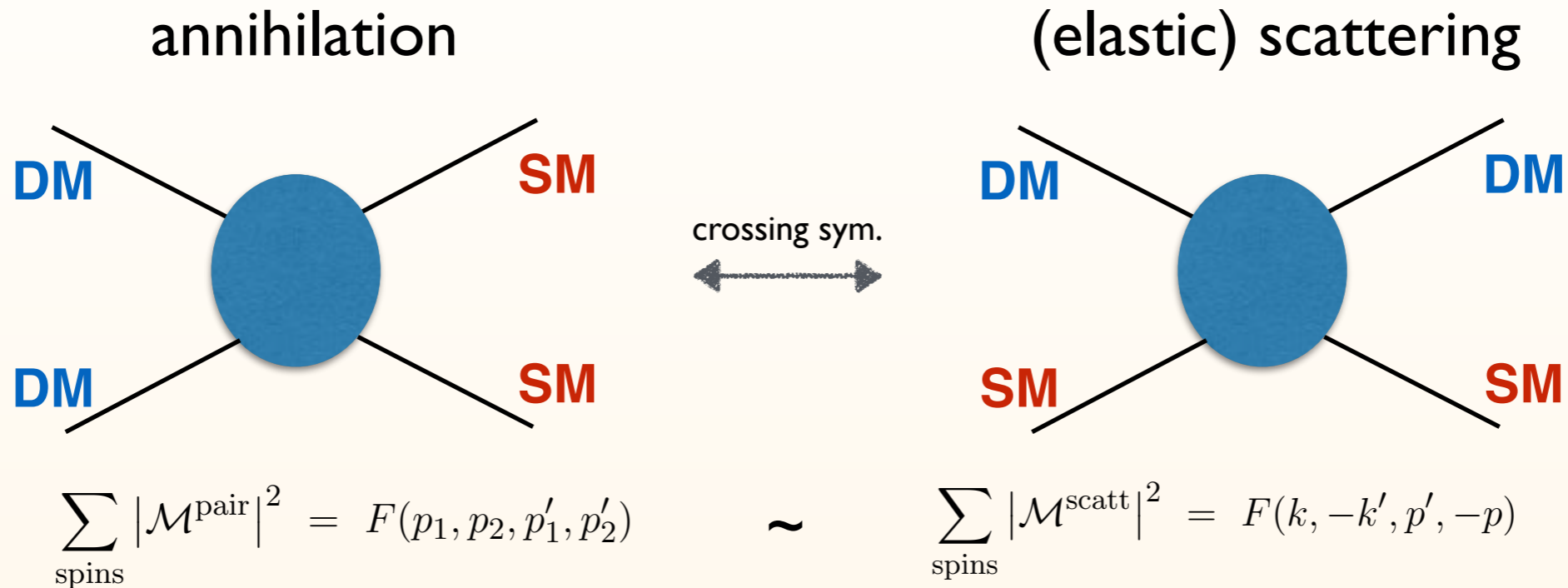


assumed in computation

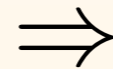


EXCEPTION *N*:  
EARLY KINETIC DECOUPLING

# FREEZE-OUT VS. DECOUPLING



Boltzmann suppression of **DM** vs. **SM**



scatterings typically more frequent

dark matter frozen-out but typically still kinetically coupled to the plasma

$$\tau_r(T_{\text{kd}}) \equiv N_{\text{coll}}/\Gamma_{\text{el}} \sim H^{-1}(T_{\text{kd}})$$

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Two consequences:

1. During freeze-out (chemical decoupling) typically:  $f_\chi \sim a(\mu) f_\chi^{\text{eq}}$
2. If kinetic decoupling much, much later: possible impact on the matter power spectrum  
i.e. kinetic decoupling can have observable consequences and affect e.g. missing satellites problem

see e.g., Bringmann, Ihle, Karsten, Valia '16

# IMPLICATIONS OF KINETIC DECOUPLING

E.g. for SUSY neutralino:

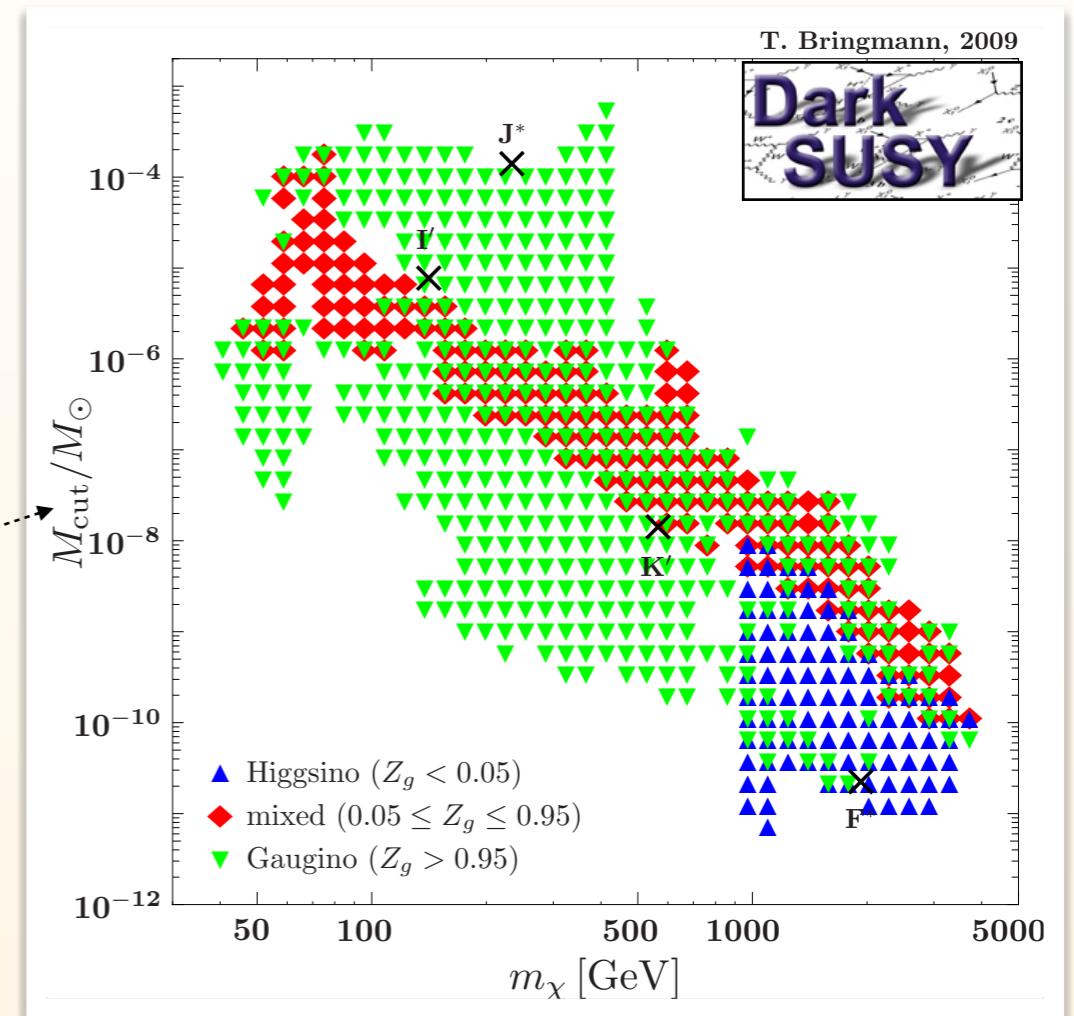
Bringmann '09

Free-streaming of DM after KD washes out **density contrasts at small scales** (similarly to baryonic oscillations)

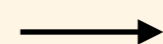
Green, Hofmann, Schwarz '05



Cut-off in the power spectrum corresponding to **smallest gravitationally bound objects**



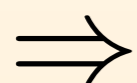
„Typical” values for **WIMPs** are relatively small



small substructures expected

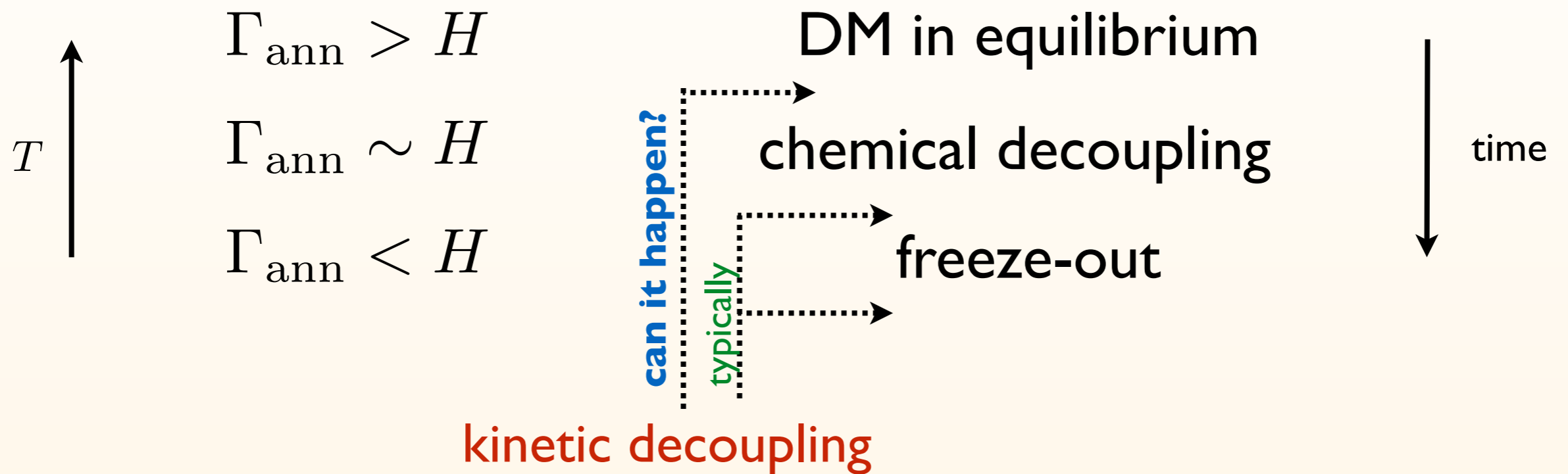


but bad for missing satellites problem



moment of KD leaves important imprint on the Universe

# A PITFALL IN A NUTSHELL



If **KD** happens around CD  $\longrightarrow$  what would be the relic density?

