DARK MATTER RELIC DENSITY

REVISITED

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based on: **T. Binder, T. Bringmann, M. Gustafsson and AH,**
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Rencontres de Moriond, 14th March 2018

MOTIVATION
THERMAL RELIC DENSITY

Theory:
(a.k.a. wishful thinking)

I. Natural

Comes out automatically from the expansion of the Universe

Naturally leads to cold DM

II. Predictive

No dependence on initial conditions

Fixes coupling(s) $\Rightarrow$ signal in DD, ID & LHC

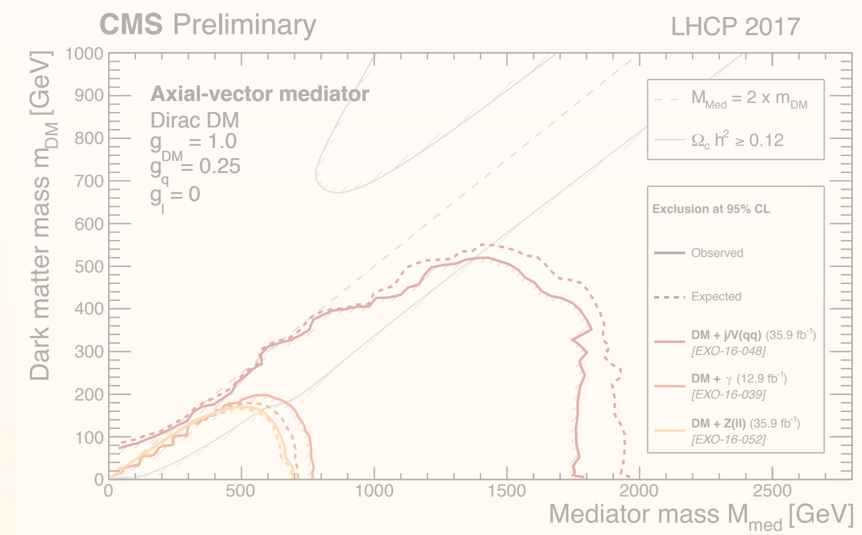
III. It is not optional

Overabundance constraint

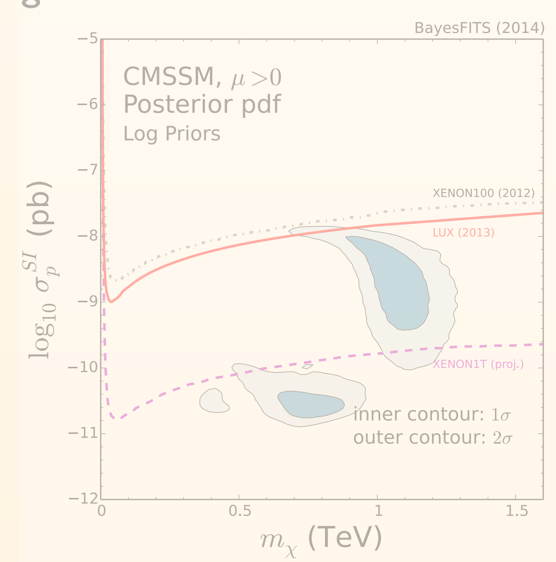
To avoid it one needs quite significant deviations from standard cosmology

Experiment:
(a.k.a. reality)

…as a constraint:



…as a target:



"(…) besides the Higgs boson mass measurement and LHC direct bounds, the constraint showing **by far the strongest impact** on the parameter space of the MSSM is the **relic density**"

Roszkowski et al. '14

…as a pin:

When a dark matter signal is (finally) found: relic abundance can pin-point the particle physics interpretation

Thermal Relic Density

STANDARD APPROACH

T ↑

$\Gamma_{\rm ann} > H$ — DM in equilibrium

$\Gamma_{\rm ann} \sim H$ — **chemical decoupling**

$\Gamma_{\rm ann} < H$ — freeze-out

time ↓

non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi] \quad \Rightarrow \quad \frac{dn_\chi}{dt} + 3Hn_\chi = C$$

Liouville operator in FRW background

the collision term

integrated

assumptions for using Boltzmann eq: classical limit, molecular chaos,...

…for derivation from thermal QFT see e.g., 1409.3049

Thermal Relic Density

the collision term

$$C_{\rm LO} = -h_\chi^2 \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\, \sigma_{\chi\bar\chi\to ij} v_{\rm rel}\, \left[f_\chi f_{\bar\chi}(1\pm f_i)(1\pm f_j) - f_i f_j (1\pm f_\chi)(1\pm f_{\bar\chi})\right]$$

assuming kinetic equilibrium at chemical decoupling: $f_\chi \sim a(\mu) f_\chi^{\rm eq}$

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\chi\bar\chi\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar\chi} - n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}\right)$$

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar\chi\to ij} v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}} \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\, \sigma_{\chi\bar\chi\to ij} v_{\rm rel}\, f_\chi^{\rm eq} f_{\bar\chi}^{\rm eq}$$

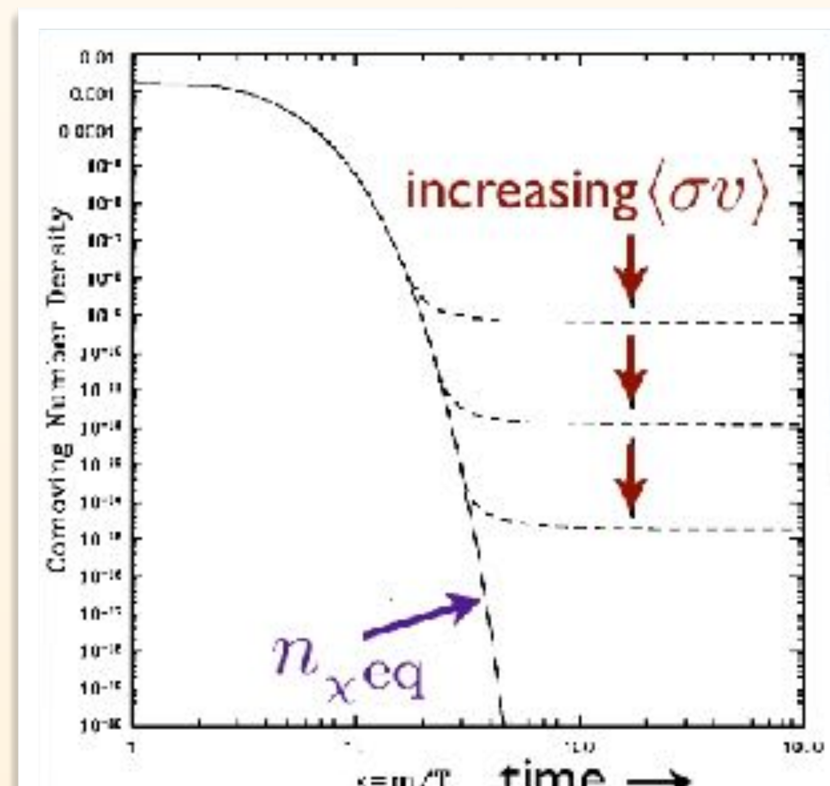


Fig.: Jungman, Kamionkowski & Griest, PR'96

Recipe:
compute annihilation cross-section,
take a thermal bath average,
throw it into BE… and voilà

THERMAL RELIC DENSITY
"EXCEPTIONS"

1. Three "exceptions" — Griest, Seckel '91
2. Non-standard cosmology — many works… very recent e.g., D'Eramo, Fernandez, Profumo '17
3. Second era of annihilation — Feng et al. '10; Bringmann et al. '12; …
4. Bound State Formation — recent e.g., Petraki at al. '15, '16; An et al. '15, '16; Cirelli et al. '16; …

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5. $3 \rightarrow 2$ and $4 \rightarrow 2$ annihilation — e.g., D'Agnolo, Ruderman '15; Cline at al. '17; Choi at al. '17; …
6. Semi-annihilation/Cannibalization — D'Eramo, Thaler '10; … e.g., Kuflik et al. '15; Pappadopulo et al. '16; …
7. Conversion driven/Co-scattering — Garny, Heisig, Lulf, Vogl '17 D'Agnolo, Pappadopulo, Ruderman '17
8. …

