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DEVELOPMENTS IN DARK MATTER RELIC ABUNDANCE CALCULATIONS

Andrzej Hryczuk

NCBJ

based on:

T. Binder, T. Bringmann, M. Gustafsson & A.H. 1706.07433, 2103.01944

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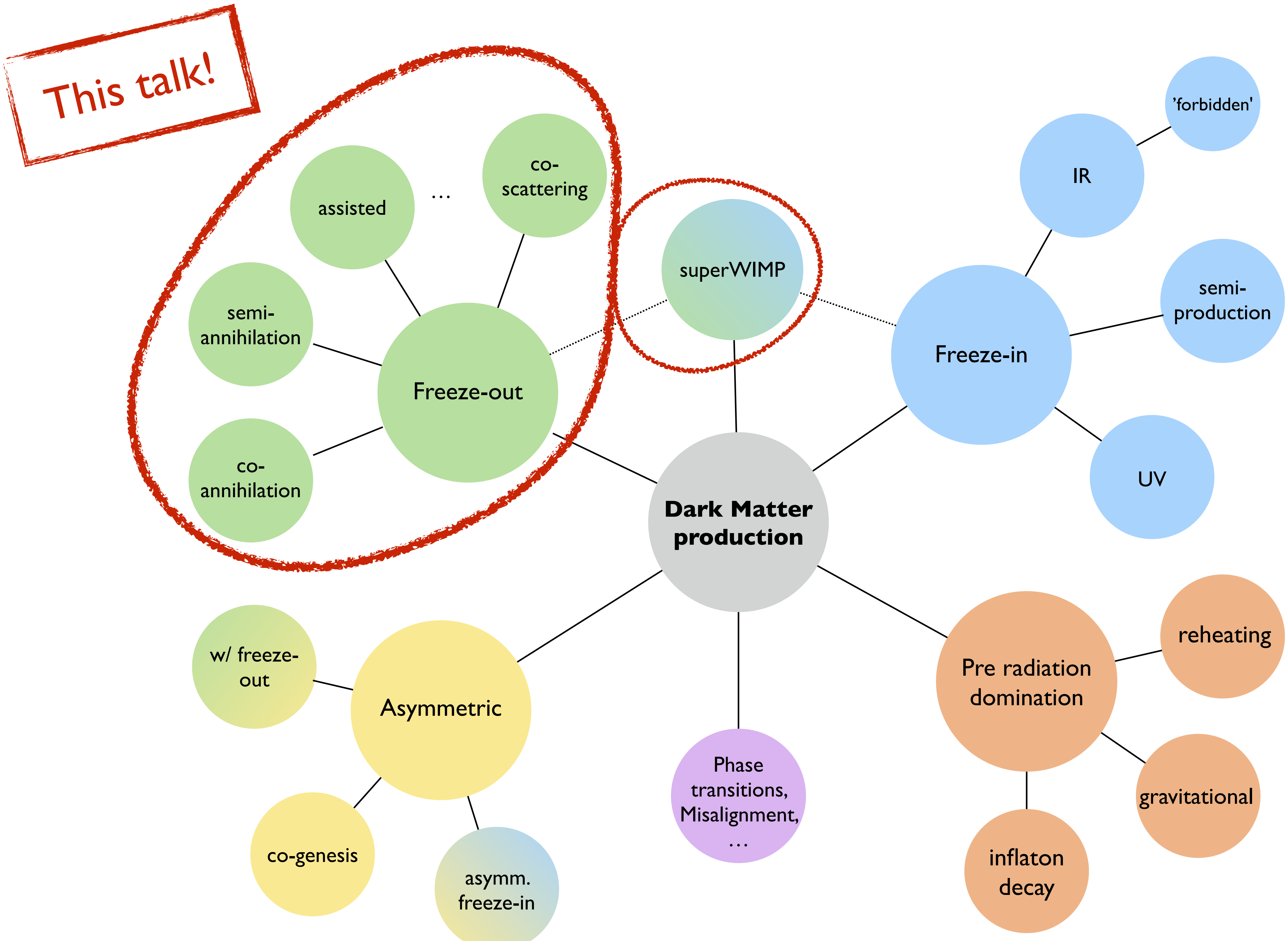
1st April 2022

Dark Matter origin



Motivation
Thermal Relic Density

Theory:

I. Natural

Comes out automatically from the expansion of the Universe

Naturally leads to cold DM

II. Predictive

No dependence on initial conditions

Fixes coupling(s) $\Rightarrow$ signal in DD, ID & LHC

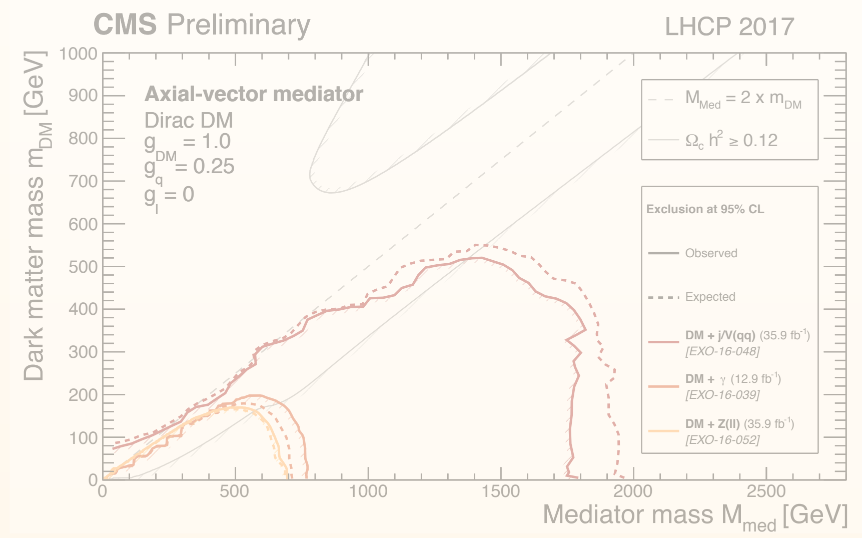
III. It is <u>not</u> optional

Overabundance constraint

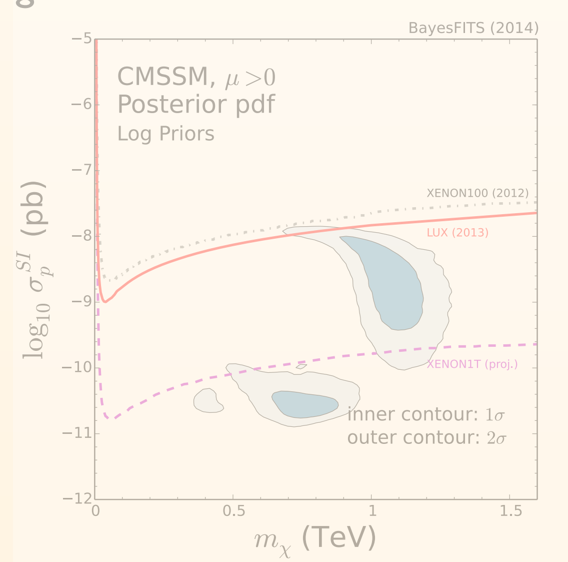
To avoid it one needs quite significant deviations from standard cosmology

Experiment:

…as a constraint:



…as a target:



"(…) besides the Higgs boson mass measurement and LHC direct bounds, the constraint showing **by far the strongest impact** on the parameter space of the MSSM is the **relic density**"

Roszkowski et al. '14

…as a pin:

When a dark matter signal is (finally) found: relic abundance can pin-point the particle physics interpretation

Thermal Relic Density

Standard Scenario

(chemical and kinetic)

T

$\Gamma_{\rm ann} > H$	DM in full equilibrium	time
$\Gamma_{\rm ann} \sim H$	chemical decoupling	
$\Gamma_{\rm ann} < H$	freeze-out	
$\Gamma_{\rm scatt} \sim H$	kinetic decoupling	

non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$\underbrace{\boxed{E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right)}}_{\text{Liouville operator in FRW background}} f_\chi = \underbrace{\mathcal{C}[f_\chi]}_{\text{the collision term}}$$

THERMAL RELIC DENSITY

STANDARD APPROACH

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq: classical limit, molecular chaos,...

...for derivation from thermal QFT see e.g., 1409.3049

$\Downarrow$ integrate over p (i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar\chi\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar\chi} - n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}\right)$$

where the thermally averaged cross section:

$$\langle\sigma_{\chi\bar\chi\to ij}v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}}\int\frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\,\sigma_{\chi\bar\chi\to ij}v_{\rm rel}\; f_\chi^{\rm eq} f_{\bar\chi}^{\rm eq}$$

$\Downarrow$

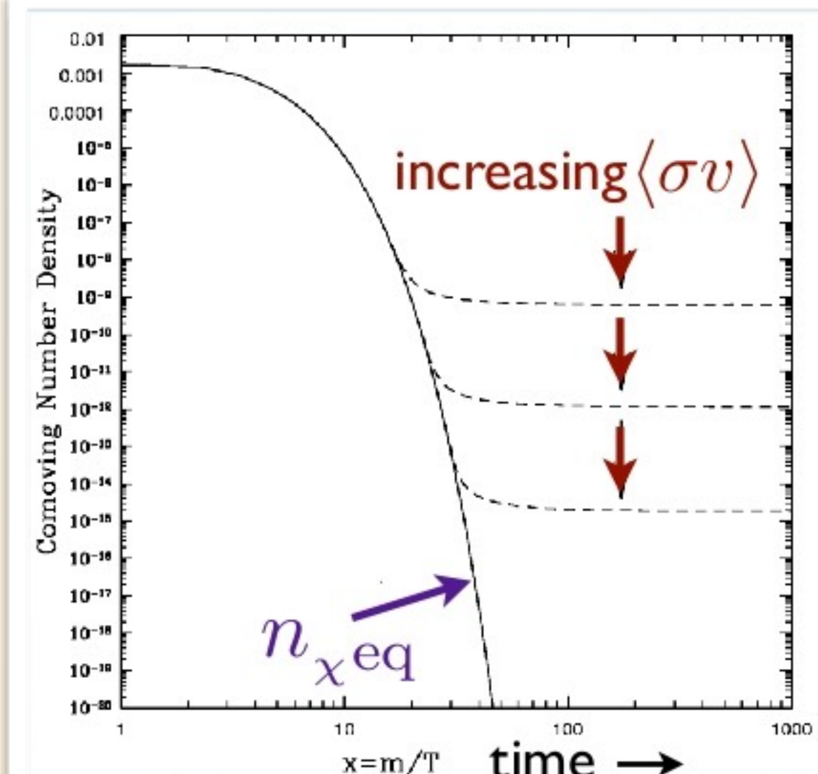


Fig.: Jungman, Kamionkowski & Griest, PR'96

Critical assumption:

kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\rm eq}$$

Historical Prelude

Three Exceptions

Griest & Seckel '91

1. Co-annihilations

if more than one state share a conserved quantum number making DM stable

$$\langle \sigma_{\text{eff}} \text{v} \rangle = \sum_{ij} \langle \sigma_{ij} \text{v}_{ij} \rangle \frac{n_i^{\text{eq}} n_j^{\text{eq}}}{n_{\text{eq}}^2}$$

with: $\sigma_{ij} = \sum_X \sigma(\chi_i \chi_j \to X)$

e.g., SUSY

2. Annihilation to forbidden channels

if DM is slightly below mass threshold for annihilation $\Longrightarrow$ „forbidden" channel can still be accessible in thermal bath

recent e.g., 1505.07107

3. Annihilation near poles

expansion in velocity (s-wave, p-wave, etc.) not safe

(more historical issue: these days most people use numerical codes)

Thermal Relic Density

modern "Exceptions"