$\downarrow$   
 how to even compute that?

$\implies$  need for refined treatment of solving the Boltzmann eq.

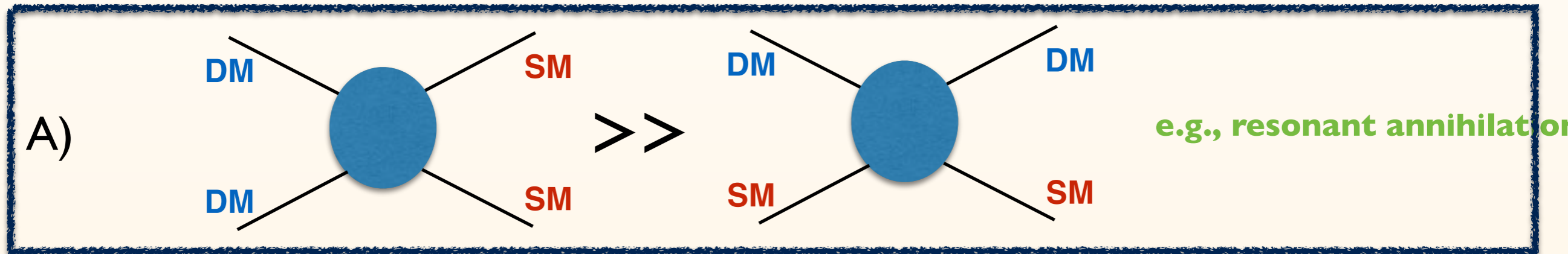
assuming kinetic equilibrium at chemical decoupling:  $f_\chi \sim a(\mu) f_\chi^{\text{eq}}$

$$C_{\text{LO}} = -\langle \sigma_{\chi\bar{\chi} \rightarrow ij} v_{\text{rel}} \rangle^{\text{eq}} (n_\chi n_{\bar{\chi}} - n_\chi^{\text{eq}} n_{\bar{\chi}}^{\text{eq}})$$

# EARLY KINETIC DECOUPLING?

A **necessary** and **sufficient** condition: scatterings weaker than annihilation  
i.e. rates around freeze-out:  $H \sim \Gamma_{\text{ann}} \gtrsim \Gamma_{\text{el}}$

Possibilities:



B) Boltzmann suppression of **SM** as strong as for **DM**  
e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure  
e.g., semi-annihilation, 3 to 2 models,...

# EARLY KD AND RESONANCE

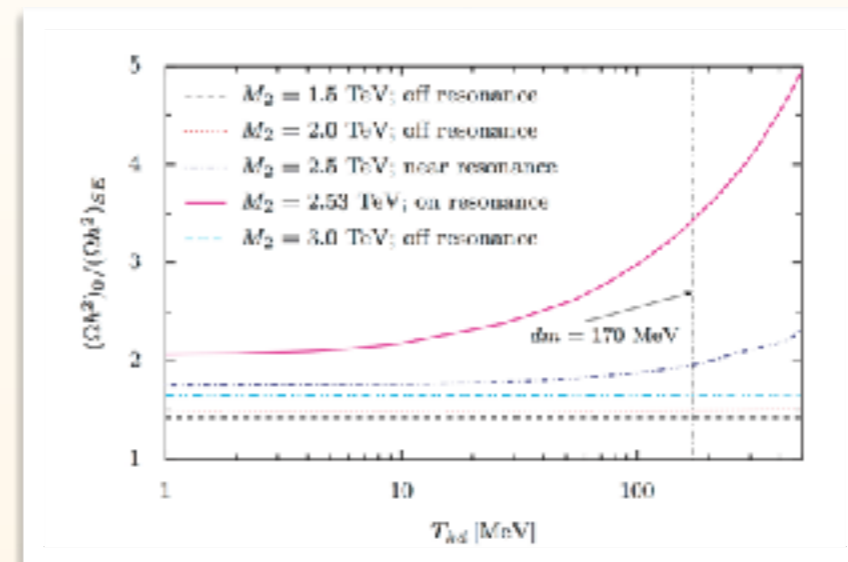
our work wasn't the first to realize that resonant annihilation can lead to early kinetic decoupling...

Feng, Kaplinghat, Yu '10 — noted that for Sommerfeld-type resonances KD can happen early

Dent, Dutta, Scherrer '10 — looked at potential effect of KD on thermal relic density

Since then people were aware of this effect and sometimes tried to estimate it assuming **instantaneous KD**, e.g., in the case of Sommerfeld effect in the MSSM:

but **no systematic studies** of decoupling process were performed, until...



AH, Iengo, Ullio '11

...models with very late KD become popular, in part to solve „missing satellites” problem

van den Aarssen et al '12; Bringmann et al '16, x2; Binder et al '16

this progress allowed for **better approach to early KD** scenarios as well and was applied to the **resonant annihilation case** in

Duch, Grzędkowski '17

... but we developed a **dedicated accurate method/code** to deal with this and other similar situations

# COMMENT:

## COMPLEMENTARITY OF DG VS. BBGH

The effect of early KD was approached from two very different perspectives:

Duch, Grządkowski '17

motivated by phenomenology of self-interacting DM

evolved into more in-depth study of effects related to general resonant annihilation: KD and energy dependent width

as an example case studied a vector DM model and looked more onto self-interactions

Binder, Bringmann, Gustafsson, AH '17

motivated by the question: can KD happen before CD?

evolved into more in-depth study of the time evolution of the phase space distribution function for a general case of velocity dependent annihilation

as an example case studied a very specific resonance model and concentrated on the relic density

# HOW TO DESCRIBE KD?

All information is in full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E (\partial_t - H\vec{p} \cdot \nabla_{\vec{p}}) f_{\chi} = \mathcal{C}[f_{\chi}]$$

contains both **scatterings** and **annihilation**

Two possible approaches:

solve numerically  
for full  $f_{\chi}(p)$

have insight on the distribution  
no constraining assumptions

numerically challenging  
typically overkill

consider system of equations  
for moments of  $f_{\chi}(p)$

partially analytic/much easier numerically  
manifestly captures all of the relevant physics

finite range of validity  
no insight on the distribution

0-th moment:  $n_{\chi}$   
2-nd moment:  $T_{\chi}$   
...



# SCATTERING

The **elastic scattering** collision term:

$$C_{\text{el}} = \frac{1}{2g_\chi} \int \frac{d^3k}{(2\pi)^3 2\omega} \int \frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}} \int \frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}}$$

$$\times (2\pi)^4 \delta^{(4)}(\tilde{p} + \tilde{k} - p - k) |\mathcal{M}|_{\chi f \leftrightarrow \chi f}^2$$

$$\times \left[ (1 \mp g^\pm)(\omega) g^\pm(\tilde{\omega}) f_\chi(\tilde{\mathbf{p}}) - (\omega \leftrightarrow \tilde{\omega}, \mathbf{p} \leftrightarrow \tilde{\mathbf{p}}) \right]$$

$\downarrow$   
 equilibrium functions for SM particles

Expanding in **NR** and small **momentum transfer**:

Bringmann, Hofmann '06

$$C_{\text{el}} \simeq \frac{m_\chi}{2} \gamma(T) \left[ T m_\chi \partial_p^2 + \left( p + 2T \frac{m_\chi}{p} \right) \partial_p + 3 \right] f_\chi$$

More generally, Fokker-Planck scattering operator  
(relativistic, but still small **momentum transfer**):

Binder et al. '16

physical interpretation:  
**scattering rate**

$$C_{\text{el}} \simeq \frac{E}{2} \nabla_{\mathbf{p}} \cdot \left[ \gamma(T, \mathbf{p}) (ET \nabla_{\mathbf{p}} + \mathbf{p}) f_\chi \right]$$

Semi-relativistic: assume that scattering  $\gamma(T, \mathbf{p})$  is momentum independent

# KINETIC DECOUPLING 101

DM temperature  
Definition:

$$T_\chi \equiv \frac{g_\chi}{3m_\chi n_\chi} \int \frac{d^3 p}{(2\pi)^3} p^2 f_\chi(p)$$

$$y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$$

actually: normalized average NR energy - equals temperature at equilibrium

First take late KD scenario and consider only **temperature evolution** -  
i.e. leave out feedback **on/from** changing **number density**:

then 2nd moment of full BE (up to terms  $p^2/m_\chi^2$ ) gives:

$$\frac{y'}{y} = -\frac{Y'}{Y} \left( 1 - \frac{\langle \sigma v_{\text{rel}} \rangle_2}{\langle \sigma v_{\text{rel}} \rangle} \right) - \left( 1 - \frac{x}{3} \frac{g'_{*S}}{g_{*S}} \right) \frac{2m_\chi c(T)}{Hx} \left( 1 - \frac{y_{\text{eq}}}{y} \right)$$