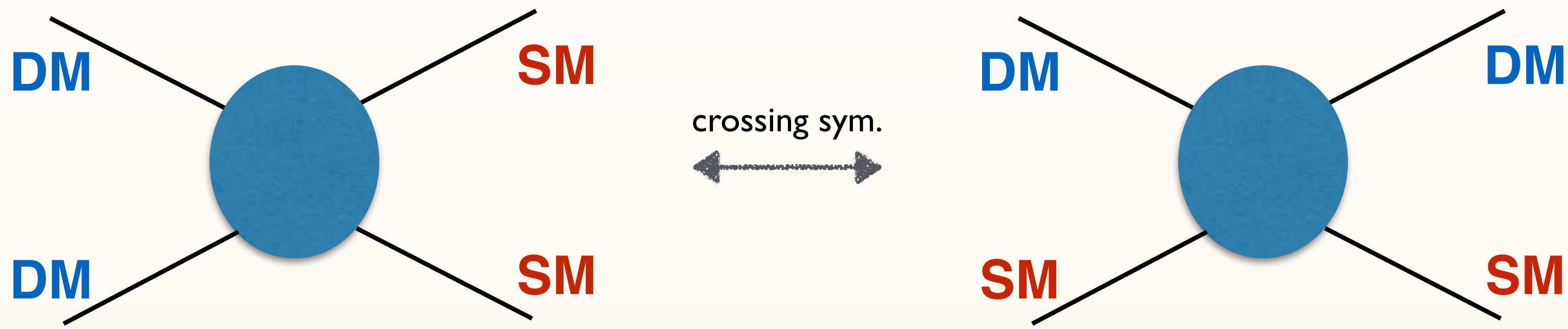
In other words: whenever studying non-minimal scenarios "exceptions" appear — but most of them come from interplay of new added effects, while do not affect the foundations of modern calculations

WHAT IF NON-MINIMAL SCENARIO?

Example: assume two particles in the dark sector: A and B

FREEZE-OUT *VS.* DECOUPLING

annihilation — (elastic) scattering



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Rightarrow$ scatterings typically more frequent

dark matter frozen-out but typically still kinetically coupled to the plasma

$$\tau_r(T_{\text{kd}}) \equiv N_{\text{coll}}/\Gamma_{\text{el}} \sim H^{-1}(T_{\text{kd}})$$

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Two consequences:

1. During freeze-out (chemical decoupling) typically: $f_\chi \sim a(\mu) f_\chi^{\text{eq}}$
2. If kinetic decoupling much, much later: possible impact on the matter power spectrum

i.e. kinetic decoupling can have observable consequences and affect e.g. missing satellites problem

see e.g., Bringmann, Ihle, Karsten, Walia '16

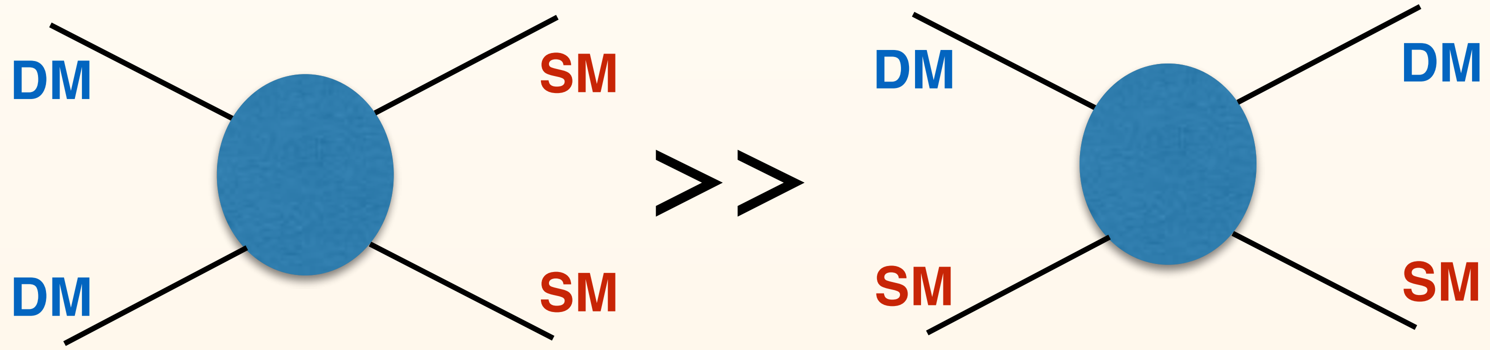
EARLY KINETIC DECOUPLING?

A necessary and sufficient condition: scatterings weaker than annihilation

i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:

A)



e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

How to describe KD?

All information is in full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

→ contains both scatterings and annihilation

Two possible approaches:

solve numerically for full $f_\chi(p)$

have insight on the distribution

no constraining assumptions

numerically challenging

typically overkill

consider system of equations for moments of $f_\chi(p)$

0-th moment: n_χ

2-nd moment: T_χ

…

partially analytic/much easier numerically

manifestly captures all of the relevant physics

finite range of validity

no insight on the distribution

KINETIC DECOUPLING 101

Consider general KD scenario, i.e. coupled temperature and number density evolution:

annihilation and production thermal averages done at different T — feedback of modified y evolution

$$Y=\frac{n_\chi}{s} \qquad y\equiv\frac{m_\chi T_\chi}{s^{2/3}}$$

$$\frac{Y'}{Y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx}sY\left(\langle\sigma v_{\rm rel}\rangle|_{x=m_\chi^2/(s^{2/3}y)}-\frac{Y_{\rm eq}^2}{Y^2}\langle\sigma v_{\rm rel}\rangle|_x\right)$$

$$\frac{y'}{y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx}\left[2m_\chi c(T)\left(1-\frac{y_{\rm eq}}{y}\right) - sY\left(\left(\langle\sigma v_{\rm rel}\rangle-\langle\sigma v_{\rm rel}\rangle_2\right)_{x=m_\chi^2/(s^{2/3}y)}-\frac{Y_{\rm eq}^2}{Y^2}\left(\langle\sigma v_{\rm rel}\rangle-\frac{y_{\rm eq}}{y}\langle\sigma v_{\rm rel}\rangle_2\right)_x\right)\right]$$

$$+\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{3m_\chi}\langle p^4/E^3\rangle_{x=m_\chi^2/(s^{2/3}y)}$$

"relativistic" term

$$T_\chi\equiv\frac{g_\chi}{3n_\chi}\int\frac{d^3p}{(2\pi)^3}\frac{p^2}{E}f_\chi(p)$$

elastic scatterings term

$$c(T)=\frac{1}{12(2\pi)^3 m_\chi^4 T}\sum_X\int dk\, k^5\omega^{-1}\, g^\pm\,(1\mp g^\pm)\int_{-4k^2}^{0}(-t)\frac{1}{8k^4}|\mathcal{M}_{\rm el}|^2$$

impact of annihilation

$$\langle\sigma v_{\rm rel}\rangle_2 \equiv \frac{g_\chi^2}{3Tm_\chi n_\chi^2}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3\tilde{p}}{(2\pi)^3}\, p^2 v_{\rm rel}\sigma_{\bar\chi\chi\to\bar X X}f(E)f(\tilde{E})$$

These equations still assume the equilibrium shape of $f_\chi(p)$ — but with variant temperature

or more accurately: that the thermal averages computed with true non-equilibrium distributions don't differ much from the above ones

NUMERICAL APPROACH

… or one can just solve full phase space Boltzmann eq.

$$
\begin{aligned}
\partial_x f_\chi(x,q) &= \frac{m_\chi^3}{\tilde{H}x^4}\frac{g_{\bar\chi}}{2\pi^2}\int d\tilde{q}\,\tilde{q}^2\,\frac{1}{2}\int d\cos\theta\; v_{\text{Møl}}\sigma_{\bar\chi\chi\to\bar{f}f} \\
&\quad\times\left[f_{\chi,\text{eq}}(q)f_{\chi,\text{eq}}(\tilde{q}) - f_\chi(q)f_\chi(\tilde{q})\right] \\
&+ \frac{2m_\chi c(T)}{2\tilde{H}x}\left[x_q\partial_q^2 + \left(q + \frac{2x_q}{q} + \frac{q}{x_q}\right)\partial_q + 3\right] f_\chi \\
&+ \tilde{g}\frac{q}{x}\partial_q f_\chi,
\end{aligned}
$$

fully general

expanded in NR and small momentum transfer (semi-relativistic!)