1. Non-standard cosmology

   many works… very recent e.g., D'Eramo, Fernandez, Profumo '17

2. Bound State Formation

   recent e.g., Petraki at al. '15, '16; An et al. '15, '16; Cirelli et al. '16; …

3. $3 \rightarrow 2$ and $4 \rightarrow 2$ annihilation

   e.g., D'Agnolo, Ruderman '15; Cline at al. '17; Choi at al. '17; …

4. Second era of annihilation

   Feng et al. '10; Bringmann et al. '12; …

5. Semi-annihilation

   D'Eramo, Thaler '10; …

6. Cannibalization

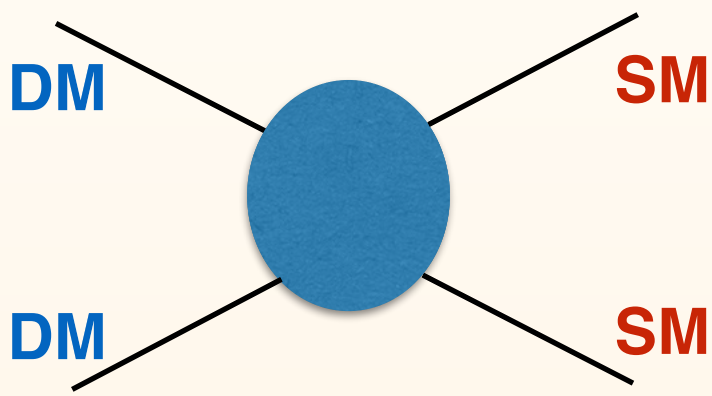
   e.g., Kuflik et al. '15; Pappadopulo et al. '16; …

7. …

…in other words: whenever studying non-minimal scenarios "exceptions" appear

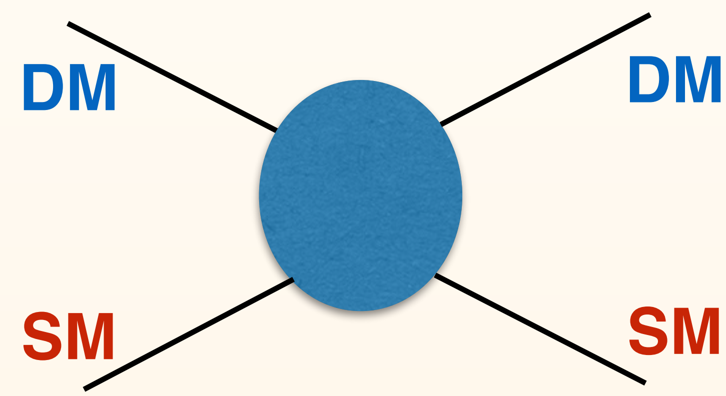
WHAT IF A NON-MINIMAL SCENARIO?

In a minimal WIMP case only two types of processes are relevant:



annihilation

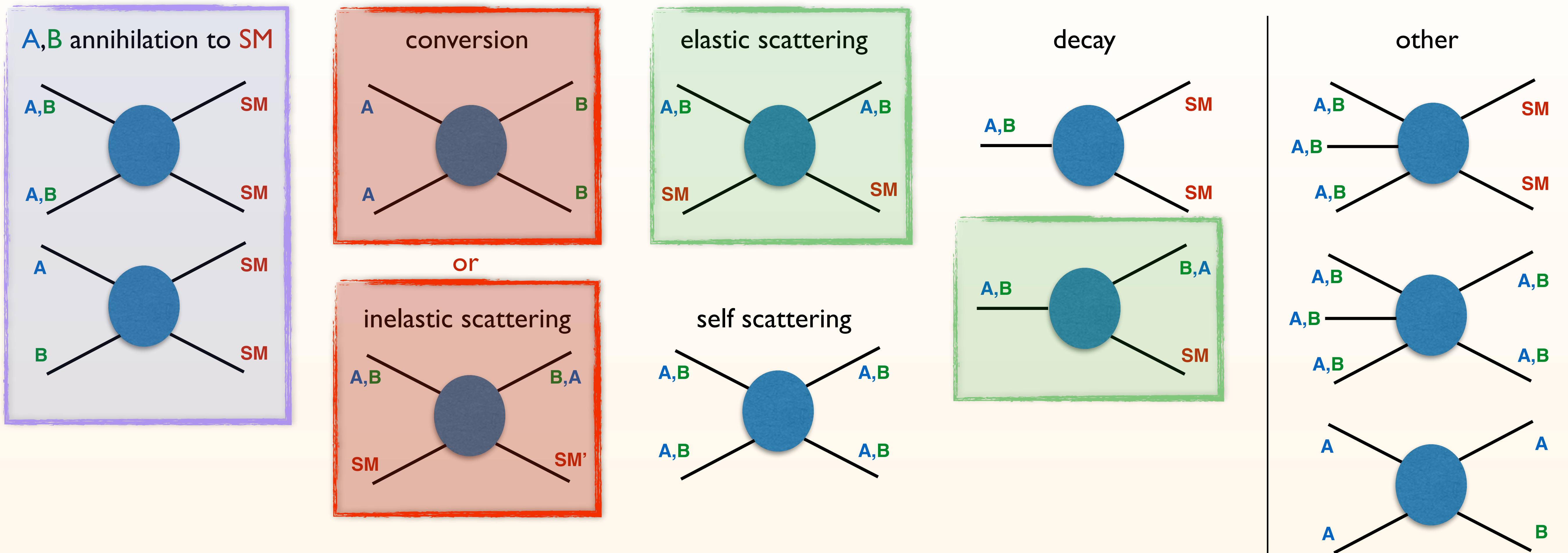
drives number density evolution



(elastic) scattering

assumed to be very efficient
(keeping the distribution to be in local thermal eq.)

WHAT IF A NON-MINIMAL SCENARIO?



+…..

typically forbidden by symmetry

Co-annihilation →

Griest, Seckel '91

due to efficient conversion processes one can trace only number density of sum of the states with shared conserved quantum number using weighted annihilation cross section

needed to be efficient for mechanism to work

drives the freeze-out (i.e. first efficient, then stops)

assumed in computation

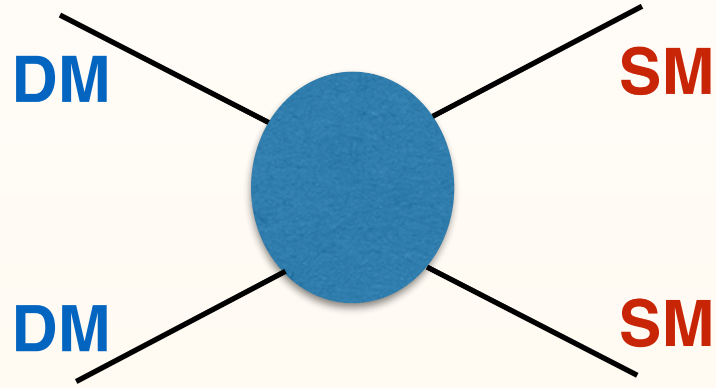
What if non-minimal scenario?

Example: assume two particles in the dark sector: A and B

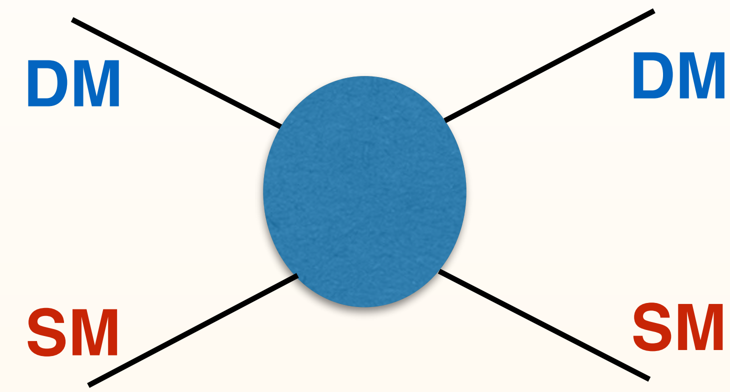
scenario / process	Co-annihilation	superWIMP	Co-decaying	Conversion-driven/ Co-scattering	Cannibal/Semi-annihilation	Forbidden-like	…
annihilation A A <-> SM SM A B <-> SM SM B B <-> SM SM	**first efficient then stops**						
conversion A A <-> B B inelastic scattering A SM <-> B SM	**has to be extremely efficient**						
elastic scattering A SM <-> A SM B SM <-> B SM	**assumed to be <u>very</u> efficient**						in all scenarios **kinetic equilibrium** assumption crucial, but not always "automatic"!
el. self-scattering A A <-> A A B B <-> B B							
decays A <-> B SM A <-> SM SM B <-> SM SM							
semi-ann/3->2 A A A <-> A A A A <-> A B A A A <-> SM A							

Freeze-out *vs.* Decoupling

annihilation | (elastic) scattering



crossing sym.



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Rightarrow$ scatterings typically more frequent

dark matter frozen-out but typically
still kinetically coupled to the plasma

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Recall: in *standard* thermal relic density calculation:

Critical assumption:
kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(\mu) f_\chi^{\text{eq}}$$

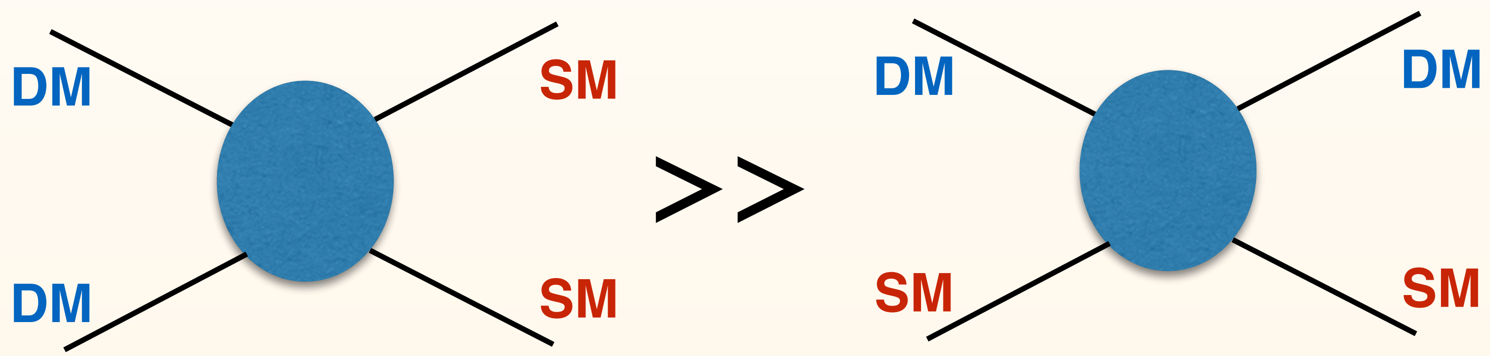
Early Kinetic Decoupling?