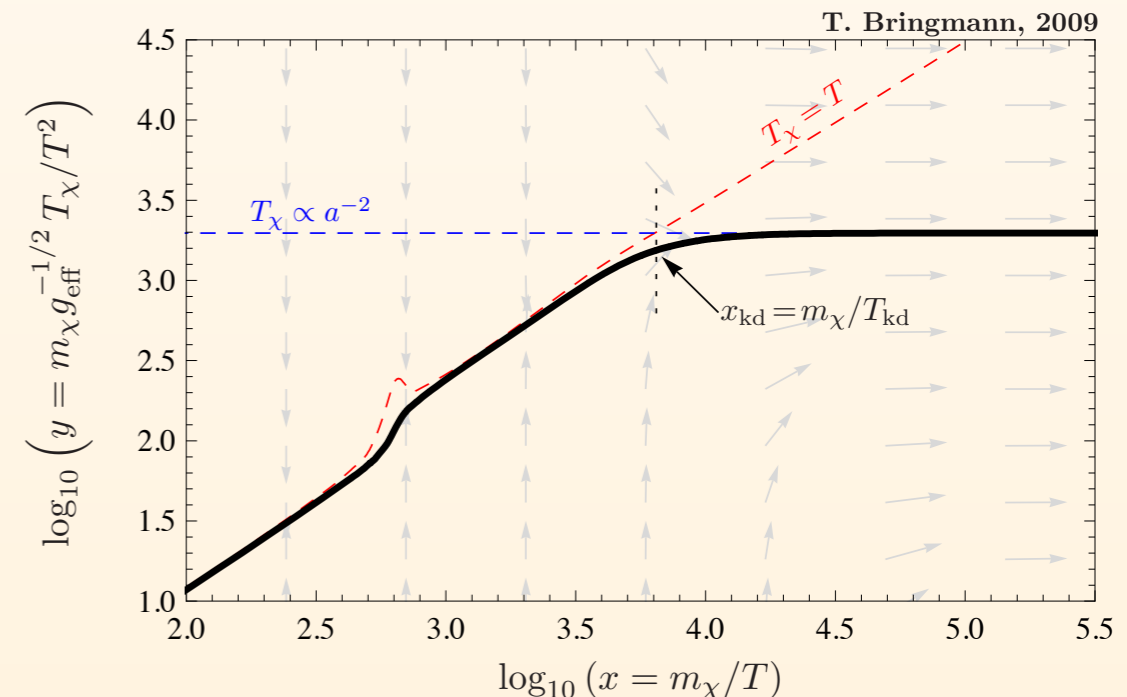
where:

$$\langle \sigma v_{\text{rel}} \rangle_2 \equiv \frac{g_\chi^2}{3Tm_\chi n_\chi^2} \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^3 \tilde{p}}{(2\pi)^3} p^2 v_{\text{rel}} \sigma_{\bar{\chi}\chi \rightarrow \bar{X}X} f(E) f(\tilde{E})$$

impact of **annihilation**

$$c(T) = \frac{1}{12(2\pi)^3 m_\chi^4 T} \sum_X \int dk k^5 \omega^{-1} g^\pm (1 \mp g^\pm) \int_{-4k^2}^0 (-t) \frac{1}{8k^4} |\mathcal{M}_{\text{el}}|^2$$

impact of elastic  
**scatterings**



# ONE STEP FURTHER...

Now consider general KD scenario, i.e. coupled **temperature** and **number density** evolution:

annihilation and production thermal averages done at different  $T$  — feedback of modified  $y$  evolution

$$\frac{Y'}{Y} = -\frac{1 - \frac{x}{3} \frac{g'_{*S}}{g_{*S}}}{Hx} sY \left( \langle \sigma v_{\text{rel}} \rangle \Big|_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\text{eq}}^2}{Y^2} \langle \sigma v_{\text{rel}} \rangle \Big|_x \right)$$

$$\frac{y'}{y} = -\frac{1 - \frac{x}{3} \frac{g'_{*S}}{g_{*S}}}{Hx} \left[ 2m_\chi c(T) \left( 1 - \frac{y_{\text{eq}}}{y} \right) - sY \left( \left( \langle \sigma v_{\text{rel}} \rangle - \langle \sigma v_{\text{rel}} \rangle_2 \right) \Big|_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\text{eq}}^2}{Y^2} \left( \langle \sigma v_{\text{rel}} \rangle - \frac{y_{\text{eq}}}{y} \langle \sigma v_{\text{rel}} \rangle_2 \right) \Big|_x \right) \right]$$

$$+ \frac{1 - \frac{x}{3} \frac{g'_{*S}}{g_{*S}}}{3m_\chi} \langle p^4/E^3 \rangle_{x=m_\chi^2/(s^{2/3}y)}$$

"relativistic" term

elastic scatterings term

impact of annihilation

$$T_\chi \equiv \frac{g_\chi}{3n_\chi} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{E} f_\chi(p)$$

These equations still assume the equilibrium shape of  $f_\chi(p)$  — but with variant temperature

or more accurately: that the thermal averages computed with true non-equilibrium distributions don't differ much from the above ones

# NUMERICAL APPROACH

... or one can just solve full phase space Boltzmann eq.

$$\begin{aligned}
 \partial_x f_\chi(x, q) &= \frac{m_\chi^3}{\tilde{H}x^4} \frac{g_{\bar{\chi}}}{2\pi^2} \int d\tilde{q} \tilde{q}^2 \frac{1}{2} \int d\cos\theta \ v_{M\emptyset} \sigma_{\bar{\chi}\chi \rightarrow \bar{f}f} \\
 &\times [f_{\chi,\text{eq}}(q) f_{\chi,\text{eq}}(\tilde{q}) - f_\chi(q) f_\chi(\tilde{q})] \\
 &+ \frac{2m_\chi c(T)}{2\tilde{H}x} \left[ x_q \partial_q^2 + \left( q + \frac{2x_q}{q} + \frac{q}{x_q} \right) \partial_q + 3 \right] f_\chi \\
 &+ \tilde{g} \frac{q}{x} \partial_q f_\chi,
 \end{aligned}$$

fully general

expanded in NR and small momentum transfer (semi-relativistic!)

discretization,  
~1000 steps

$$\begin{aligned}
 \partial_x f_i &= \\
 &\frac{m_\chi^3}{\tilde{H}x^4} \frac{g_{\bar{\chi}}}{2\pi^2} \sum_{j=1}^{N-1} \frac{\Delta\tilde{q}_j}{2} \left[ \tilde{q}_j^2 \langle v_{M\emptyset} \sigma_{\bar{\chi}\chi \rightarrow \bar{f}f} \rangle_{i,j}^\theta (f_i^{\text{eq}} f_j^{\text{eq}} - f_i f_j) \right. \\
 &+ \left. \tilde{q}_{j+1}^2 \langle v_{M\emptyset} \sigma_{\bar{\chi}\chi \rightarrow \bar{f}f} \rangle_{i,j+1}^\theta (f_i^{\text{eq}} f_{j+1}^{\text{eq}} - f_i f_{j+1}) \right] \\
 &+ \frac{2m_\chi c(T)}{2\tilde{H}x} \left[ x_{q,i} \partial_q^2 + \left( q_i + \frac{2x_{q,i}}{q_i} + \frac{q_i}{x_{q,i}} \right) \partial_q + 3 \right] f_i \\
 &+ \tilde{g} \frac{q_i}{x} \partial_q f_i,
 \end{aligned}$$

Solved numerically with MatLab

Note:

can be extended to e.g. self-scatterings

very stiff, care needed with numerics

**EXAMPLE:**  
**SCALAR SINGLET DM**

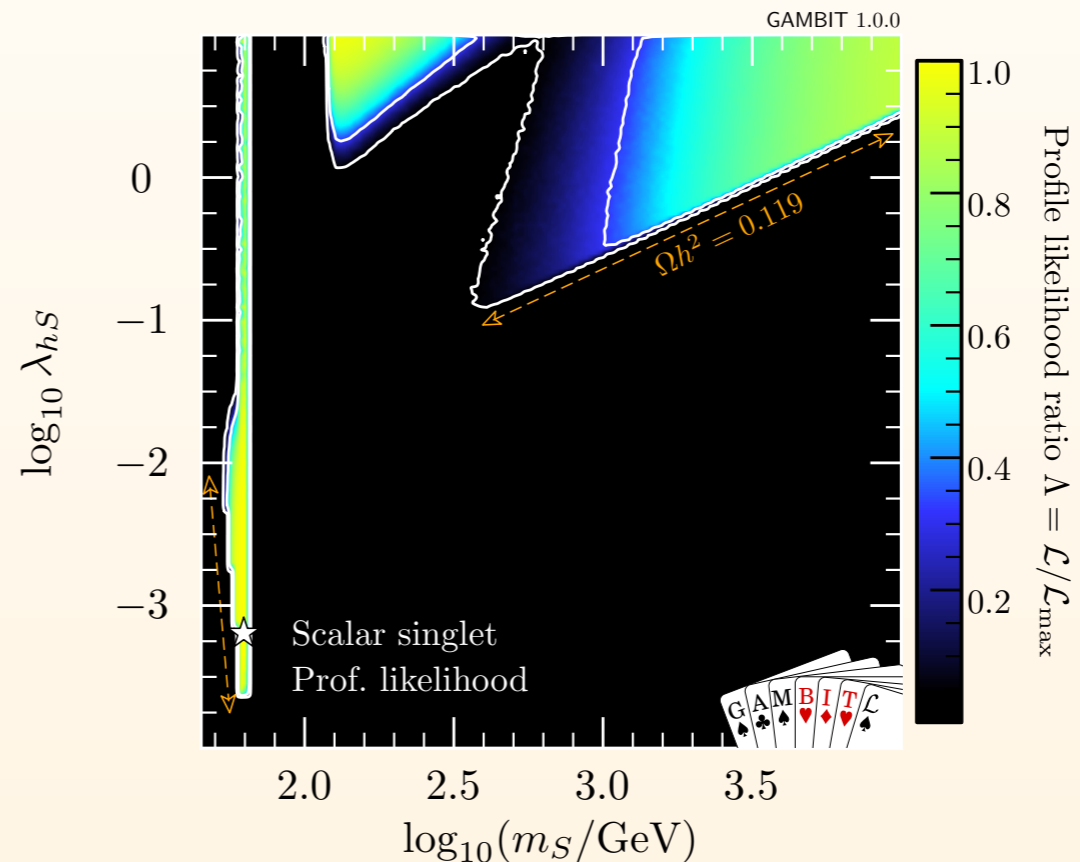
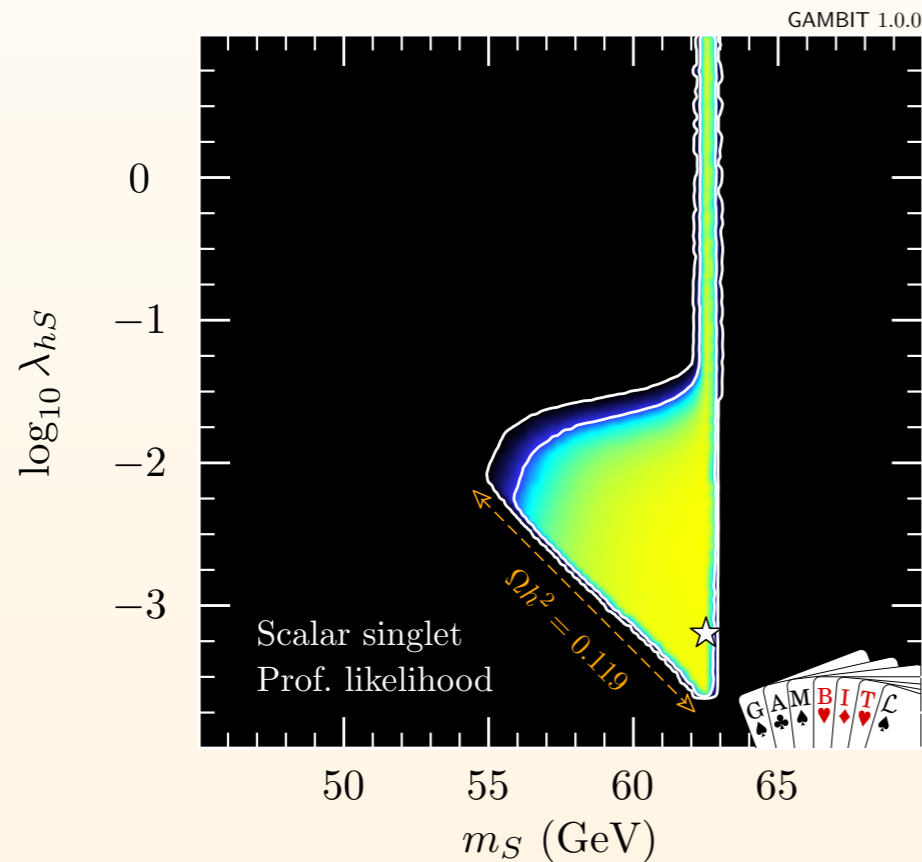
# SCALAR SINGLET DM