discretization, ~1000 steps

$$
\begin{aligned}
\partial_x f_i &= \\
&\frac{m_\chi^3}{\tilde{H}x^4}\frac{g_{\bar\chi}}{2\pi^2}\sum_{j=1}^{N-1}\frac{\Delta\tilde{q}_j}{2}\Big[\tilde{q}_j^2\,\langle v_{\text{Møl}}\sigma_{\bar\chi\chi\to\bar{f}f}\rangle_{i,j}^\theta\left(f_i^{\text{eq}}f_j^{\text{eq}} - f_i f_j\right) \\
&+ \tilde{q}_{j+1}^2\,\langle v_{\text{Møl}}\sigma_{\bar\chi\chi\to\bar{f}f}\rangle_{i,j+1}^\theta\left(f_i^{\text{eq}}f_{j+1}^{\text{eq}} - f_i f_{j+1}\right)\Big] \\
&+ \frac{2m_\chi c(T)}{2\tilde{H}x}\left[x_{q,i}\partial_q^2 + \left(q_i + \frac{2x_{q,i}}{q_i} + \frac{q_i}{x_{q,i}}\right)\partial_q + 3\right] f_i \\
&+ \tilde{g}\frac{q_i}{x}\partial_q f_i,
\end{aligned}
$$

Solved numerically with MatLab

Note:

can be extended to e.g. self-scatterings

very stiff, care needed with numerics

Example:

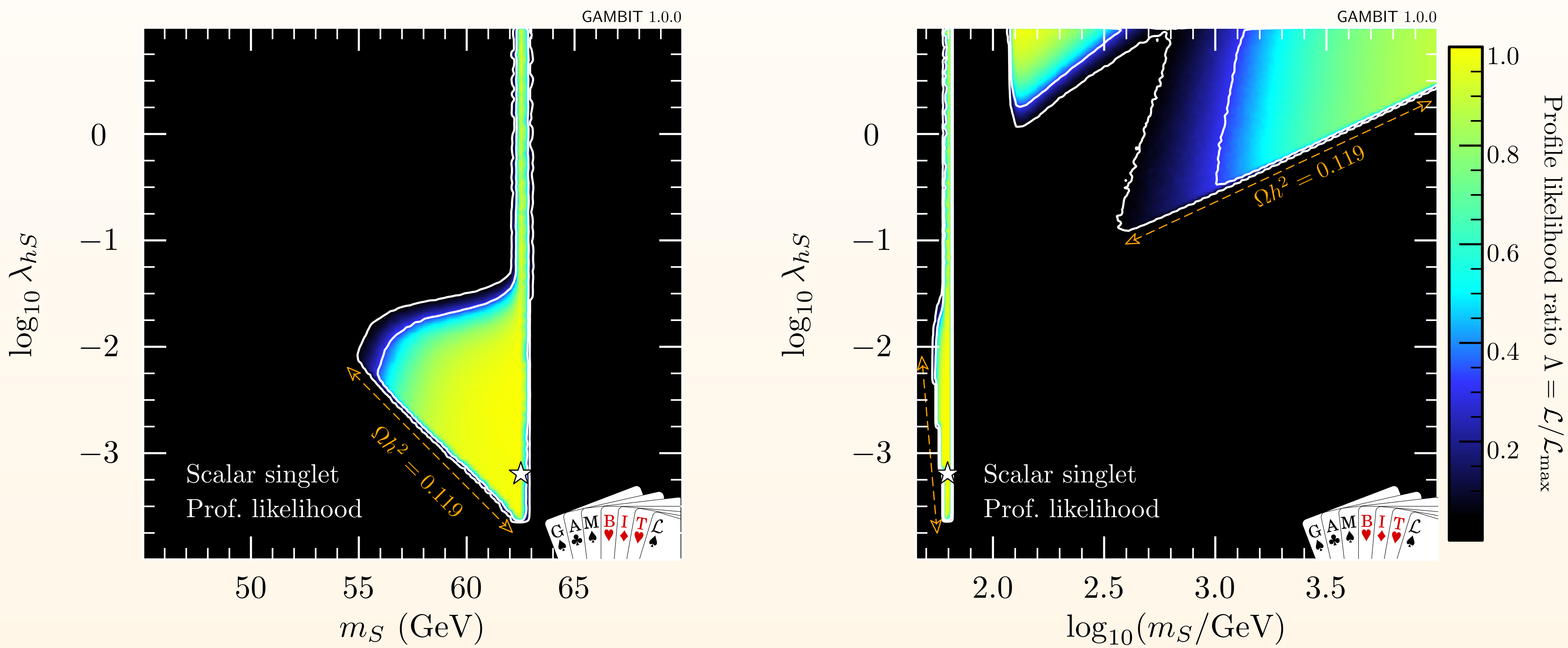
Scalar Singlet DM

SCALAR SINGLET DM

To the SM Lagrangian add one singlet scalar field *S* with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_s S^2 |H|^2$$

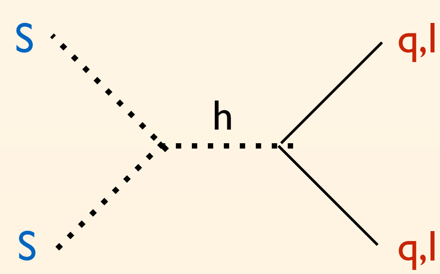
$$m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$



GAMBIT collaboration
1705.07931

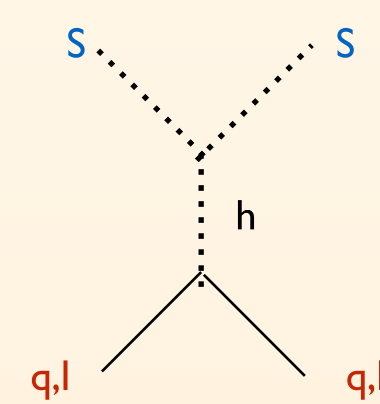
Annihilation processes:

resonant



El. scattering processes:

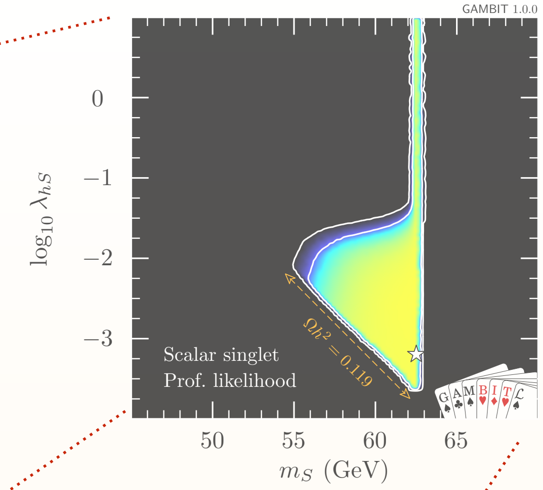
non-resonant



Hierarchical Yukawa couplings: strongest coupling to more Boltzmann suppressed quarks/leptons