A necessary and sufficient condition: <u>scatterings weaker than annihilation</u>

i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:

A)



e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

D) Multi-component dark sectors

e.g., additional sources of DM from late decays, …

How to go beyond kinetic equilibrium?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

contains both scatterings and annihilations

Two possible approaches:

fBE

cBE

0-th moment: n_χ
2-nd moment: T_χ
$\ldots$

solve numerically for full $f_\chi(p)$

have insight on the distribution
no constraining assumptions

numerically challenging
often an overkill

consider system of equations for moments of $f_\chi(p)$

partially analytic/much easier numerically
manifestly captures all of the relevant physics

finite range of validity
no insight on the distribution

Kinetic Decoupling 101

DM temperature Definition:

$$T_\chi \equiv \frac{g_\chi}{3 m_\chi n_\chi} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} p^2 f_\chi(p) \qquad y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$$

actually: normalized average NR energy - equals temperature at equilibrium

First take consider only temperature evolution - then 2nd moment of full BE (up to terms p^2/m_χ^2) gives:

$$\boxed{\frac{y'}{y} = -\cancel{\frac{Y'}{Y}\left(1 - \frac{\langle \sigma v_{\rm rel}\rangle_2}{\langle \sigma v_{\rm rel}\rangle}\right)} - \left(1 - \frac{x}{3}\frac{g_{*S}'}{g_{*S}}\right)\frac{2 m_\chi c(T)}{H x}\left(1 - \frac{y_{\rm eq}}{y}\right)}$$

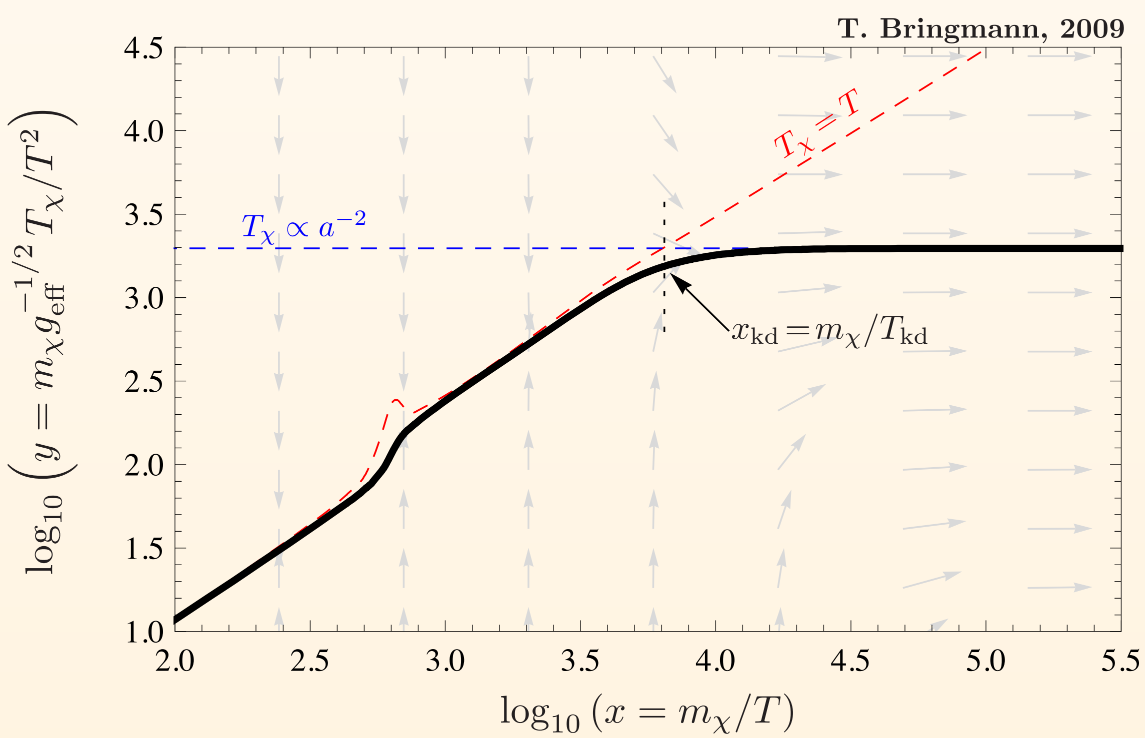
where:

$$\langle \sigma v_{\rm rel}\rangle_2 \equiv \frac{g_\chi^2}{3 T m_\chi n_\chi^2} \int \frac{d^3 p}{(2\pi)^3} \int \frac{d^3 \tilde{p}}{(2\pi)^3}\, p^2 v_{\rm rel} \sigma_{\bar{\chi}\chi \to \bar{X}X} f(E) f(\tilde{E})$$

impact of annihilation

$$c(T) = \frac{1}{12 (2\pi)^3 m_\chi^4 T} \sum_X \int dk\, k^5 \omega^{-1} g^{\pm} \left(1 \mp g^{\pm}\right) \int_{-4k^2}^{0} (-t) \frac{1}{8 k^4} |\mathcal{M}_{\rm el}|^2$$

impact of elastic scatterings



ONE STEP FURTHER…

Now consider general KD scenario, i.e. coupled temperature and number density evolution:

annihilation and production thermal averages done at different T — feedback of modified y evolution

$$\frac{Y'}{Y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx} sY \left(\langle \sigma v_{\rm rel} \rangle|_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\rm eq}^2}{Y^2} \langle \sigma v_{\rm rel} \rangle|_x \right)$$

$$\frac{y'}{y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx} \left[\underline{2m_\chi c(T)\left(1-\frac{y_{\rm eq}}{y}\right)} - sY \left(\left(\langle \sigma v_{\rm rel} \rangle - \langle \sigma v_{\rm rel} \rangle_2 \right)_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\rm eq}^2}{Y^2} \left(\langle \sigma v_{\rm rel} \rangle - \frac{y_{\rm eq}}{y} \langle \sigma v_{\rm rel} \rangle_2 \right)_x \right) \right]$$

$$+ \underline{\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{3m_\chi} \langle p^4/E^3 \rangle_{x=m_\chi^2/(s^{2/3}y)}}$$

"relativistic" pressure term

$$T_\chi \equiv \frac{g_\chi}{3n_\chi} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{E} f_\chi(p)$$

elastic scatterings term

impact of annihilation

These equations still assume the equilibrium shape of $f_\chi(p)$ — but with variant temperature

or more accurately: that the thermal averages computed with true non-equilibrium distributions don't differ much from the above ones

Numerical approach

… or one can just solve full phase space Boltzmann eq.

$$\begin{aligned}\partial_x f_\chi(x,q) &= \frac{m_\chi^3}{\tilde{H}x^4}\frac{g_{\bar\chi}}{2\pi^2}\int d\tilde q\,\tilde q^2\,\frac{1}{2}\int d\cos\theta\; v_{\mathrm{Møl}}\sigma_{\bar\chi\chi\to\bar f f}\\ &\quad\times\left[f_{\chi,\mathrm{eq}}(q)f_{\chi,\mathrm{eq}}(\tilde q)-f_\chi(q)f_\chi(\tilde q)\right]\\ &+\frac{2m_\chi c(T)}{2\tilde H x}\left[x_q\partial_q^2+\left(q+\frac{2x_q}{q}+\frac{q}{x_q}\right)\partial_q+3\right]f_\chi\\ &+\tilde g\frac{q}{x}\partial_q f_\chi,\end{aligned}$$

fully general

expanded in NR and small momentum transfer (semi-relativistic!)

discretization, ~1000 steps

$$\begin{aligned}\partial_x f_i &= \\ &\frac{m_\chi^3}{\tilde{H}x^4}\frac{g_{\bar\chi}}{2\pi^2}\sum_{j=1}^{N-1}\frac{\Delta\tilde q_j}{2}\Big[\tilde q_j^2\,\langle v_{\mathrm{Møl}}\sigma_{\bar\chi\chi\to\bar f f}\rangle_{i,j}^{\theta}\left(f_i^{\mathrm{eq}}f_j^{\mathrm{eq}}-f_if_j\right)\\ &+\tilde q_{j+1}^2\,\langle v_{\mathrm{Møl}}\sigma_{\bar\chi\chi\to\bar f f}\rangle_{i,j+1}^{\theta}\left(f_i^{\mathrm{eq}}f_{j+1}^{\mathrm{eq}}-f_if_{j+1}\right)\Big]\\ &+\frac{2m_\chi c(T)}{2\tilde H x}\left[x_{q,i}\partial_q^2+\left(q_i+\frac{2x_{q,i}}{q_i}+\frac{q_i}{x_{q,i}}\right)\partial_q+3\right]f_i\\ &+\tilde g\frac{q_i}{x}\partial_q f_i,\end{aligned}$$

Solved numerically

Note:

can be extended to e.g. self-scatterings

very stiff, care needed with numerics

NEW TOOL!

GOING BEYOND THE STANDARD APPROACH

- Home
- Downloads
- Contact

DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.
DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,**
  Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]

Currently, an user guide can be found in the Appendix A of this reference.
Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for
any (user defined) model*

Interplay between chemical and kinetic decoupling

Prediction for the DM
phase space distribution

Late kinetic decoupling
and impact on cosmology

see e.g., 1202.5456

…

(only) prerequisite:
Wolfram Language (or *Mathematica*)

*at the moment for a single DM species and w/o co-annihlations… but stay tuned for extensions!

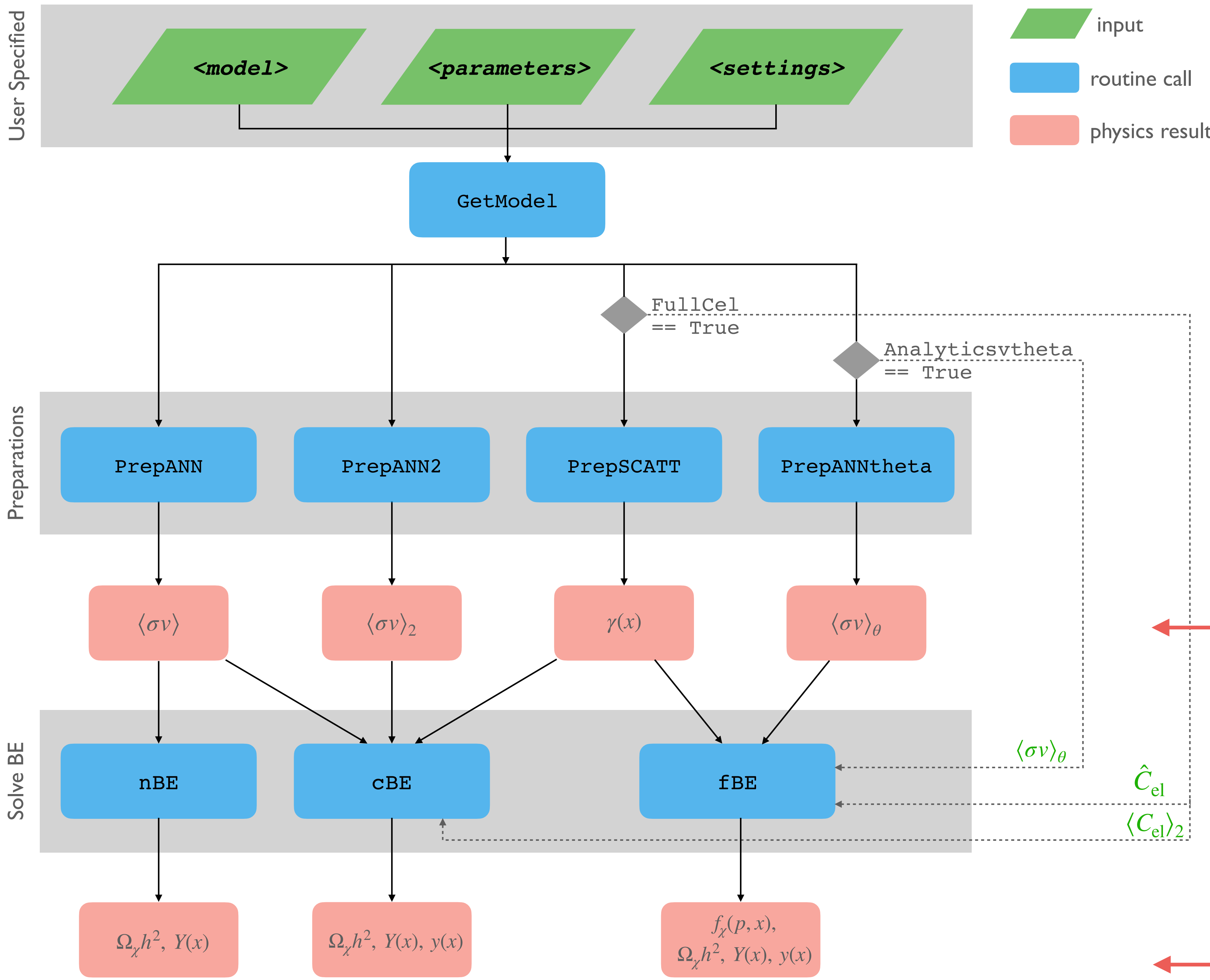
DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

https://drake.hepforge.org

Few words about the Code

written in *Wolfram Language*, lightweight, modular and simple to use both via script and front end usage



<u>Input</u>: model file (few models implemented, user extendable), parameter values and settings choices

<u>Preparation phase</u>: all needed thermal averages etc.

[slowest part (~ few sec - some min), can be bypassed by user own routines/results…]

← (intermediate) results for rates

<u>Final phase</u>: tailored solvers for Boltzmann eq.