## VERY SHORT INTRODUCTION

To the SM Lagrangian add one singlet scalar field  $S$  with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2} \partial_\mu S \partial^\mu S - \frac{1}{2} \mu_S^2 S^2 - \frac{1}{2} \lambda_s S^2 |H|^2$$

$$m_s = \sqrt{\mu_S^2 + \frac{1}{2} \lambda_s v_0^2}$$



GAMBIT collaboration  
1705.07931

Most of the parameter space excluded, but... even such a simple model is hard to kill

# SCALAR SINGLET DM

## ANNIHILATION VS. SCATTERINGS

$$\sigma v_{\text{rel}} = \frac{2\lambda_s^2 v_0^2}{\sqrt{s}} |D_h(s)|^2 \Gamma_h(\sqrt{s})$$

with:

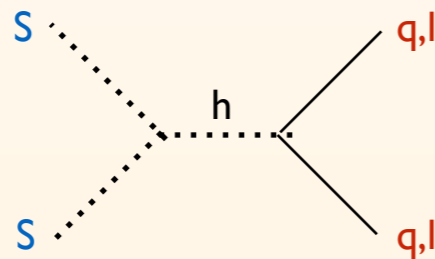
$$|D_h(s)|^2 \equiv \frac{1}{(s - m_h^2)^2 + m_h^2 \Gamma_h^2(m_h)}$$

tabulated  
Higgs width

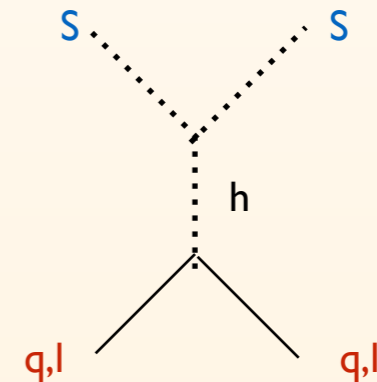
$$\langle |\mathcal{M}|^2 \rangle_t = \sum_f \frac{N_f \lambda_S^2 m_f^2}{8k^4} \left[ \frac{2k_{\text{cm}}^2 - 2m_f^2 + m_h^2}{1 + m_h^2/(4k_{\text{cm}}^2)} - (m_h^2 - 2m_f^2) \log(1 + 4k_{\text{cm}}^2/m_h^2) \right].$$

**Hierarchical Yukawa couplings:** strongest coupling to more Boltzmann suppressed quarks/leptons

Annihilation  
processes:  
**resonant**



El. scattering  
processes:  
**non-resonant**



Freeze-out at few GeV  $\rightarrow$  what is the abundance of heavy quarks in QCD plasma?

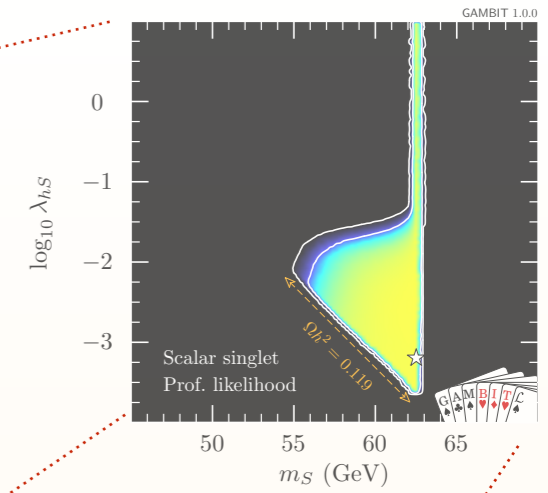
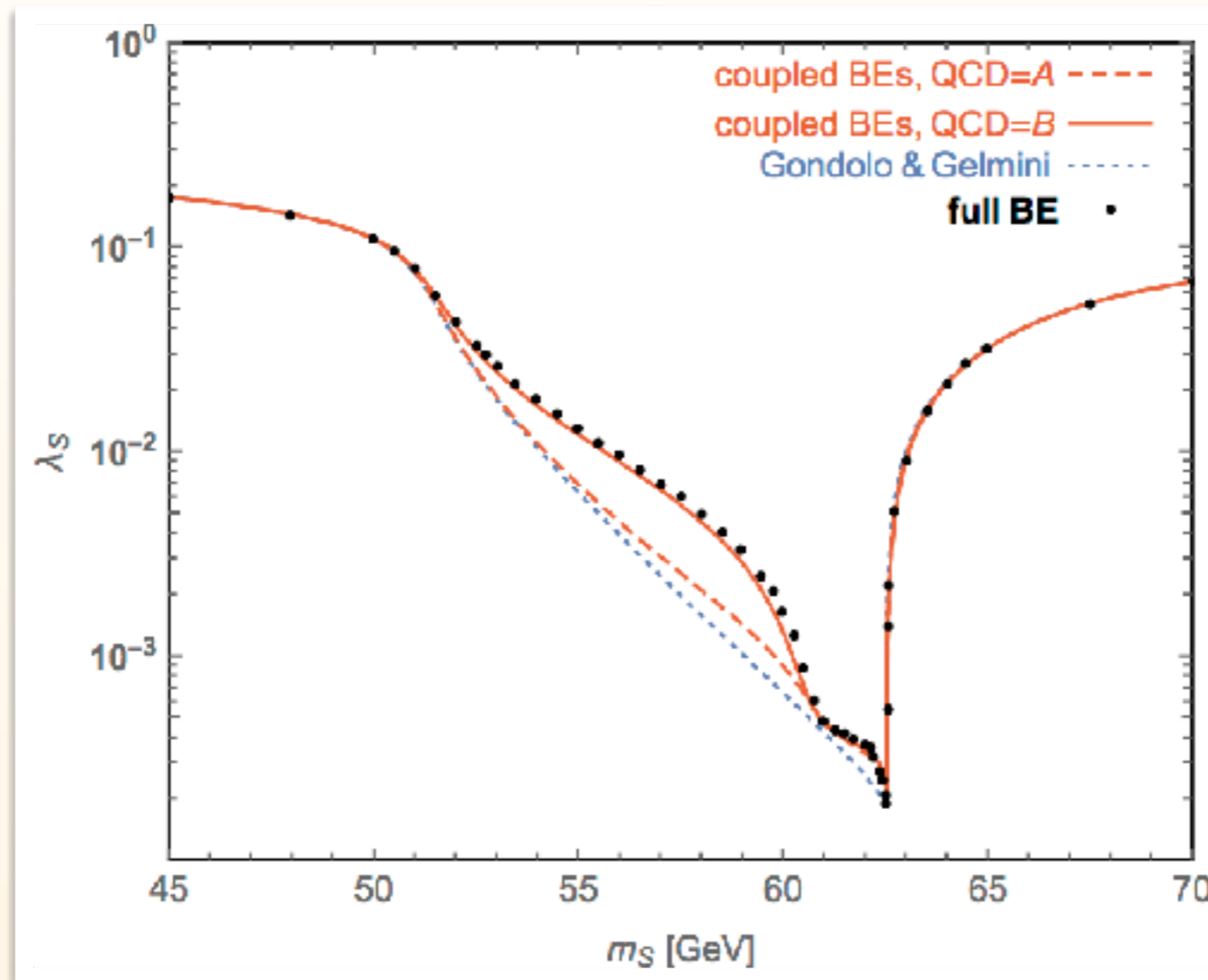
two scenarios:

QCD = A - all quarks are free and present in the plasma down to  $T_c = 154$  MeV

QCD = B - only light quarks contribute to scattering and only down to  $4T_c$

# RESULTS

## RD CONTOURS



essentially the only region left for this model

Significant modification of the **observed relic density contour** in the Scalar Singlet DM model

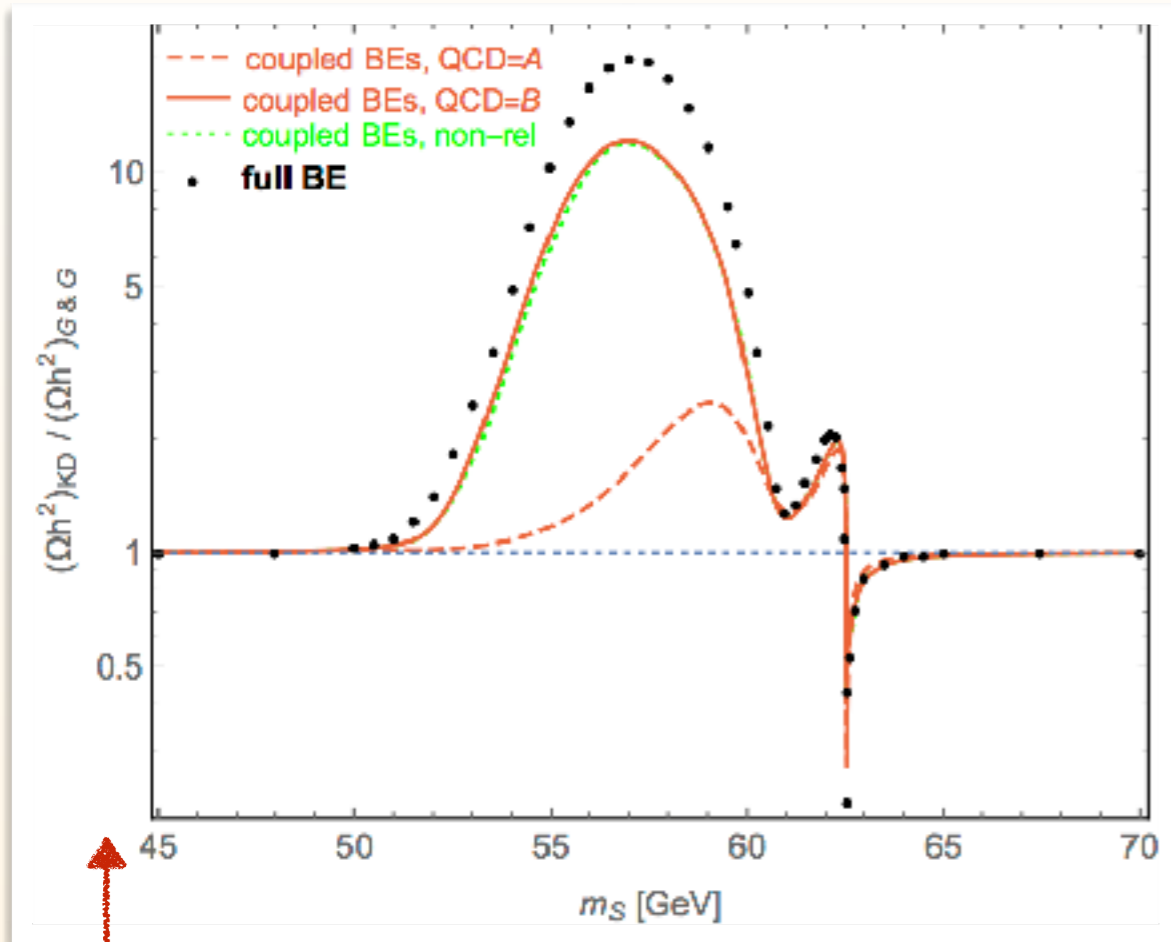
→ **larger coupling** needed → better chance for closing the last window



# RESULTS

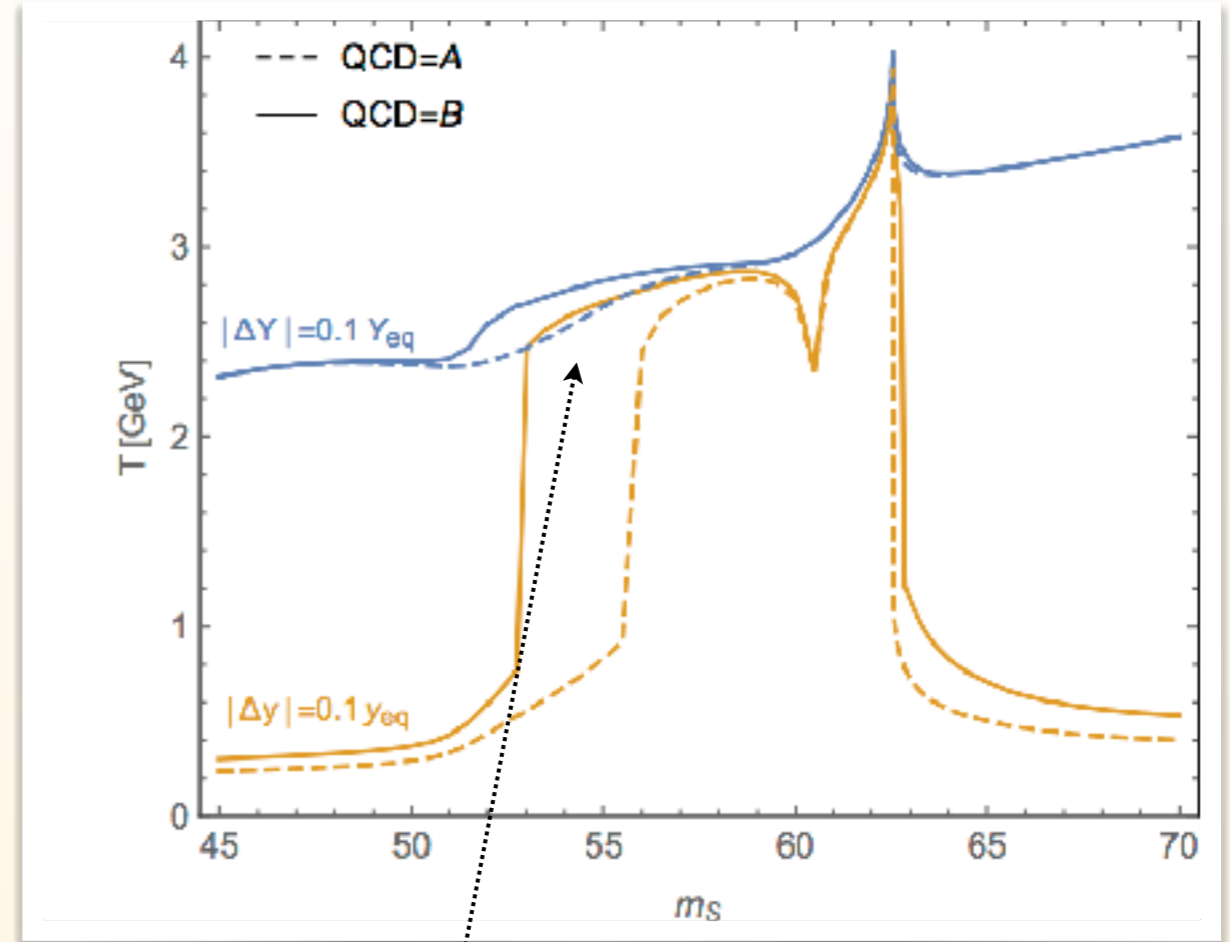
## EFFECT

effect on relic density:



effect on relic density:  
up to  $O(\sim 10)$

kinetic and chemical decoupling:



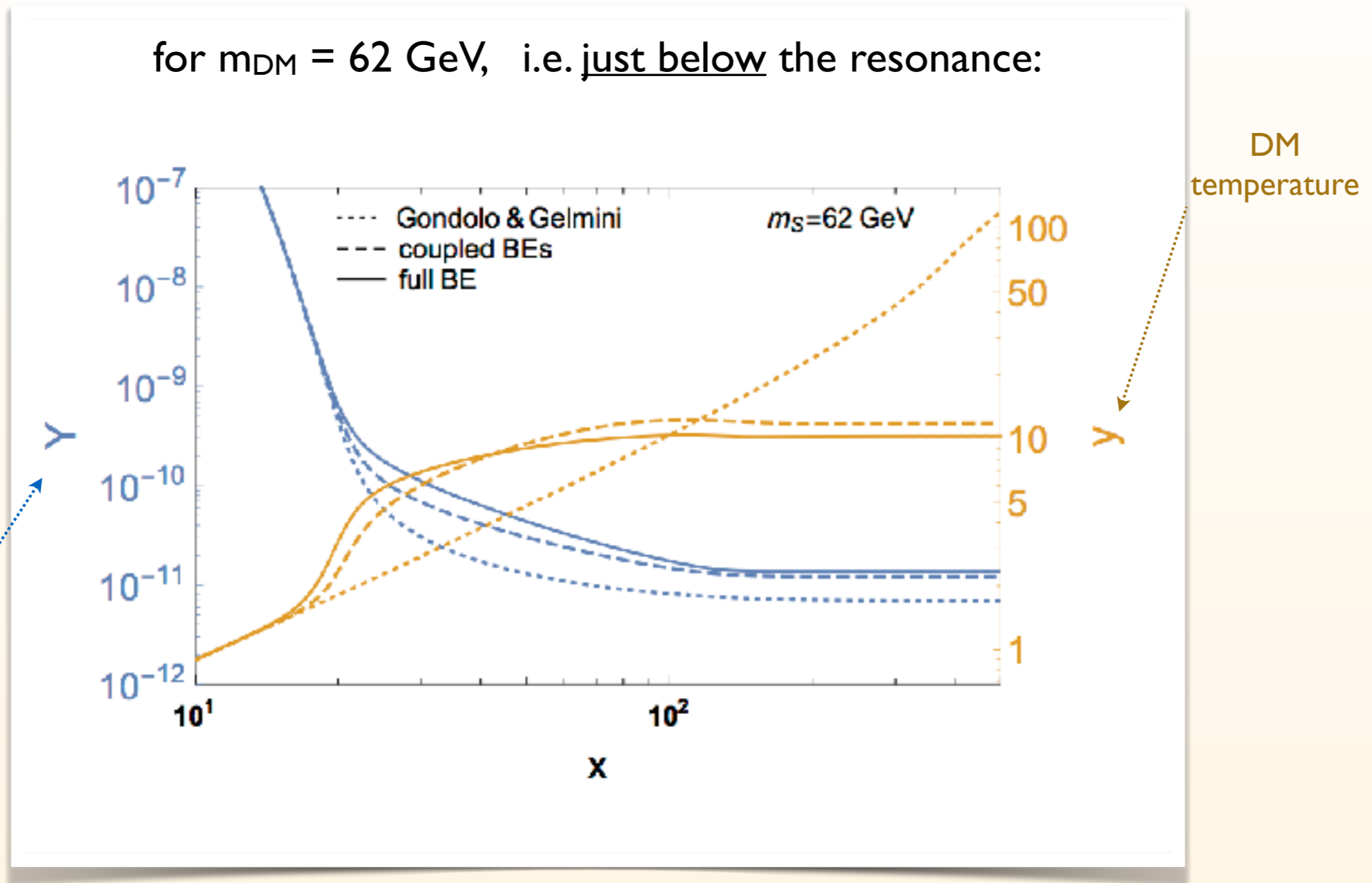
ratio approaches 1,  
but does not reach it!