RESULTS

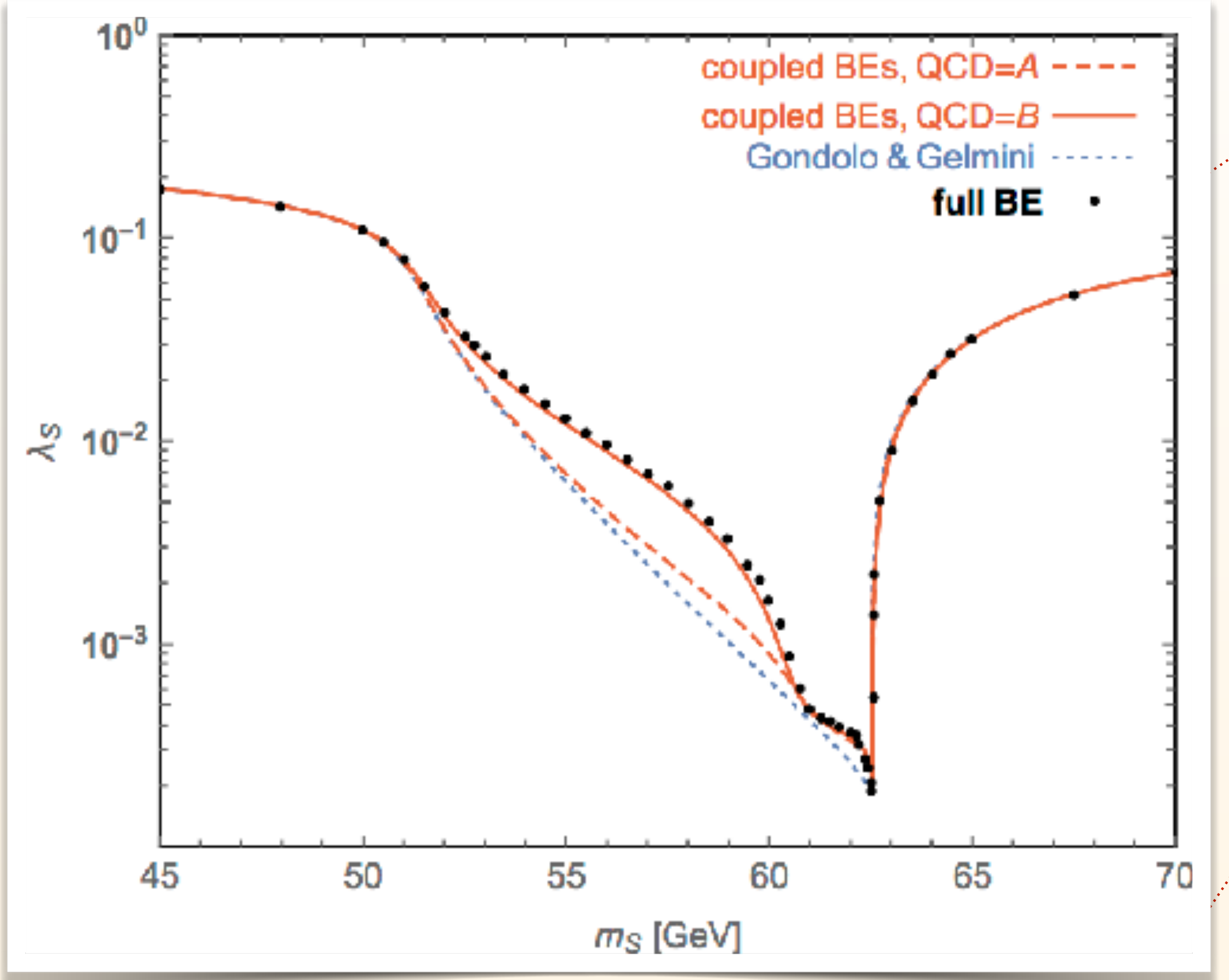
RD CONTOURS



essentially the only region left for this model

QCD = A - all quarks are free and present down to T_c =154 MeV

QCD = B - only light quarks in the plasma and only down to $4T_c$

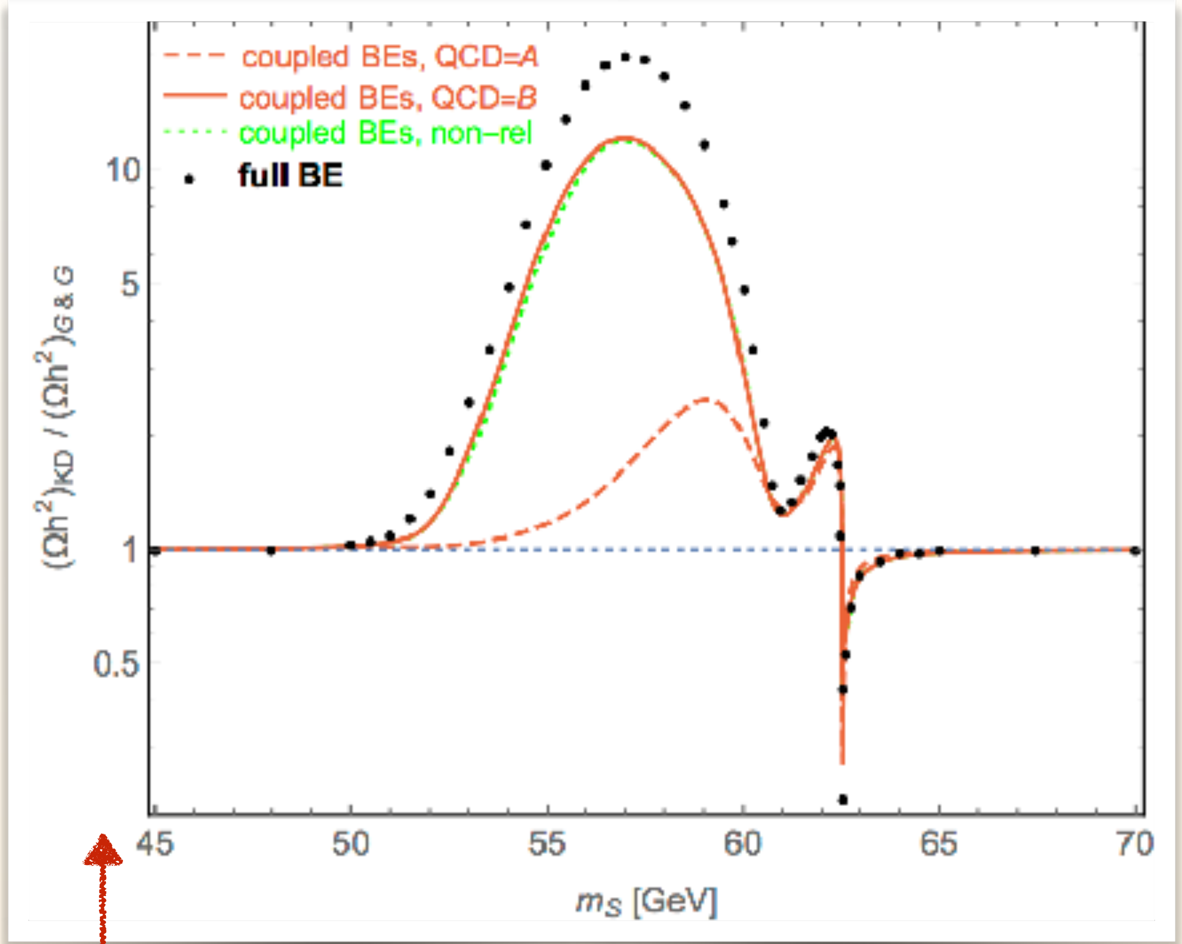


Significant modification of the observed relic density contour in the Scalar Singlet DM model

→ larger coupling needed → better chance for closing the last window

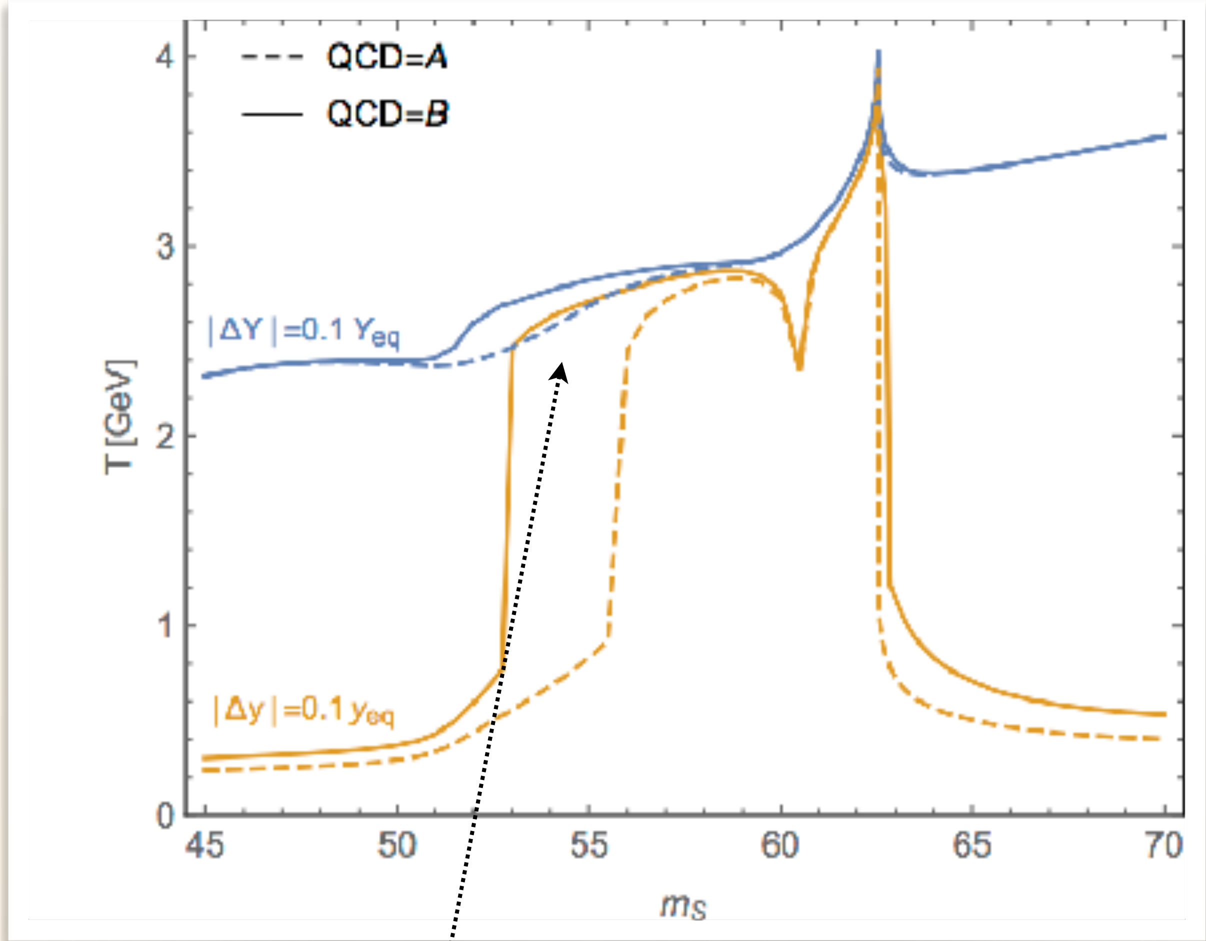
RESULTS
EFFECT

effect on relic density:



effect on relic density:
up to O(~10)

kinetic and chemical decoupling:



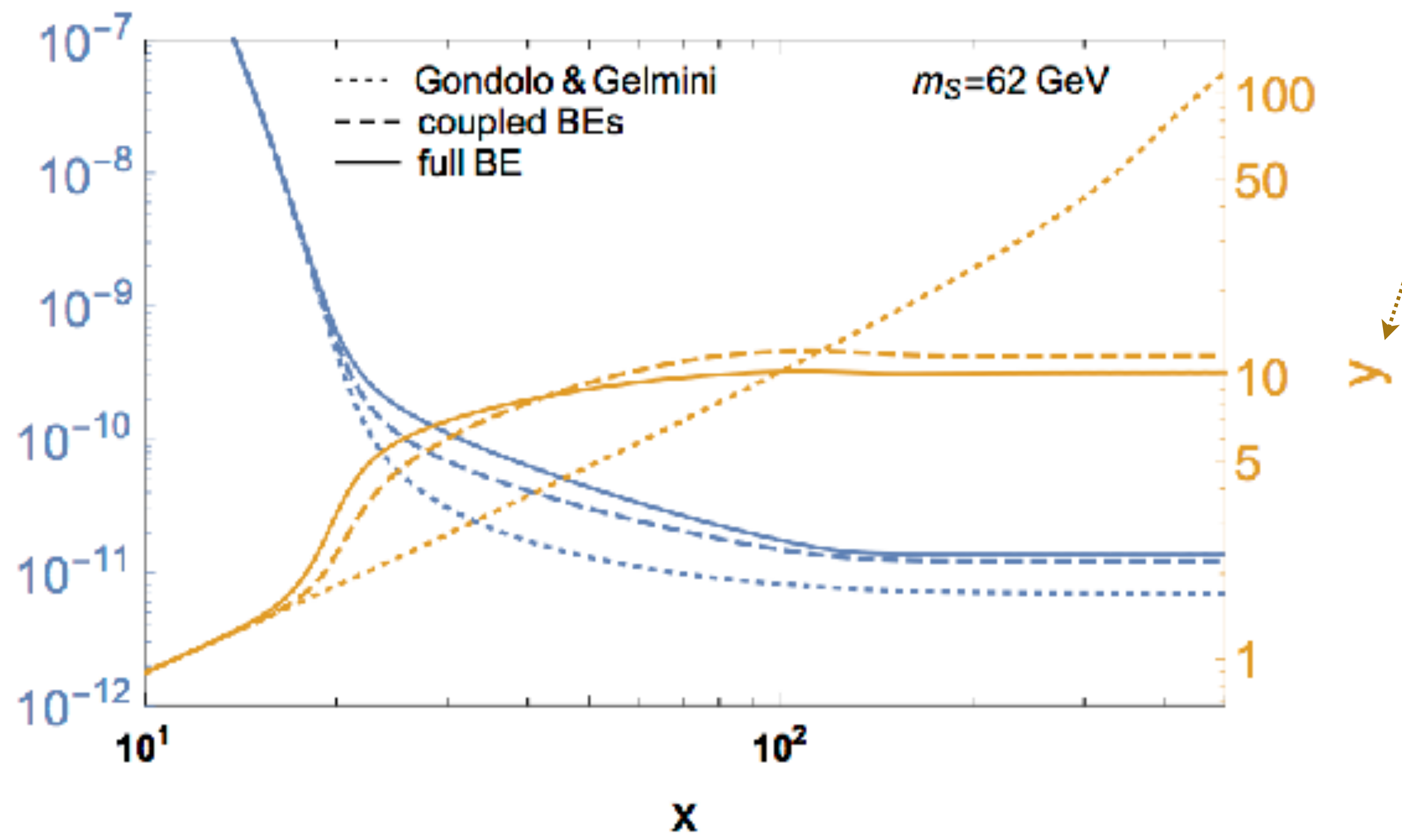
ratio approaches 1,
but does not reach it!

Why such non-trivial shape of the effect of early kinetic decoupling?

→ we'll inspect the y and Y evolution…

Density and T_{DM} evolution

for m_{DM} = 62 GeV, i.e. just below the resonance:



DM temperature

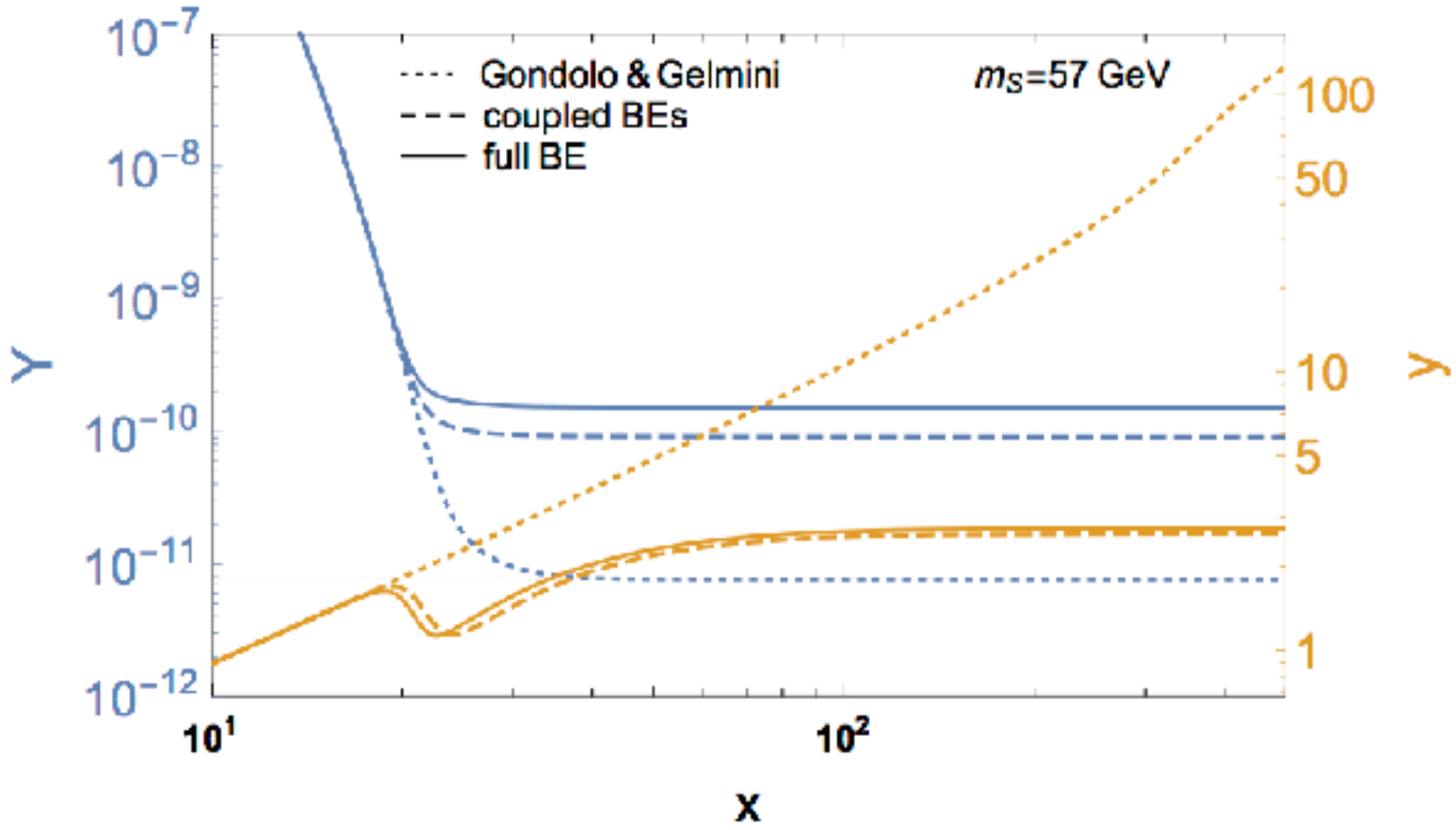
co-moving number density

Resonant annihilation **most effective** for **low momenta**

⟶ DM fluid goes through "heating" phase before leaves kinetic equilibrium

Density and T_{DM} evolution

for m_{DM} = 57 GeV,   i.e. further away from the resonance:



Resonant annihilation **most effective** for **high momenta**

⟶ DM fluid goes through fast "cooling" phase

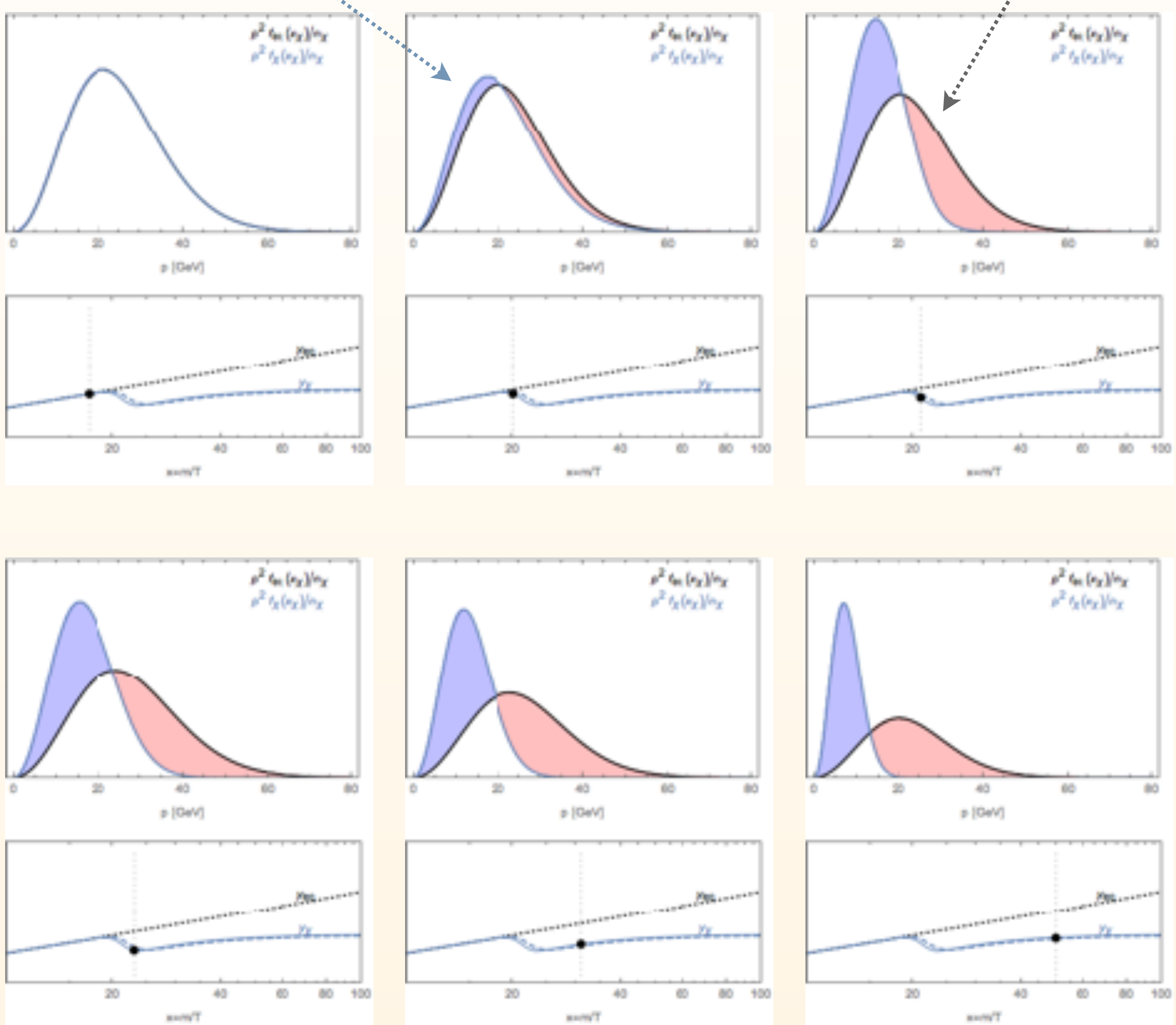
after that when T_{DM} drops to much annihilation not effective anymore

FULL PHASE-SPACE EVOLUTION

blue - full solution for f_{DM} at T_{DM}

m_{DM} = 58 GeV

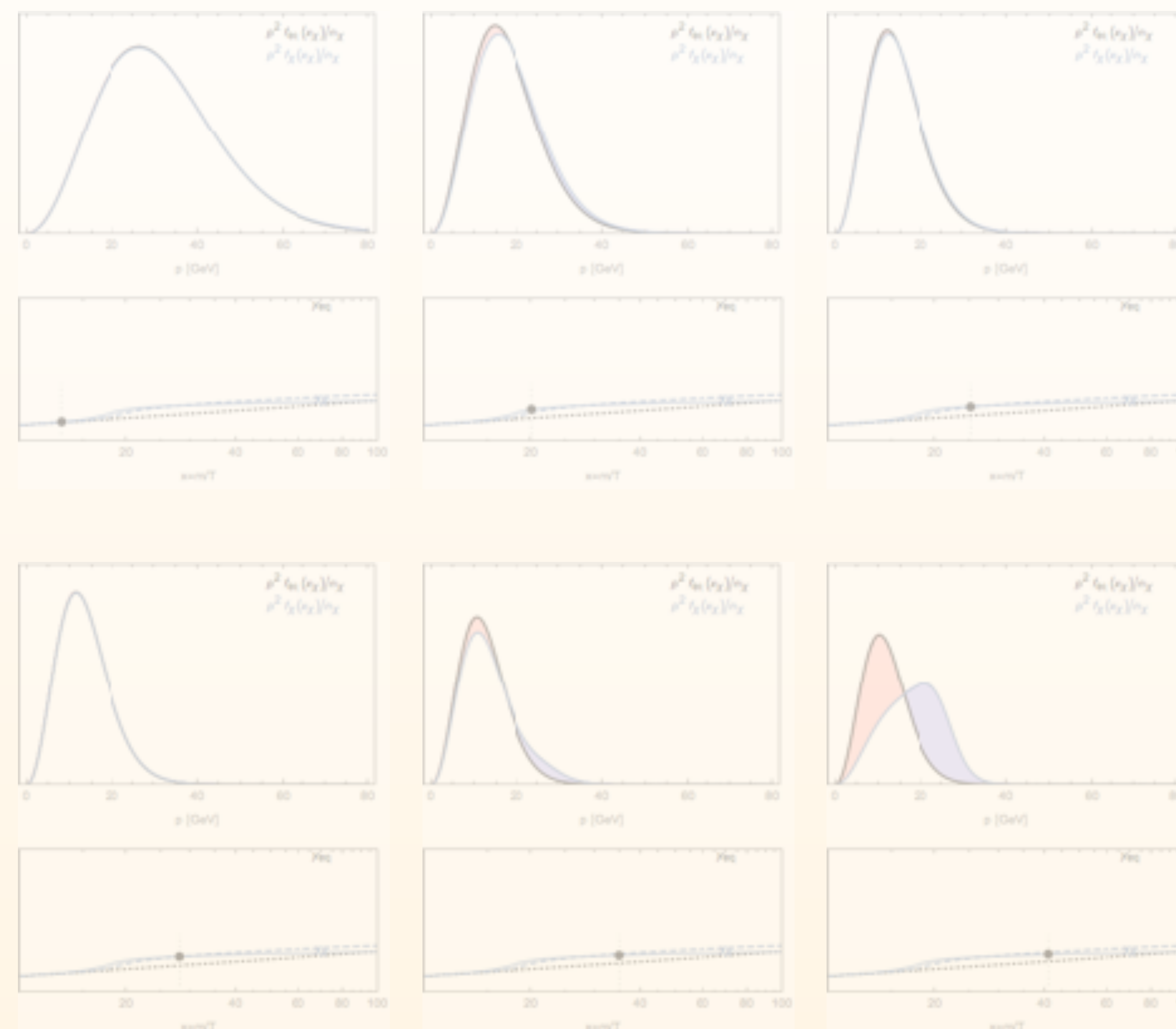
black - equilibrium at T_{DM}



significant deviation from equilibrium shape already around freeze-out

⟶ effect on relic density largest, both from different T and f_{DM}

m_{DM} = 62.5 GeV



large deviations only at later times, around freeze-out not far from eq. shape

⟶ effect on relic density ~only from different T

WHAT NEXT?