[~ few seconds]

← (final) results for $\Omega h^2, f_{DM}(p)$

Dark matter Relic Abundance beyond Kinetic Equilibrium

https://drake.hepforge.org

SNAPSHOTS FROM AN EXAMPLE NOTEBOOK

1. Load DRAKE

```
Needs["DRAKE`"]
```

2. Initialize model

```
GetModel["WIMP", "bm1", "settings_bm1"]
----------------- Model: WIMP-like toy model ----------------
{-------- card: bm1 mDM=100.  gDM=1  sv0=1.6877e-9  xkd=25.--------}
```

3. Run

nBE

```
PrepANN;
nBE
Oh2nBE = 0.12
```

cBE

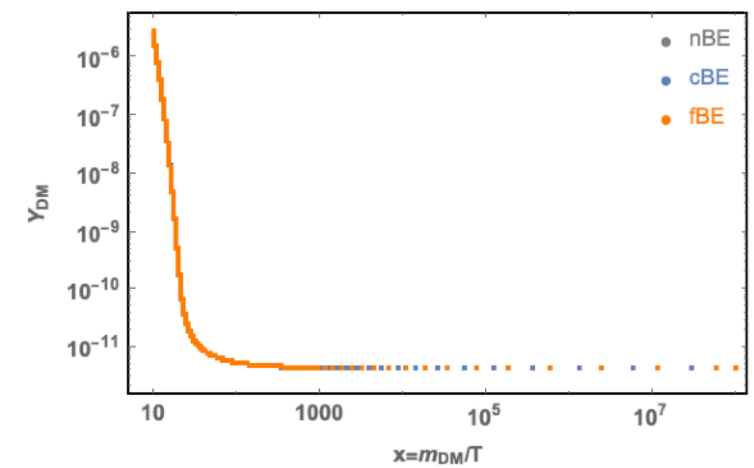
```
If[! scatttype == "gamma(x)" && ! FullCel, PrepSCATT];
(* PrepANN; *) (* uncomment if not called earlier *)
PrepANN2;
cBE
Oh2cBE = 0.120013
```

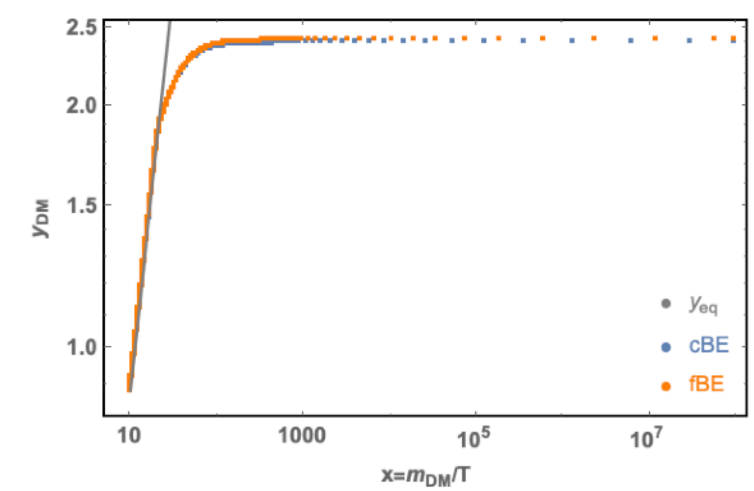
fBE

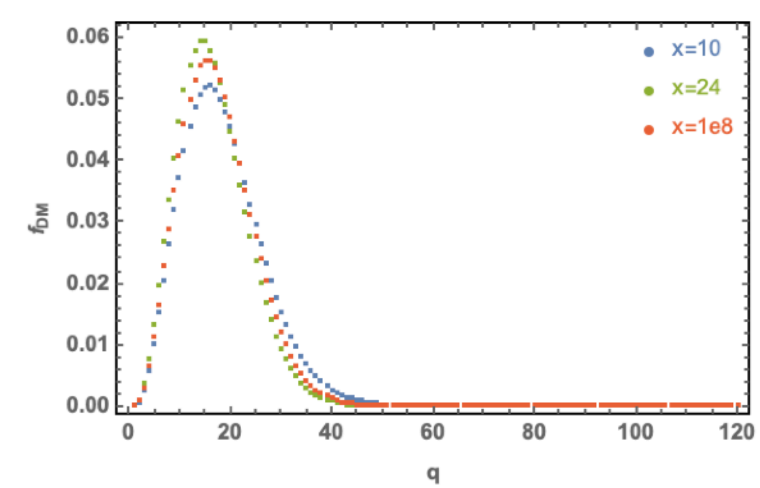
```
PrepANNtheta; RegArrayGen[tsvtheta];
fBE
Oh2fBE = 0.120037
```

4. Print plots

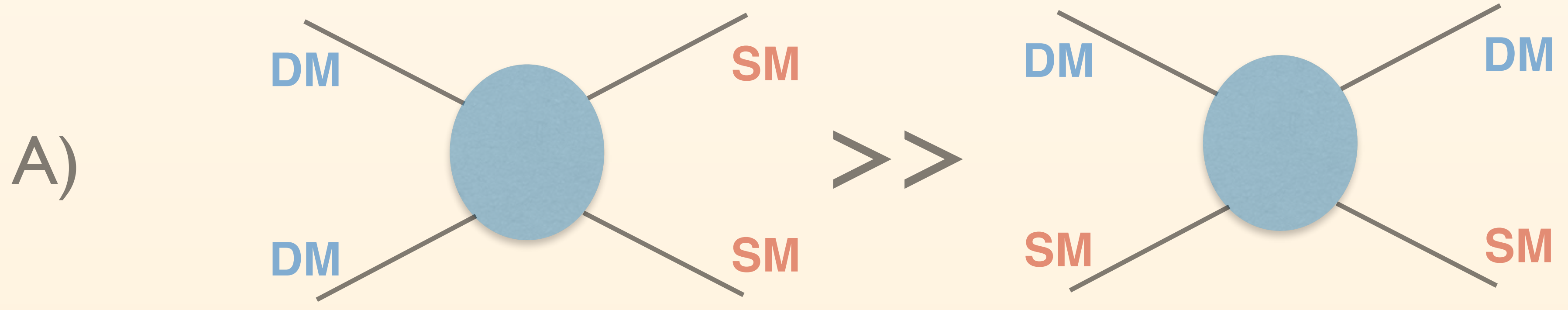
```
(* Print out result plots *)
MakePlots
```







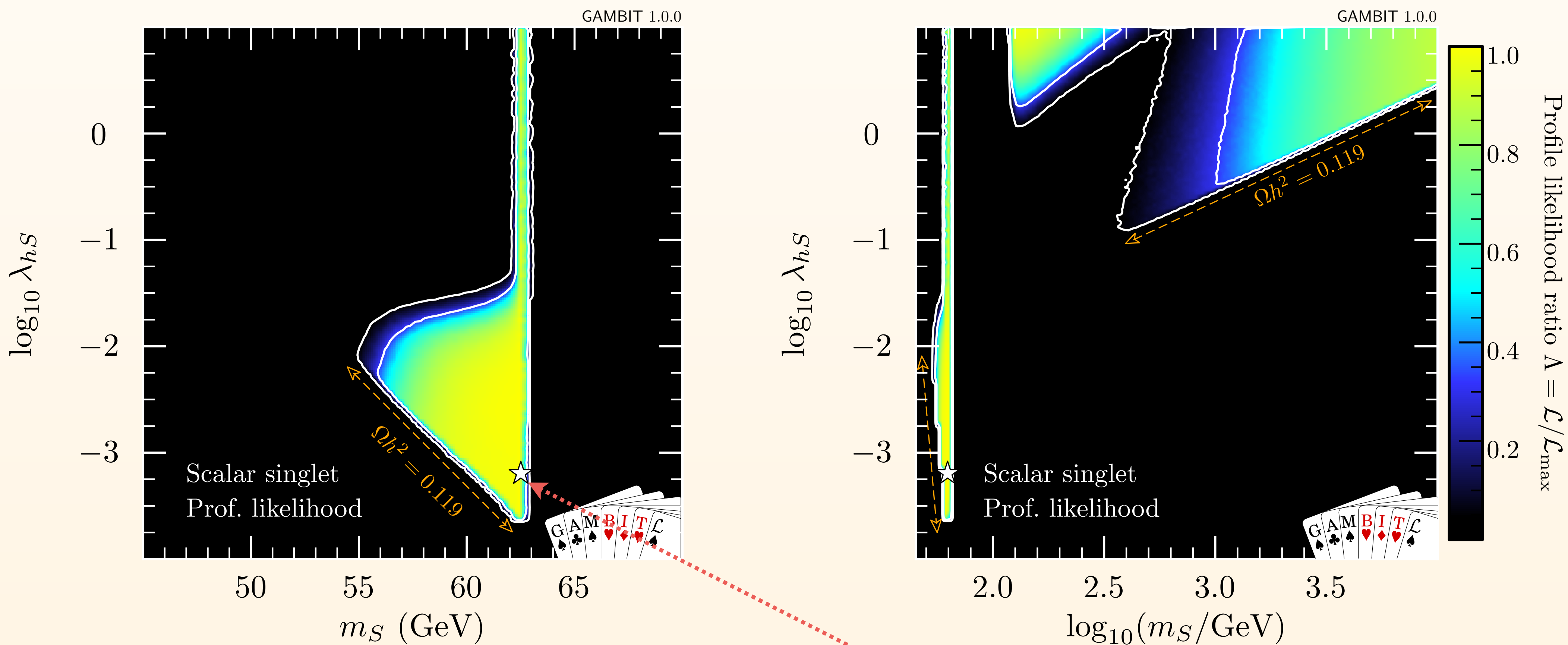
EXAMPLE A:
SCALAR SINGLET DM



EXAMPLE A
SCALAR SINGLET DM

To the SM Lagrangian add one singlet scalar field *S* with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_s S^2 |H|^2 \qquad m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$



GAMBIT collaboration
1705.07931

Most of the parameter space excluded, but… even such a simple model is hard to kill

best fit point hides in the resonance region!

SCALAR SINGLET DM

ANNIHILATION VS. SCATTERINGS

$$\sigma v_{\rm rel} = \frac{2\lambda_s^2 v_0^2}{\sqrt{s}} |D_h(s)|^2 \Gamma_h(\sqrt{s})$$

tabulated
Higgs width

with:

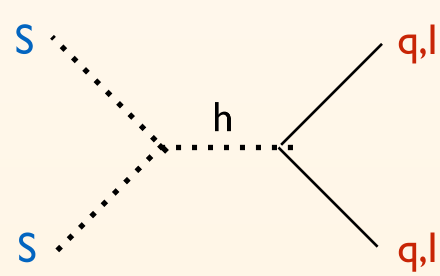
$$|D_h(s)|^2 \equiv \frac{1}{(s-m_h^2)^2 + m_h^2 \Gamma_h^2(m_h)} \,.$$

$$\left\langle |\mathcal{M}|^2 \right\rangle_t = \sum_f \frac{N_f \lambda_S^2 m_f^2}{8k^4} \left[\frac{2k_{\rm cm}^2 - 2m_f^2 + m_h^2}{1 + m_h^2/(4k_{\rm cm}^2)} - \left(m_h^2 - 2m_f^2\right) \log\left(1 + 4k_{\rm cm}^2/m_h^2\right) \right].$$

Hierarchical Yukawa couplings: strongest coupling to more Boltzmann suppressed quarks/leptons

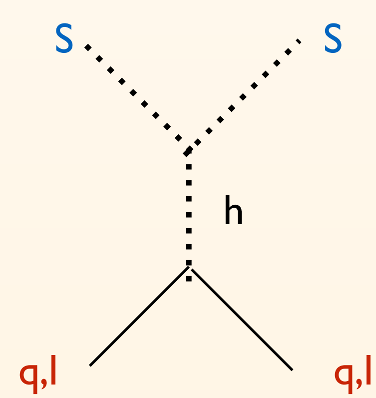
Annihilation processes:

resonant



El. scattering processes:

non-resonant



Freeze-out at few GeV $\longrightarrow$ what is the abundance of heavy quarks in QCD plasma?