Why such **non-trivial shape** of the effect of early kinetic decoupling?

↳ we'll inspect the  $y$  and  $Y$  evolution...

# DENSITY AND $T_{\text{DM}}$ EVOLUTION

for  $m_{\text{DM}} = 62 \text{ GeV}$ , i.e. just below the resonance:

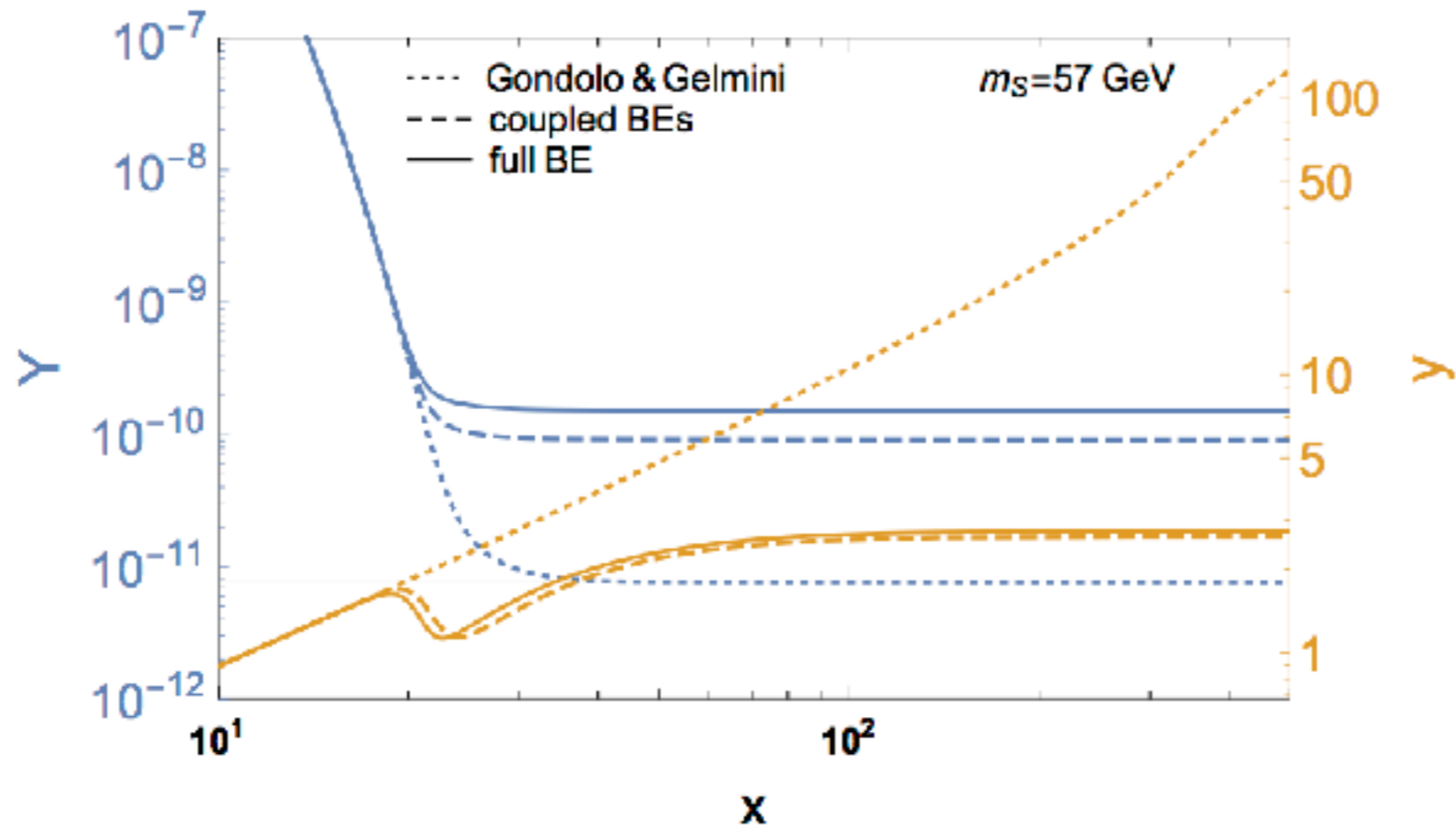


**Resonant annihilation most effective for low momenta**

→ DM fluid goes through "heating" phase before leaves kinetic equilibrium

# DENSITY AND $T_{\text{DM}}$ EVOLUTION

for  $m_{\text{DM}} = 57 \text{ GeV}$ , i.e. further away from the resonance:

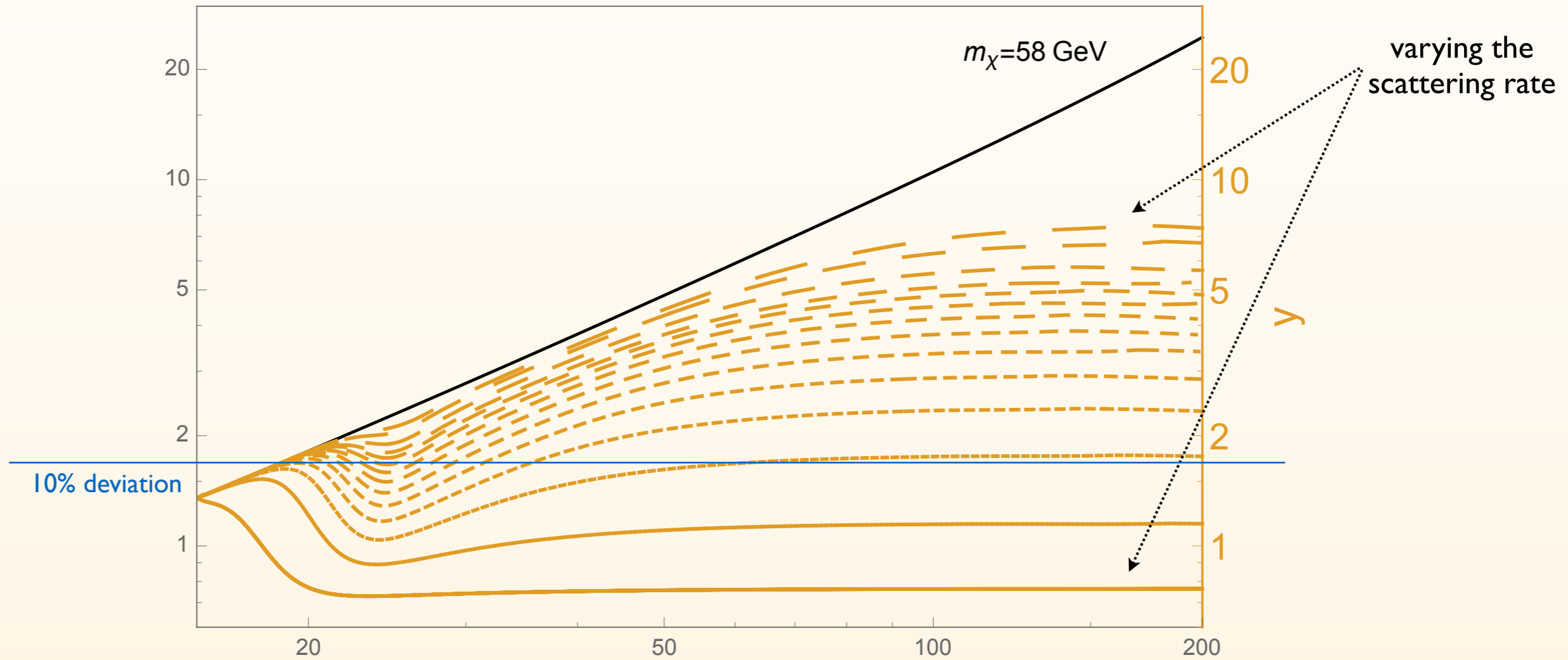
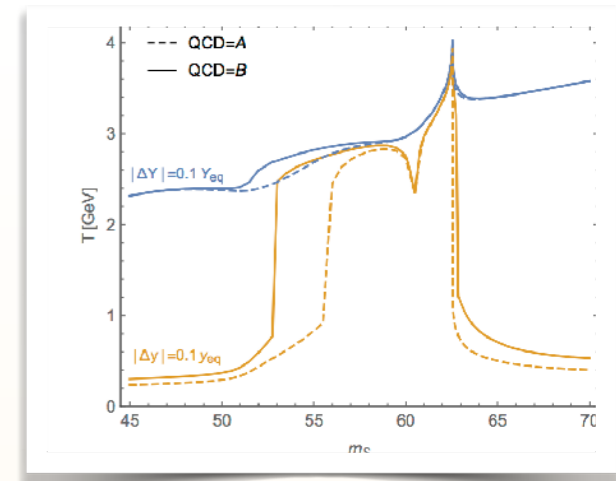