1. Extend the numerical full phase space BE code to the case of scattering on heavy particles

(no small momentum transfer!)

2. Prepare a public release (and study some more examples)

stay tuned for this!

3. Work on extension to self-scattering

(none of the particles in scattering term has equilibrium phase space density)

4. At later stage: inelastic scatterings, semi-annihilation, cannibal, …

KD BEFORE CD?

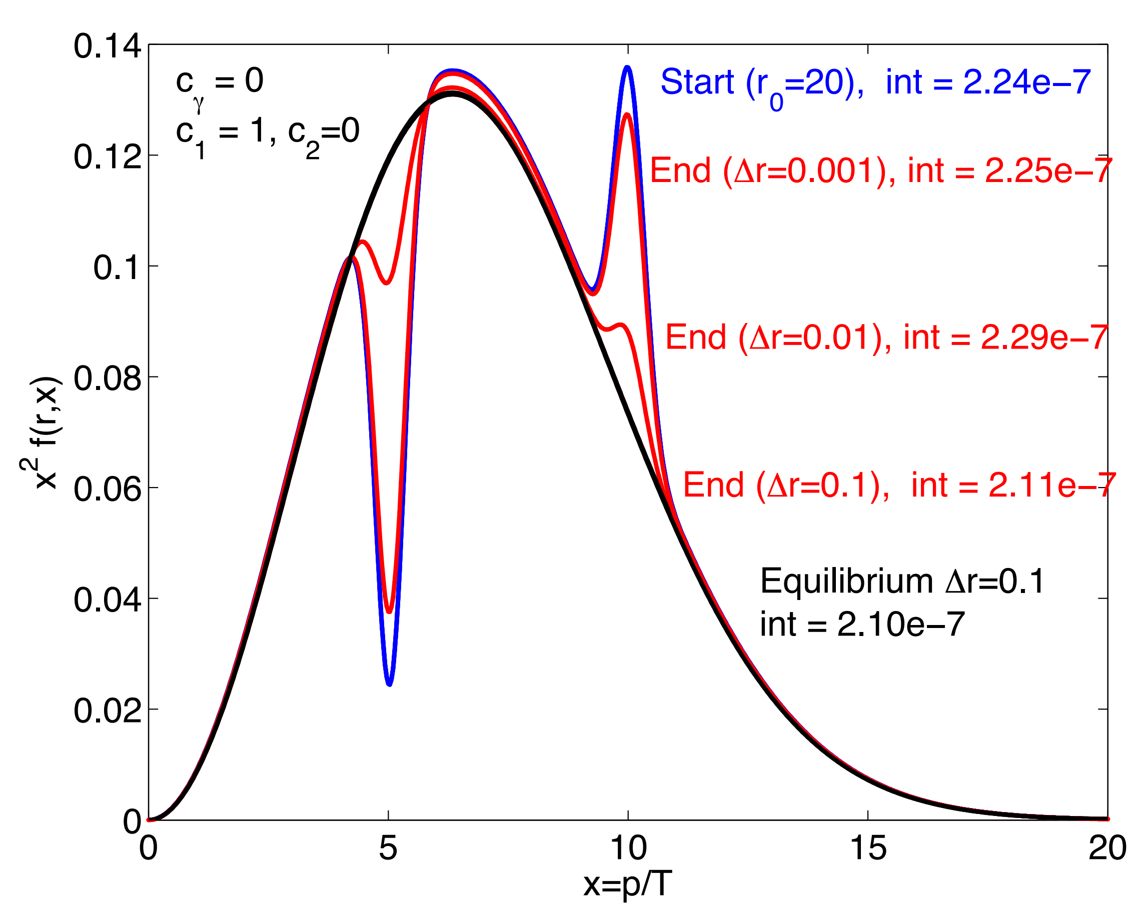
Obvious issue:

How to define exactly the kinetic and chemical decouplings and what is the significance of such definitions?

⇓

Improved question:

Can kinetic decoupling happen much earlier than chemical?



we have already seen that even if scatterings were very inefficient compared to annihilation, departure from equilibrium for both Y and y happened around the same time…

← turn off scatterings and take s-wave annihilation; look at local disturbance

annihilation/production precesses drive to restore kinetic equilibrium!

CONCLUSIONS

1. One needs to remember that kinetic equilibrium is a necessary assumption for standard relic density calculations

2. Coupled system of Boltzmann equations for 0th and 2nd moments allow for a very accurate treatment of the kinetic decoupling and its effect on relic density

3. In special cases the full phase space Boltzmann equation can be necessary — especially if one wants to trace DM temperature as well

4. A public release of the full phase space Boltzmann code coming soon

BACKUP

SCATTERING

The elastic scattering collision term:

$$C_{\rm el} = \frac{1}{2g_\chi}\int\frac{d^3k}{(2\pi)^3 2\omega}\int\frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}}\int\frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}}$$
$$\times(2\pi)^4\delta^{(4)}(\tilde{p}+\tilde{k}-p-k)|\mathcal{M}|^2_{\chi f\leftrightarrow\chi f}$$
$$\times\left[(1\mp g^\pm)(\omega)\,g^\pm(\tilde{\omega})f_\chi(\tilde{\mathbf{p}})-(\omega\leftrightarrow\tilde{\omega},\mathbf{p}\leftrightarrow\tilde{\mathbf{p}})\right]$$

→ equilibrium functions for SM particles

Expanding in **NR** and small **momentum transfer**:

Bringmann, Hofmann '06

$$C_{\rm el}\simeq\frac{m_\chi}{2}\gamma(T)\left[Tm_\chi\partial_p^2+\left(p+2T\frac{m_\chi}{p}\right)\partial_p+3\right]f_\chi$$

physical interpretation: scattering rate

More generally, Fokker-Planck scattering operator (relativistic, but still small **momentum transfer**):

Binder et al. '16

$$C_{\rm el}\simeq\frac{E}{2}\nabla_{\mathbf{p}}\cdot\left[\gamma(T,\mathbf{p})\left(ET\nabla_{\mathbf{p}}+\mathbf{p}\right)f_\chi\right]$$

Semi-relativistic: assume that scattering $\gamma(T,\mathbf{p})$ is momentum independent

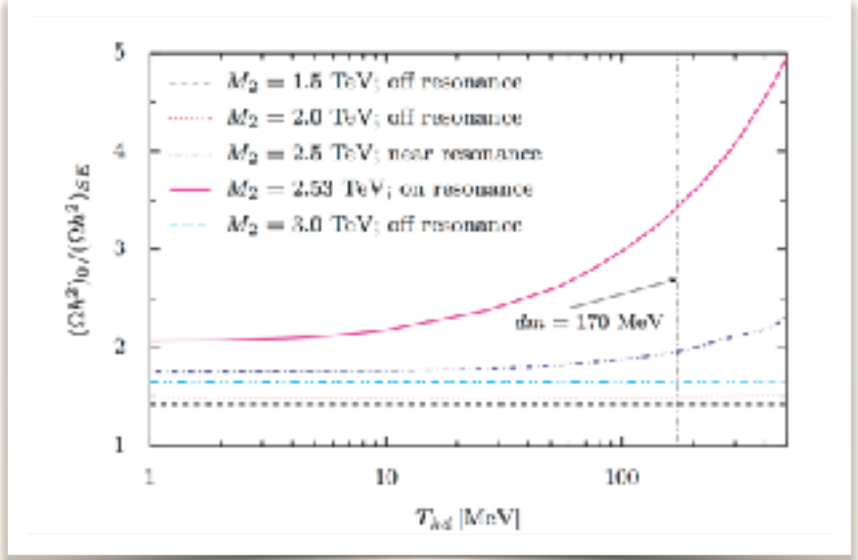
EARLY KD AND RESONANCE

our work wasn't the first to realize that resonant annihilation can lead to early kinetic decoupling…

Feng, Kaplinghat, Yu '10 — noted that for Sommerfeld-type resonances KD can happen early

Dent, Dutta, Scherrer '10 — looked at potential effect of KD on thermal relic density

Since then people were aware of this effect and sometimes tried to estimate it assuming instantaneous KD, e.g., in the case of Sommerfeld effect in the MSSM:



AH, Iengo, Ullio '11

but no systematic studies of decoupling process were performed, until…

…models with very late KD become popular, in part to solve „missing satellites" problem