two scenarios:

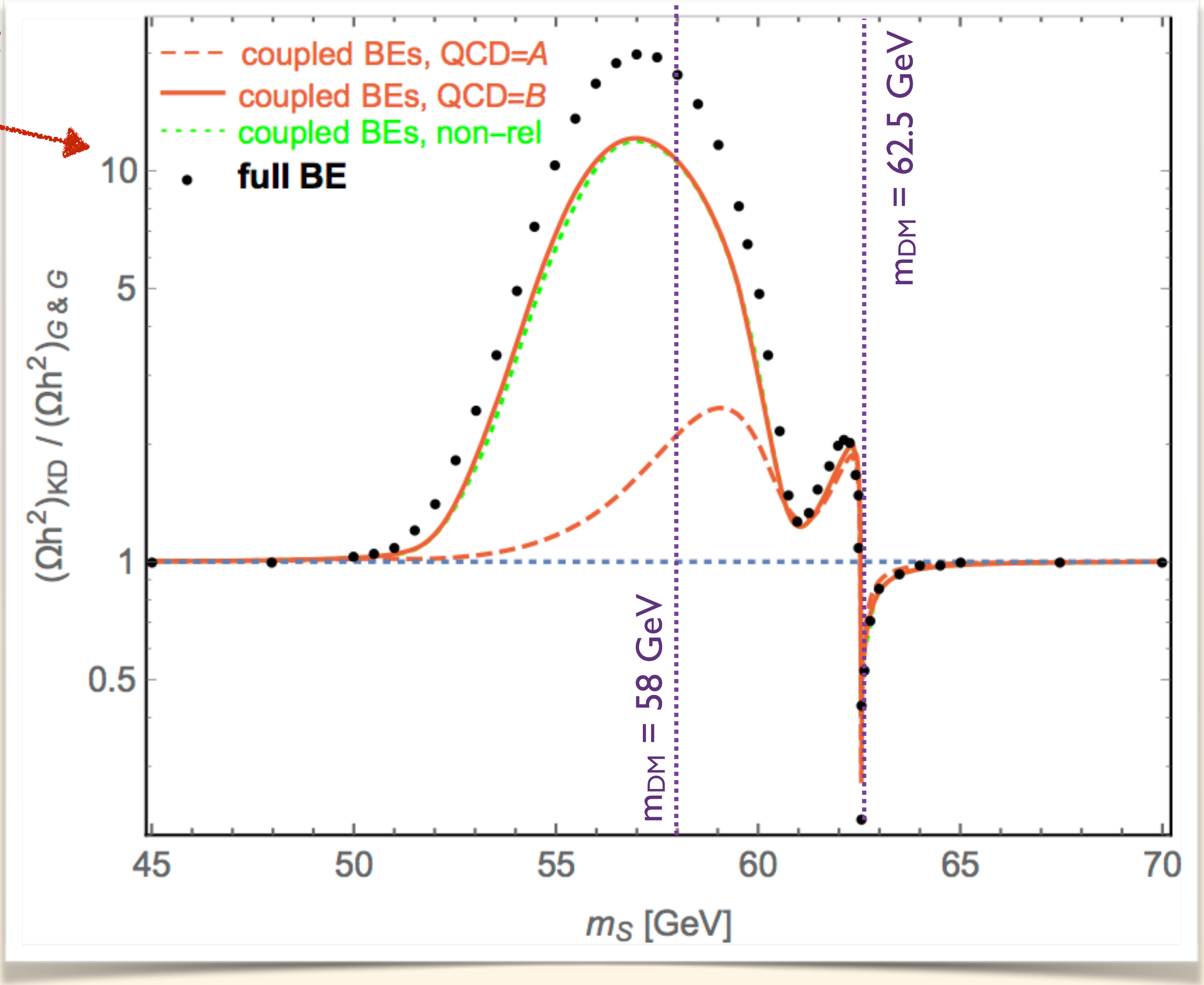
QCD = A - all quarks are free and present in the plasma down to T_c =154 MeV

QCD = B - only light quarks contribute to scattering and only down to $4T_c$

RESULTS

EFFECT ON THE Ωh^2

effect on relic density:
up to O(~10)



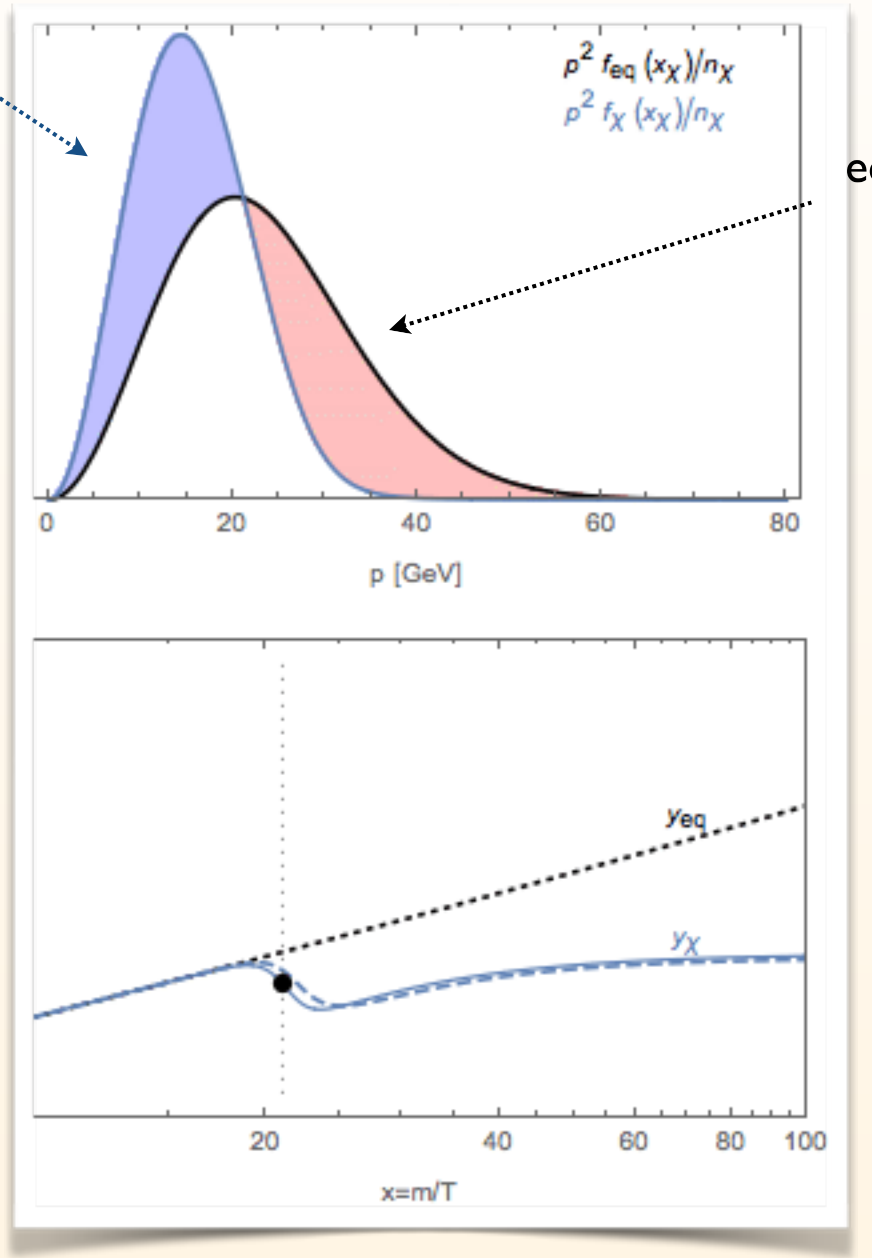
[… Freeze-out at few GeV ⟶ what is the abundance of heavy quarks in QCD plasma?

two scenarios: QCD = A - all quarks are free and present in the plasma down to T_c = 154 MeV
QCD = B - only light quarks contribute to scattering and only down to $4T_c$ …]