**Resonant annihilation most effective for high momenta**

→ DM fluid goes through fast "cooling" phase

after that when  $T_{\text{DM}}$  drops to much annihilation not effective anymore

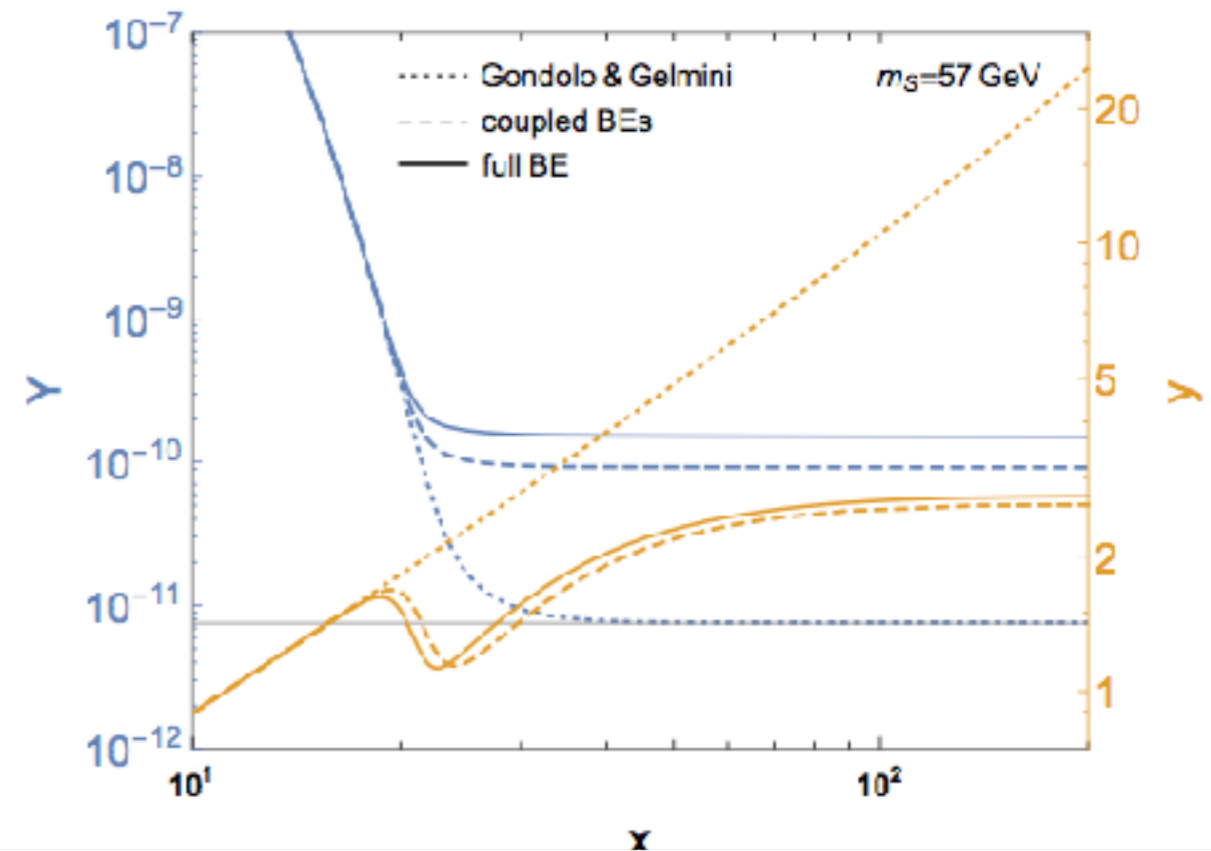
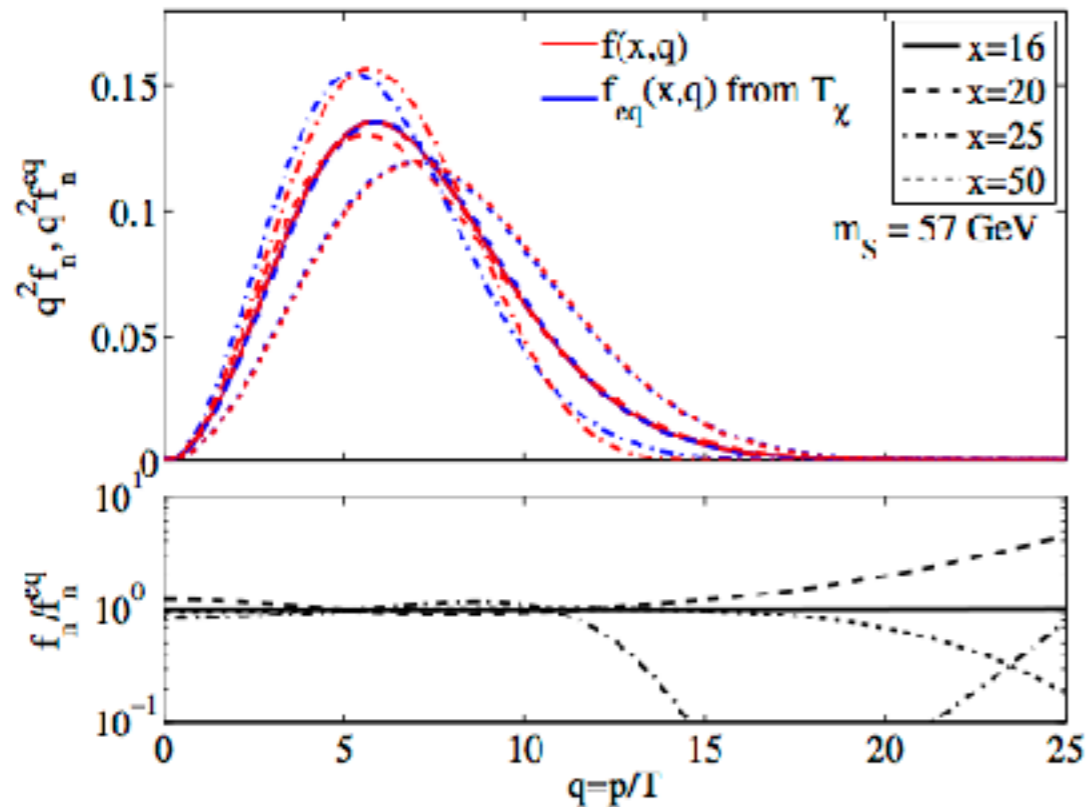
# WHY SPIKES IN $T_{KD}$ ?



Effect resembling first order „phase transition” —  
**artificial** as dependent on a particular choice of  $T_{KD}$  definition

# FULL PHASE-SPACE BE SOLVER

Solutions for full **phase-space distribution function**:



Results of both approaches compatible:  
 some **deviation from equilibrium shape** mildly affects the  $Y$  and  $y$  evolution