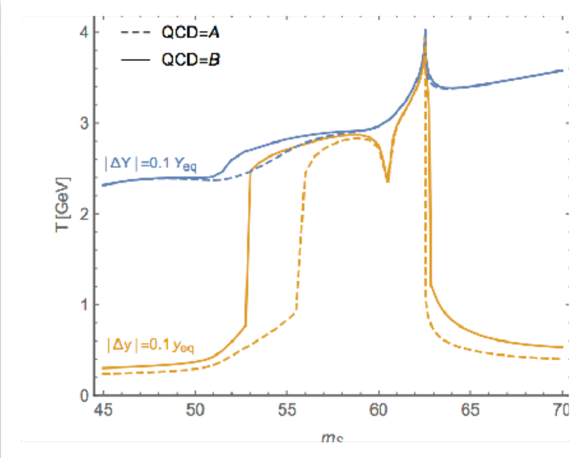
van den Aarssen et al '12; Bringmann et al '16, x2; Binder et al '16

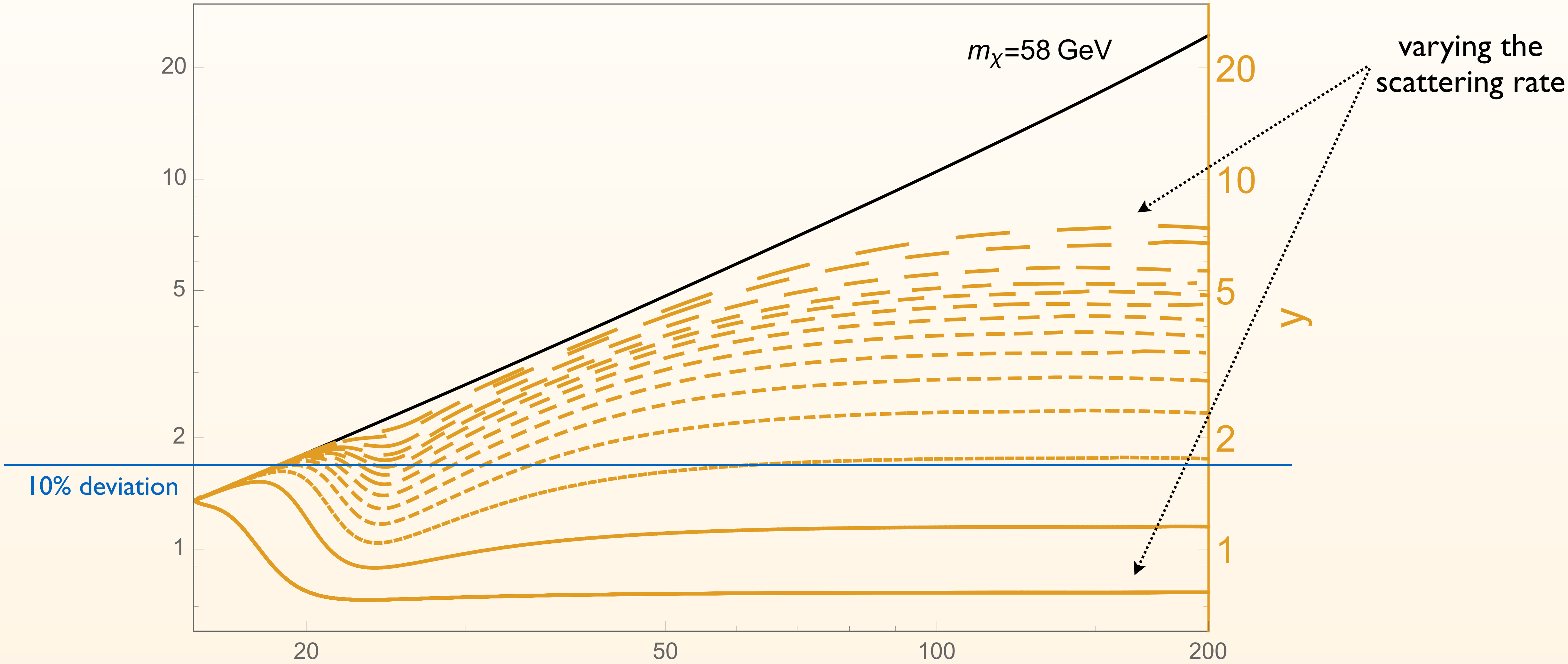
this progress allowed for better approach to early KD scenarios as well and was applied to the resonant annihilation case in

Duch, Grządkowski '17

… but we developed a dedicated accurate method/code to deal with this and other similar situations

WHY SPIKES IN T_{KD}?

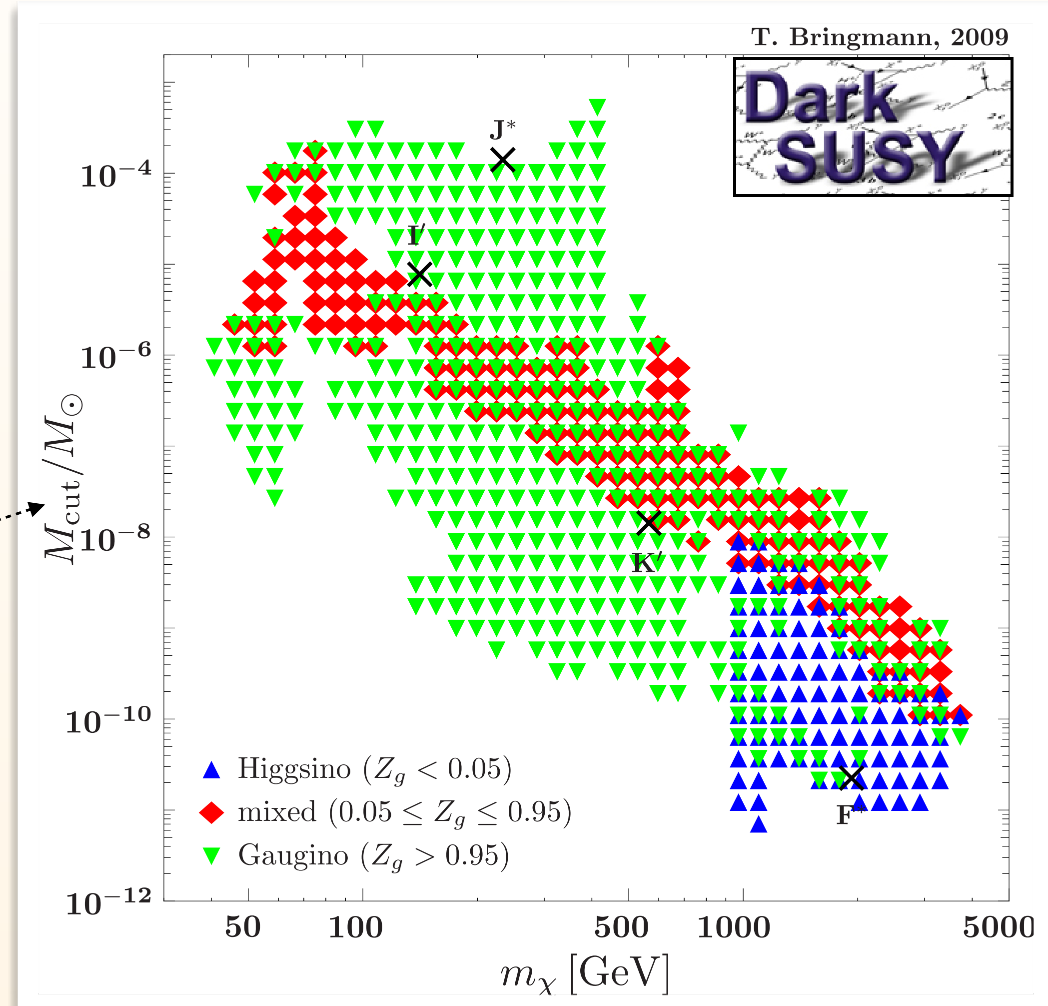




Effect resembling first order „phase transition” —
artificial as dependent on a particular choice of T_{KD} definition

Implications of Kinetic Decoupling

E.g. for SUSY neutralino: Bringmann '09



Free-streaming of DM after KD washes out density contrasts at small scales (similarly to baryonic oscillations)

Green, Hofmann, Schwarz '05

Cut-off in the power spectrum corresponding to smallest gravitationally bound objects

„Typical" values for WIMPs are relatively small → small substructures expected

→ but bad for missing satellites problem

$\Longrightarrow$ moment of KD leaves important imprint on the Universe