FULL PHASE-SPACE EVOLUTION

m_{DM} = 58 GeV

blue - full solution for f_{DM} at T_{DM}

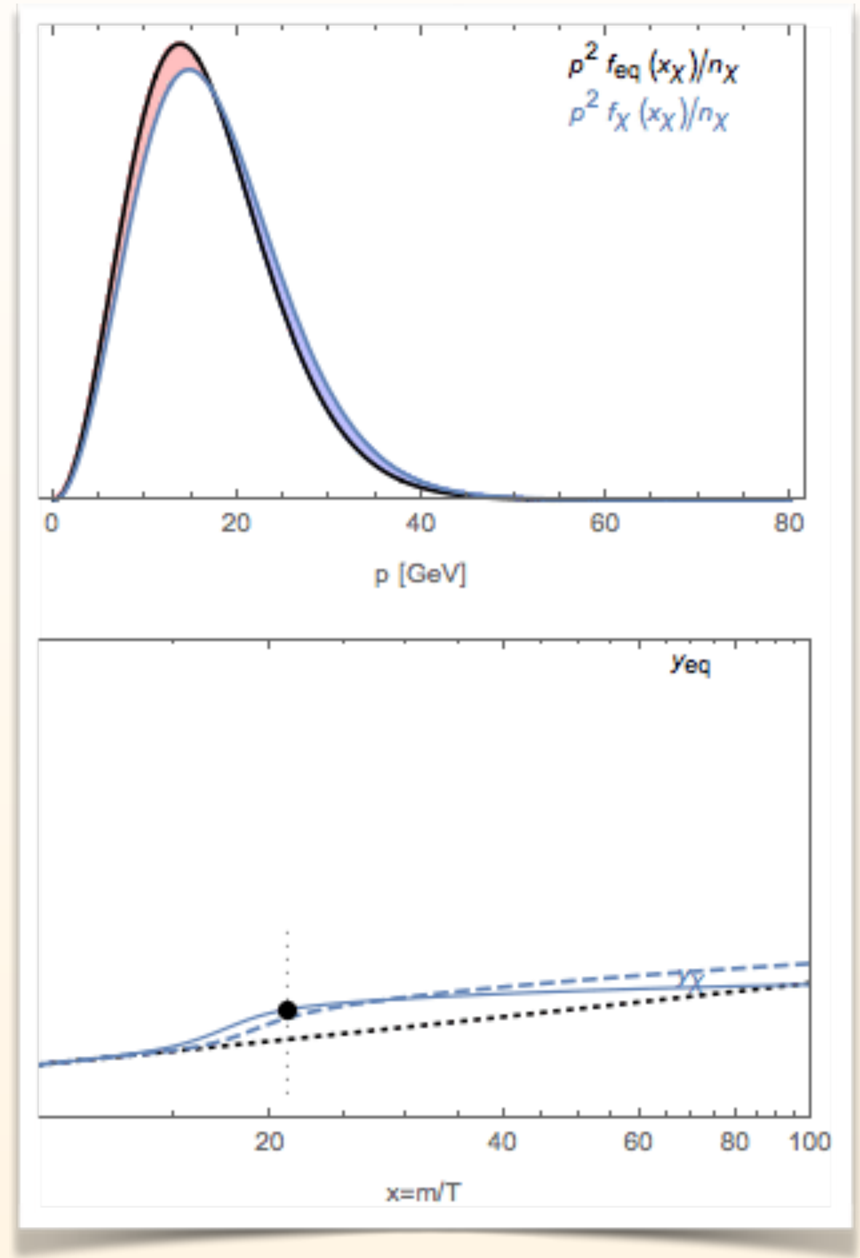


black - equilibrium at T_{DM}

significant deviation from equilibrium shape already around freeze-out

⟶ effect on relic density largest, both from different T and f_{DM}

m_{DM} = 62.5 GeV



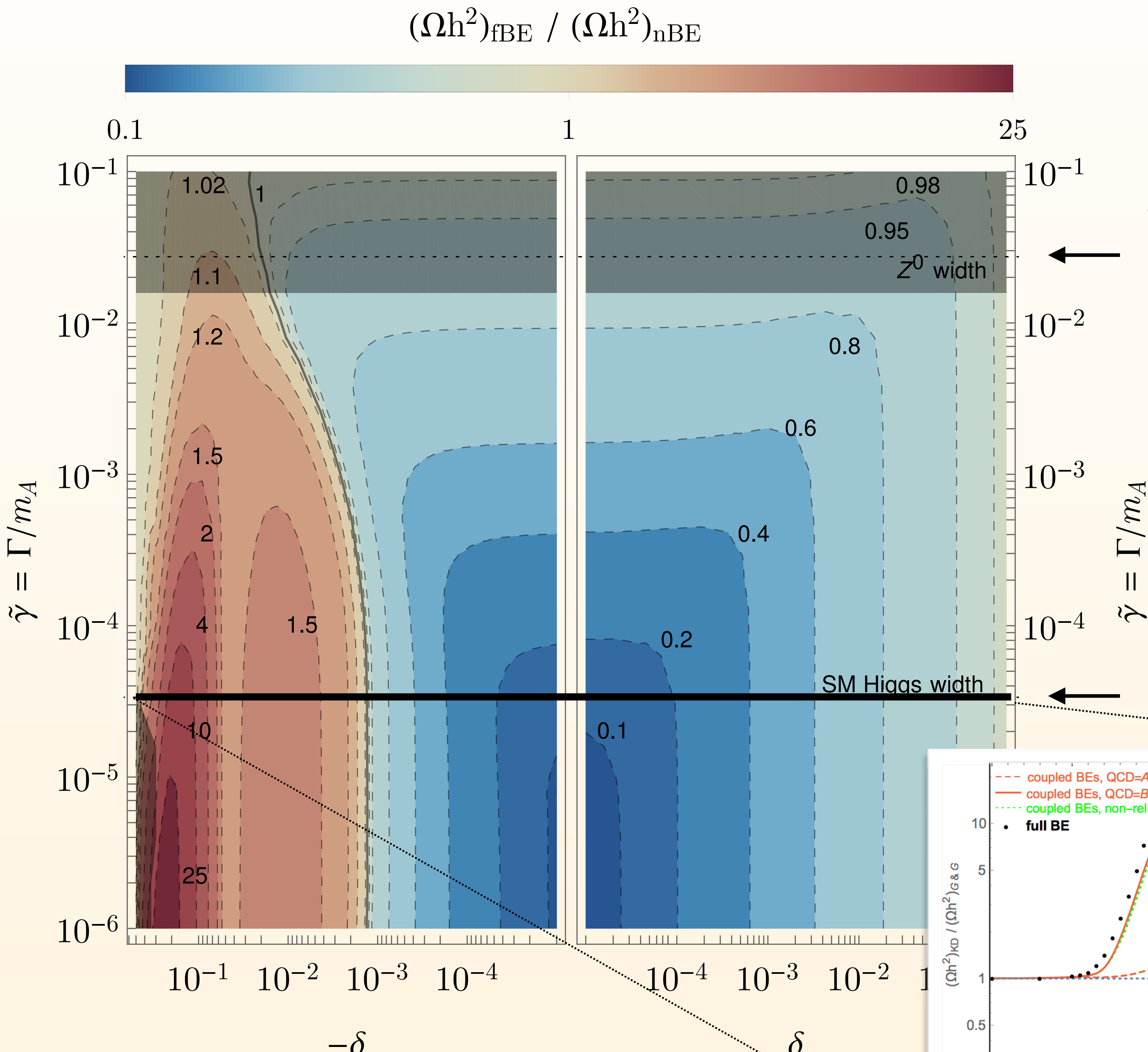
large deviations at later times, around freeze-out not far from eq. shape

⟶ effect on relic density ~only from different T

GENERIC RESONANT ANNIHILATION

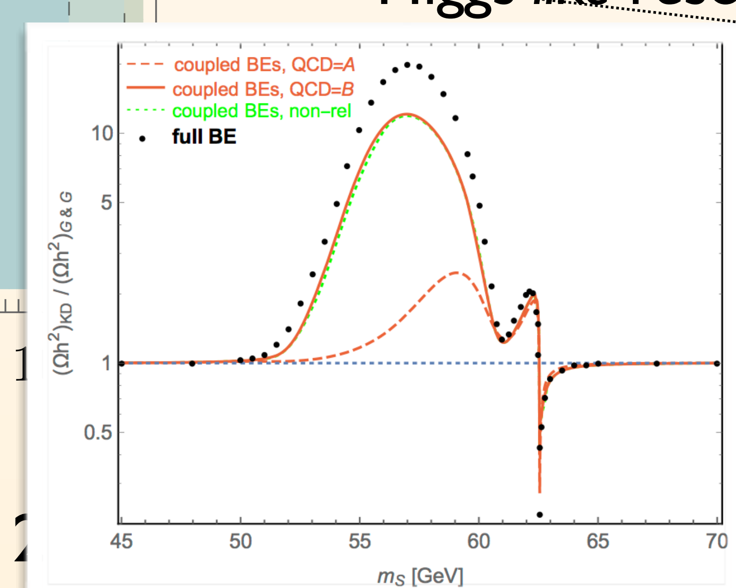
EXAMPLE EFFECT OF EARLY KD ON RELIC DENSITY

resonance **width** normalized to m_{DM}



can reach O(10%) even for *Z-like* resonance

very large effect for Higgs-*like* resonance



distance in mass from the exact condition 2

cBE vs. fBE

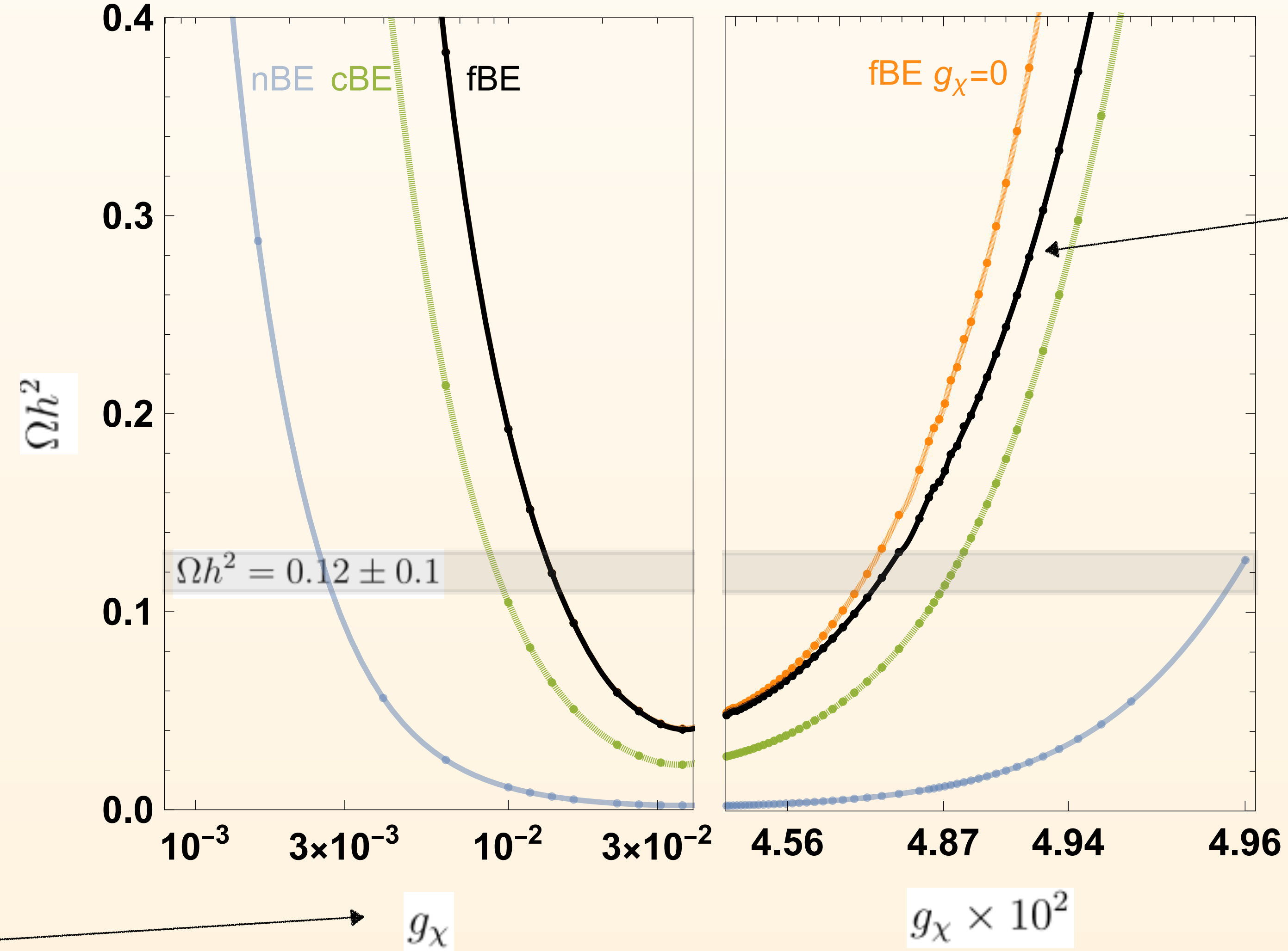
which is more accurate?!

They correspond to the opposite limits of self-interaction strengths:

very efficient - **cBE**

inefficient - f**BE**

Which limit is closer to reality depends on the model, but (from what we looked at) it seems that fBE is typically more accurate, unless self-scattering is tuned up, e.g:



black line gives the result including self-scattering processes! (being between pure fBE and cBE)

coupling to mediator; governs self-scatterings

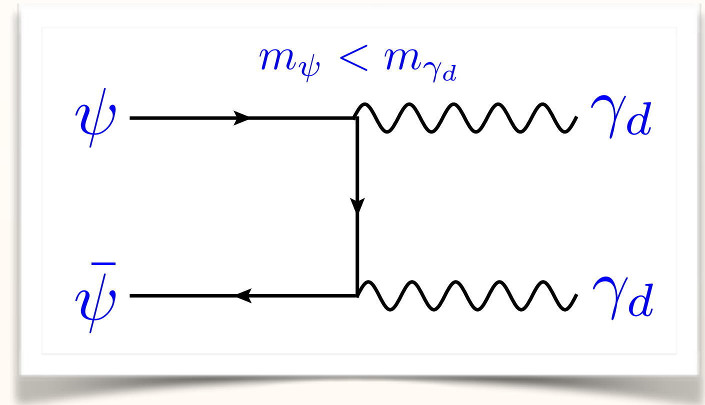
Example B:
Forbidden DM

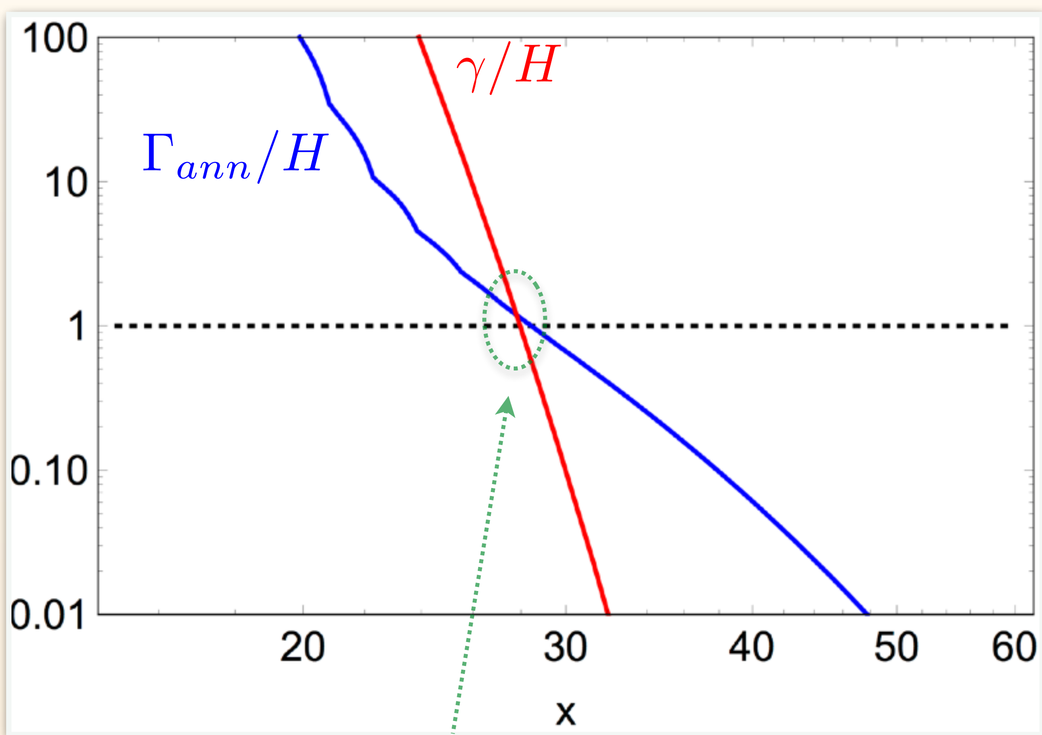
B) Boltzmann suppression of SM as strong as for DM