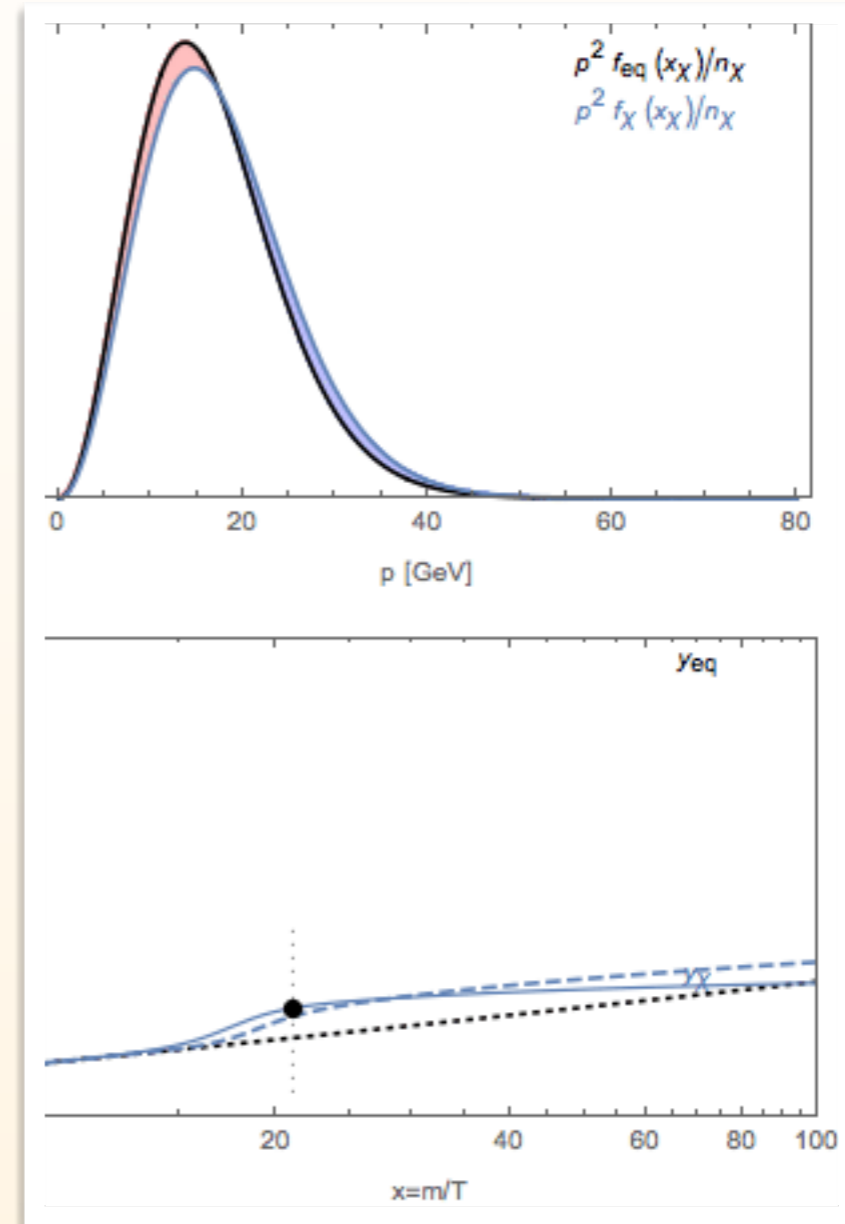
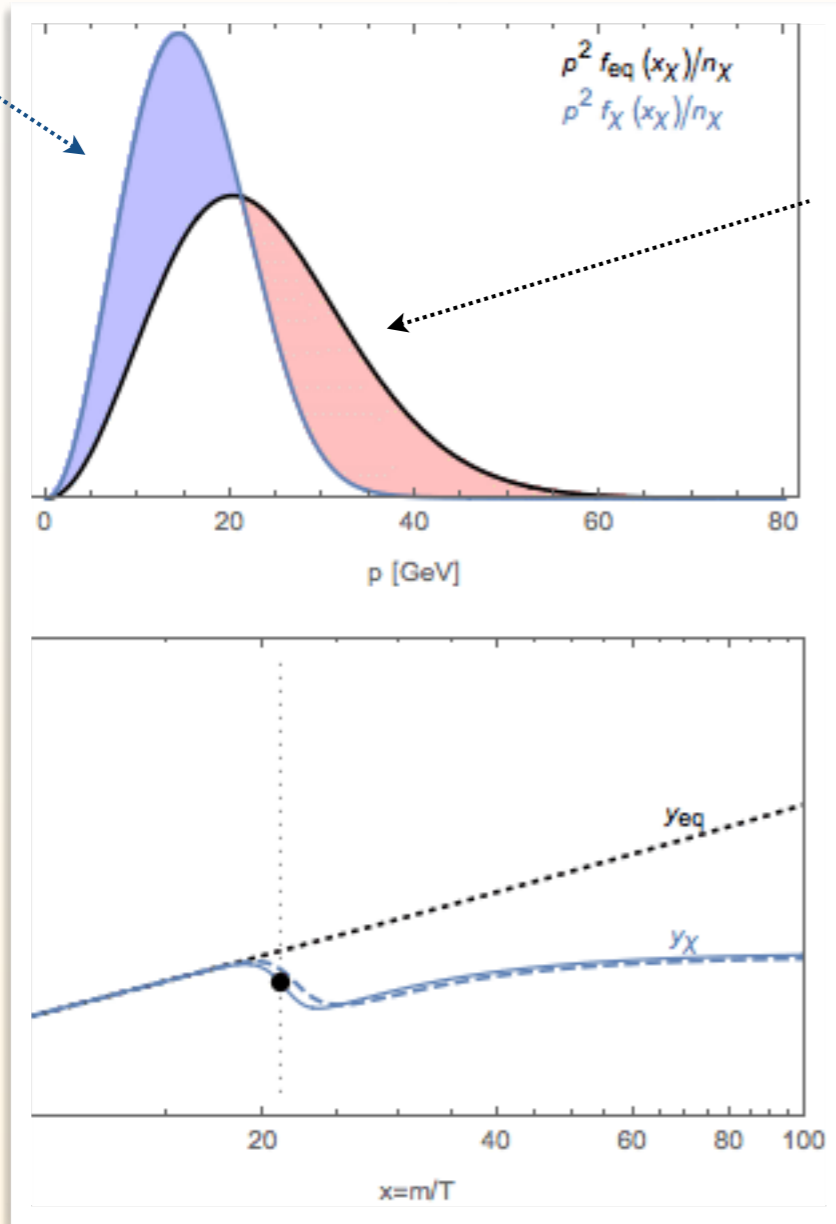
Allows to study the evolution of  $f_\chi(p)$  and the interplay between scatterings and annihilation!

# FULL PHASE-SPACE EVOLUTION

$m_{DM} = 58 \text{ GeV}$

$m_{DM} = 62.5 \text{ GeV}$

blue - full solution for  $f_{DM}$  at  $T_{DM}$



significant deviation from equilibrium shape **already around freeze-out**

large deviations **at later times**, around freeze-out not far from eq. shape

→ effect on relic density largest, **both from different T and  $f_{DM}$**

→ effect on relic density **~only from different T**

# KD BEFORE CD?

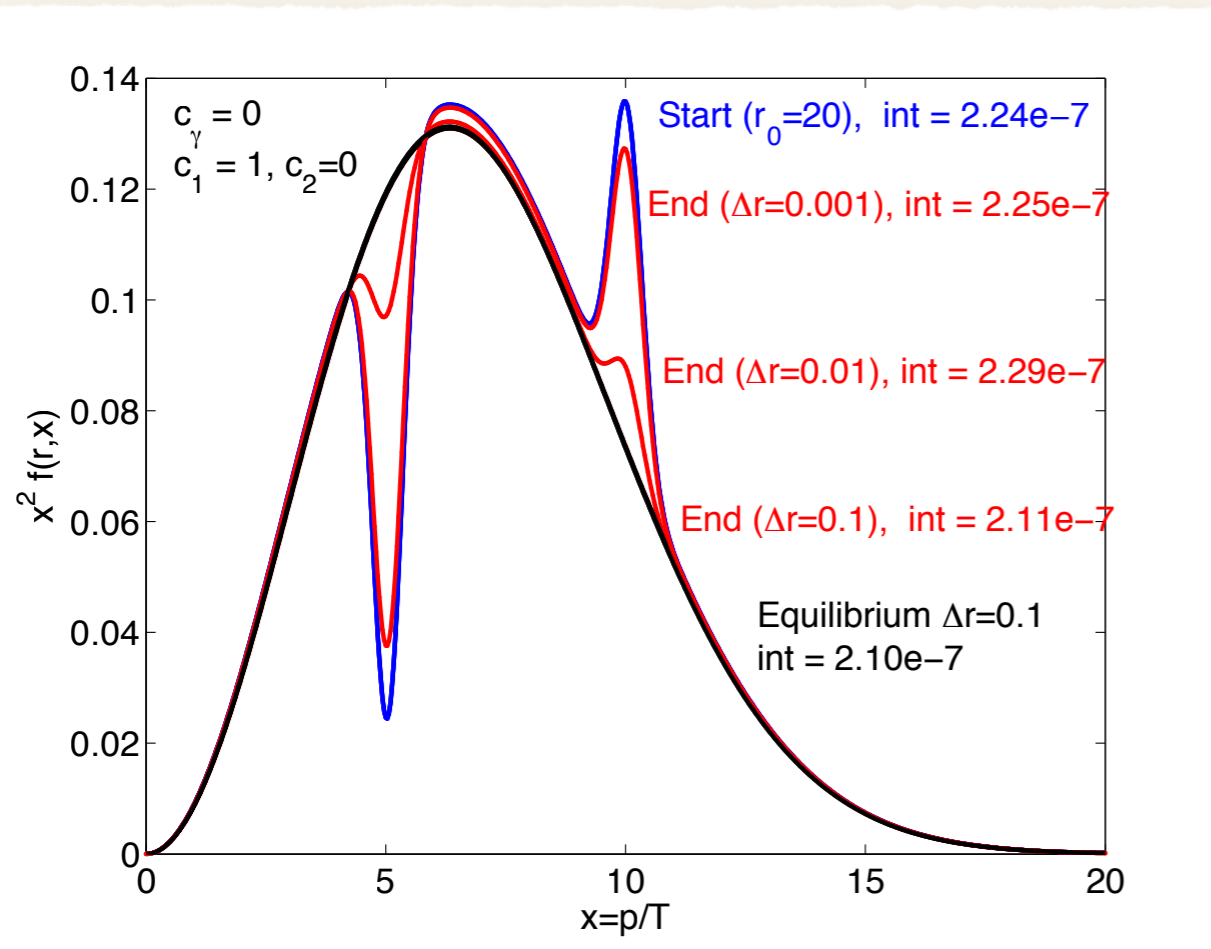
Obvious issue:

How to define exactly the **kinetic** and **chemical** decouplings and what is the significance of such definitions?



Improved question:

Can **kinetic** decoupling happen much earlier than **chemical**?



we have already seen that even if scatterings were very inefficient compared to annihilation, departure from equilibrium for both  $Y$  and  $y$  happened **around the same time**...

← turn off scatterings and take s-wave annihilation;  
look at local disturbance

annihilation/production processes drive to  
restore **kinetic equilibrium**!

# CONCLUSIONS

1. One needs to remember that **kinetic equilibrium** is a necessary assumption for standard relic density calculations
2. Coupled **system of Boltzmann equations for 0th and 2nd moments** allow for a very accurate treatment of the kinetic decoupling and its effect on relic density
3. In special cases the **full phase space Boltzmann equation** can be necessary — especially if one wants to trace DM temperature as well

## **Exception N:**

kinetic decoupling can happen together with freeze-out...



# TAKEAWAY MESSAGE

**When computing relic density of dark matter one needs carefully to check if the standard treatment is sufficient for the case at hand**

*”Everything should be made as simple as possible, but no simpler.”*

attributed to\* Albert Einstein

\*The published quote reads:

”It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.”

„On the Method of Theoretical Physics” ,The Herbert Spencer Lecture, delivered at Oxford (10 June 1933); also published in *Philosophy of Science*, Vol. 1, No. 2 (April 1934), pp. 163-169., p. 165

God Jul