EXAMPLE B
FORBIDDEN DARK MATTER

DM is a thermal relic that annihilates <u>only</u> to heavier states
(forbidden in zero temperature)

…, D'Agnolo, Ruderman '15, …





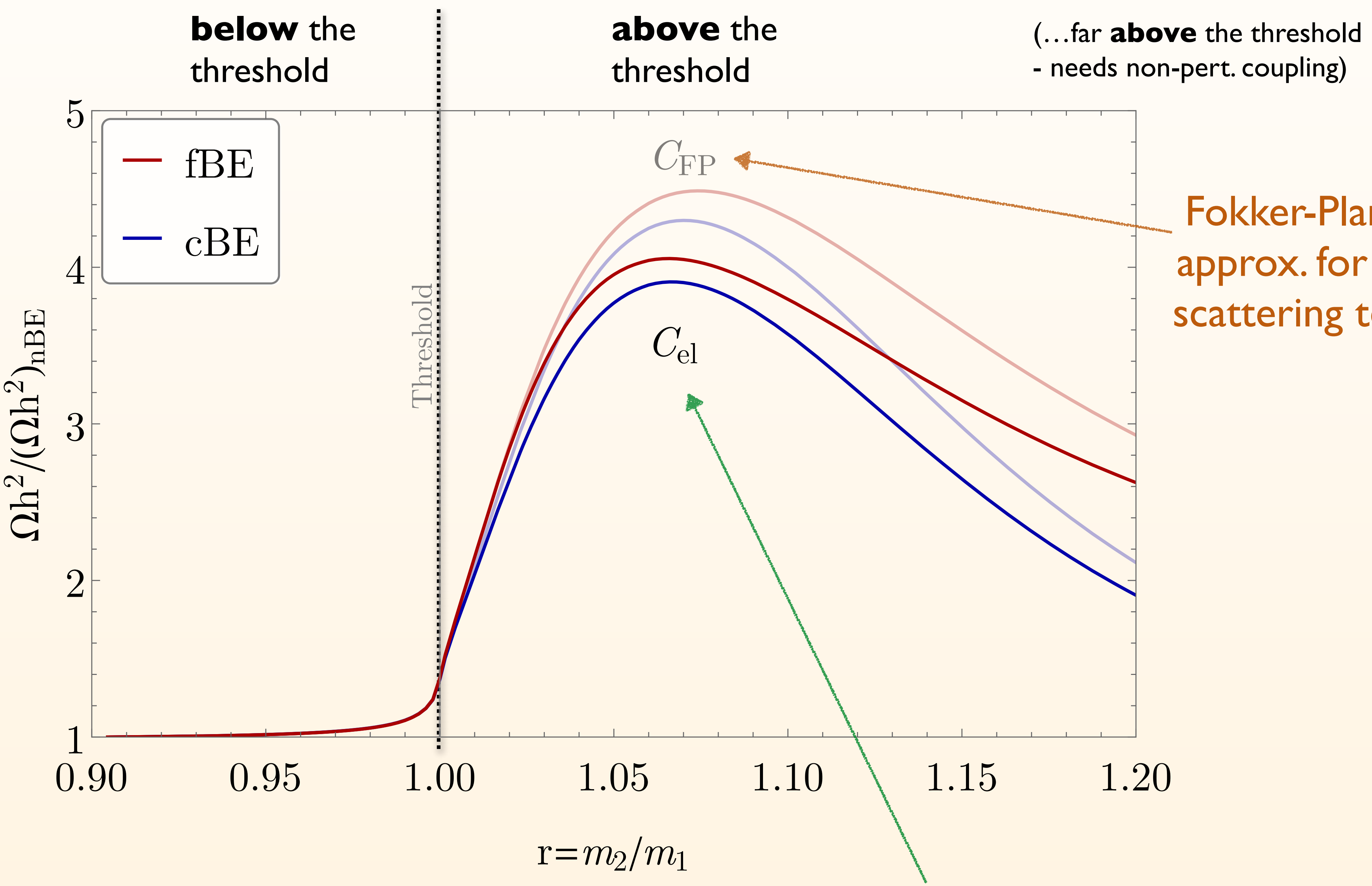
kinetic and chemical
decoupling close

Annihilation threshold → velocity dependence

"heavy" SM particle → scattering rate low

Forbidden Dark Matter

Example effect of early KD on relic density



effect on relic density:
up to O(~few)

Fokker-Planck approx. for the scattering term

full (numerical) scattering term

EXAMPLE C:
SEMI-ANNIHILATION

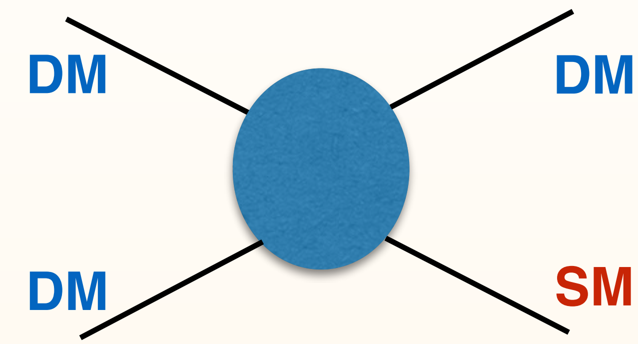
C) Scatterings and annihilation have different structure

Dark Matter Semi-Annihilation

and its simplest realization

DM is a thermal relic but with freeze-out governed by the semi-annihilation process

D’Eramo, Thaler ’10; …



Z_3 complex scalar singlet:

$$V = \mu_H^2 |H|^2 + \lambda_H |H|^4 + \mu_S^2 |S|^2 + \lambda_S |S|^4 + \lambda_{SH} |S|^2 |H|^2 + \frac{\mu_3}{2}(S^3 + S^{\dagger 3}).$$

just above the Higgs threshold semi-annihilation dominant!

Belanger, Kannike, Pukhov, Raidal ’13

very weak elastic scatterings → semi-annihilation by itself does not equilibrate DM

↓

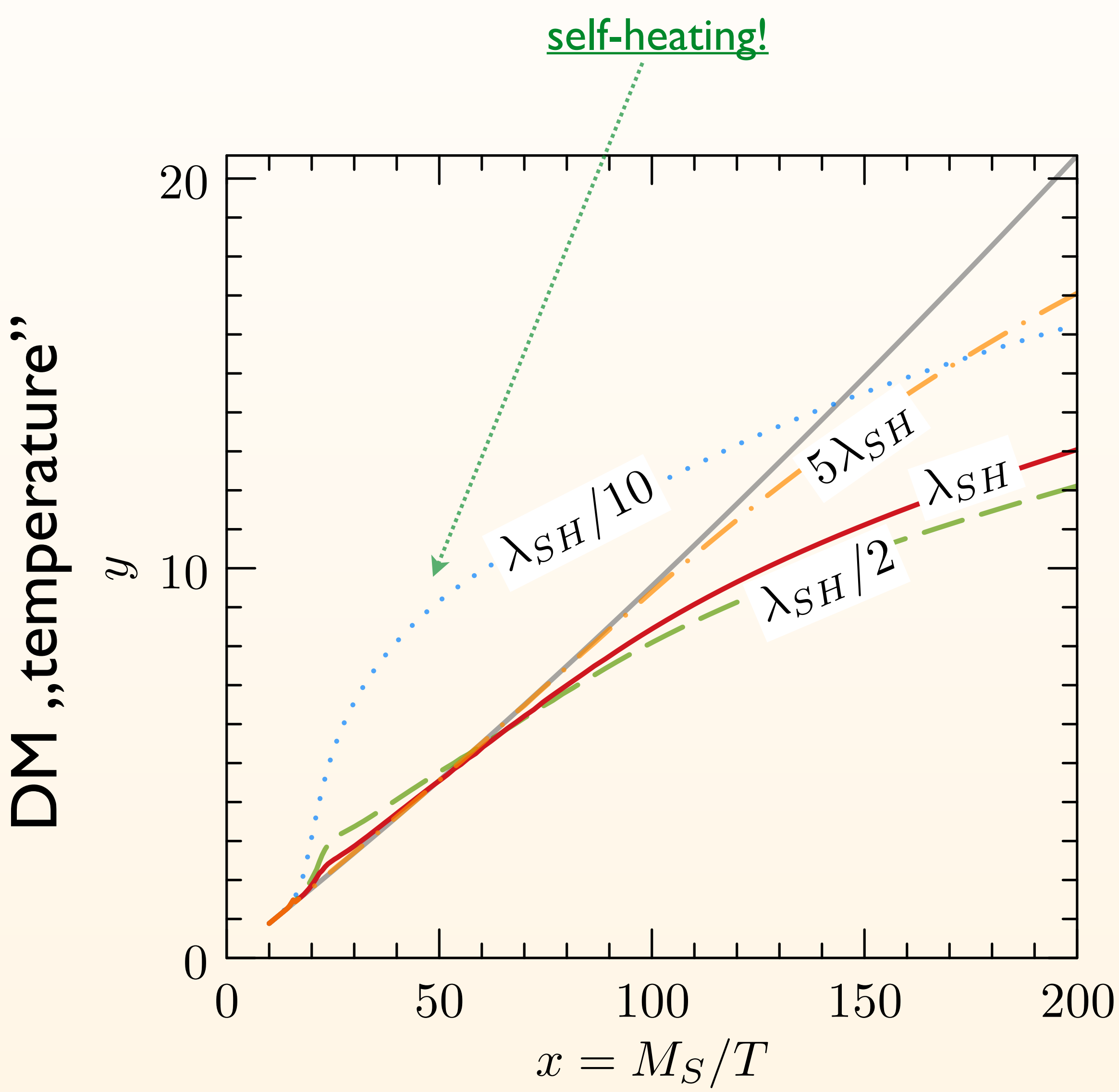
but rather leads to self-heating! ← **implications for ID**

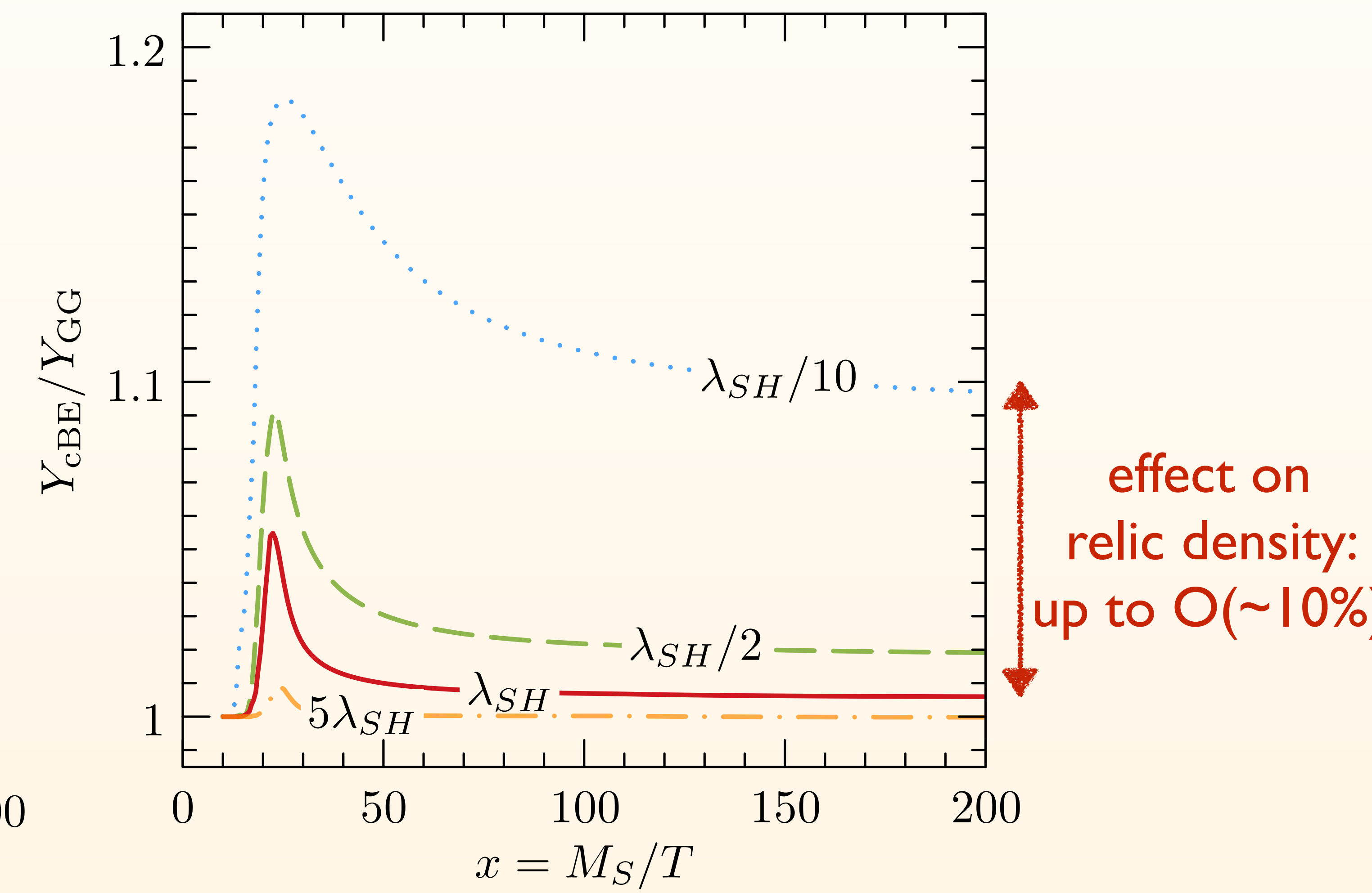
see also Cai, Spray 1807.00832

Semi-Annihilation

Example effect on early KD on relic density

A. Hektor, AH and K. Kannike 1901.08074





Note: here the final effect is relatively mild (though still larger than the observational error), but only because in the simplest model the velocity dependence of annihilation is mild as well…

*calculated with the (modified) coupled BEs method

EXAMPLE D:
WHEN ADDITIONAL INFLUX OF DM ARRIVES

D) Multi-component dark sectors

Sudden injection of more DM particles distorts $f_\chi(p)$
(e.g. from a decay or annihilation of other states)

- this can modify the annihilation rate (if still active)
- how does the thermalization due to elastic scatterings happen?

EXAMPLE EVOLUTION

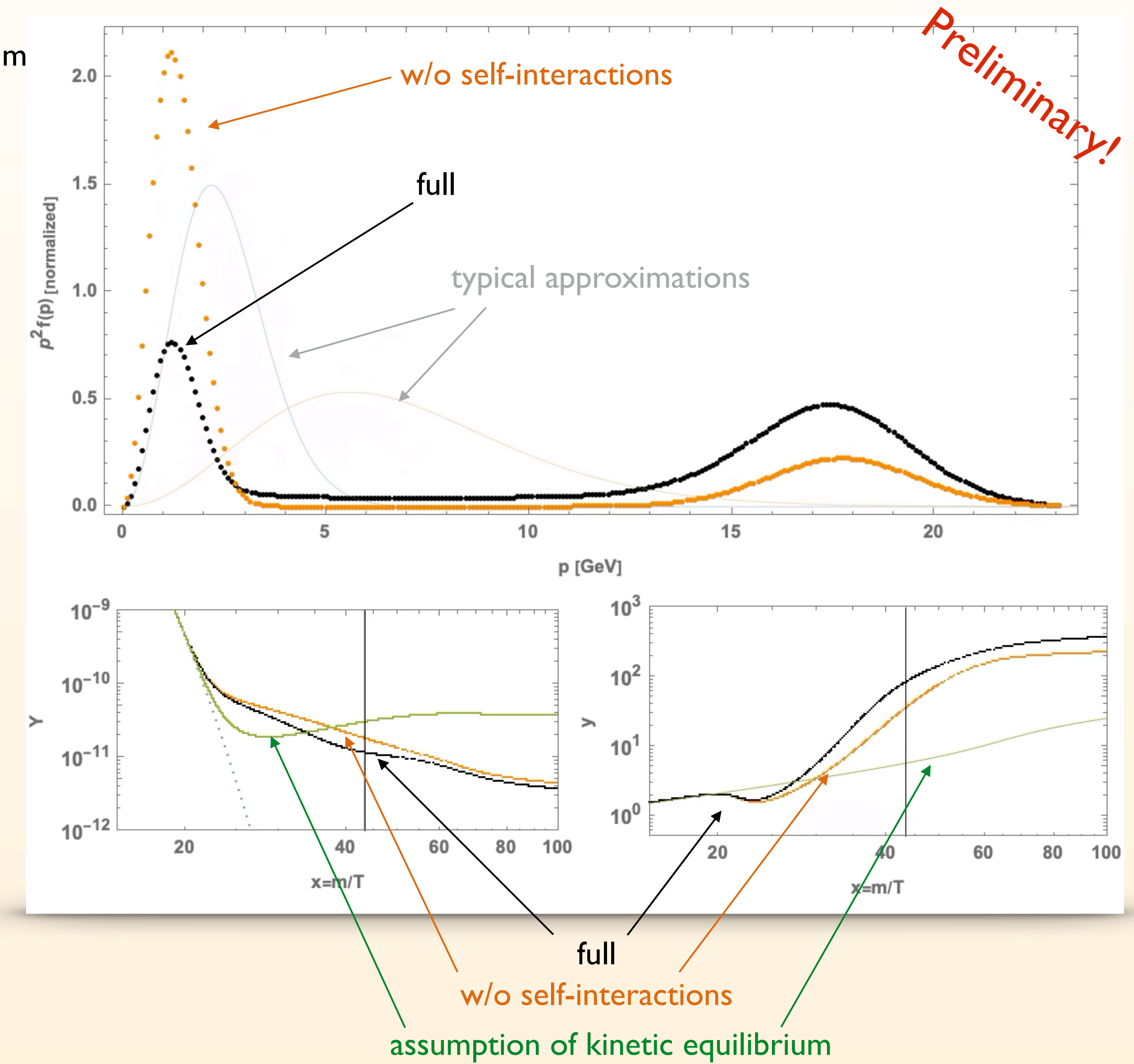
1) DM produced via:
- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

2) DM annihilation has a threshold
e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$

$p^2 f(p) \sim$ momentum distribution

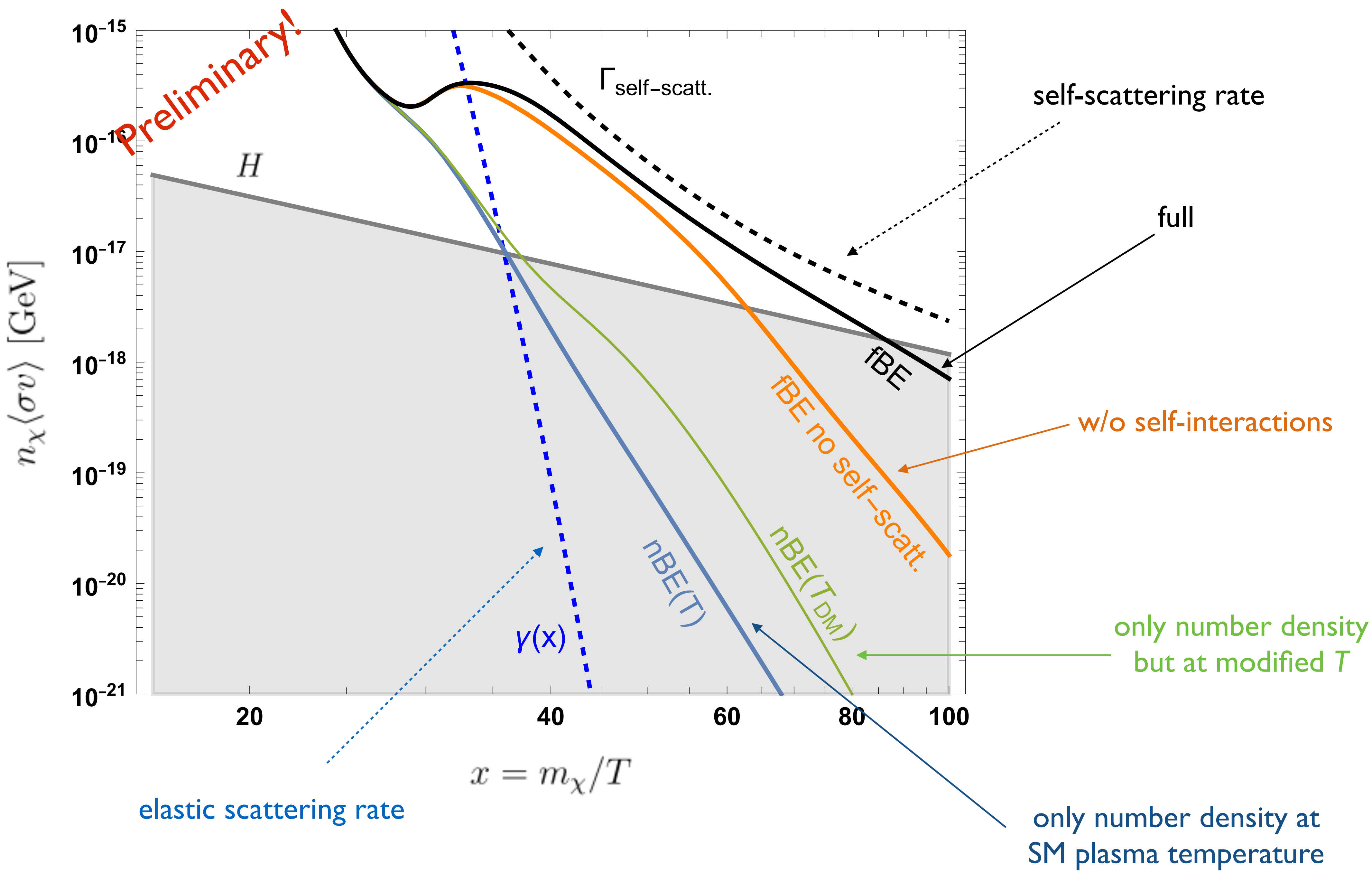
$Y \sim$ number density

$y \sim$ temperature



WHY DOES INJECTING MORE DM PARTICLES CAN LEAD TO DECREASE OF THE RELIC ABUNDANCE?

Let's look on the annihilation rate for different cases:



SUMMARY

1. Kinetic equilibrium is a necessary (often implicit) assumption for standard relic density calculations in all the numerical tools…
…while it is not always warranted!

2. Introduced coupled system of Boltzmann eqs. for 0th and 2nd moments (cBE) allows for much more accurate treatment while the full phase space Boltzmann equation (fBE) can be also successfully solved for higher precision and/or to obtain result for $f_{\rm DM}(p)$

3. Introduced DRAKE : a new tool to extend the current capabilities to the regimes beyond kinetic equilibrium

4. Future developments and applications:

new processes (e.g., freeze-in, semi-annihilations), imprint on power spectrum, …