NATIONAL SCIENCE CENTRE
POLAND
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ELASTIC SELF-SCATTERINGS

IN THE CALCULATION OF DARK MATTER RELIC ABUNDANCE

Andrzej Hryczuk

NCBJ

Review Part: a personal selection of recent ideas in the field

New Results Part:

A.H. & M. Laletin 2204.07078

+ some work in progress with **S. Chatterjee**
+ some older results based on:

A.H. & M. Laletin 2104.05684

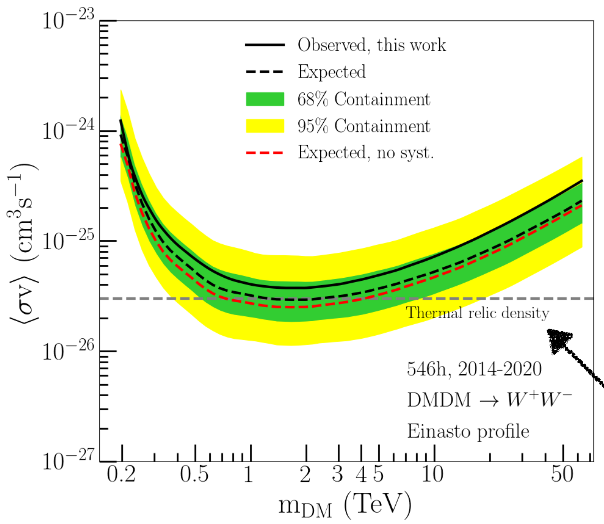
T. Binder, T. Bringmann, M. Gustafsson & A.H. 1706.07433, 2103.01944

Particle Astrophysics in Poland, Kraków

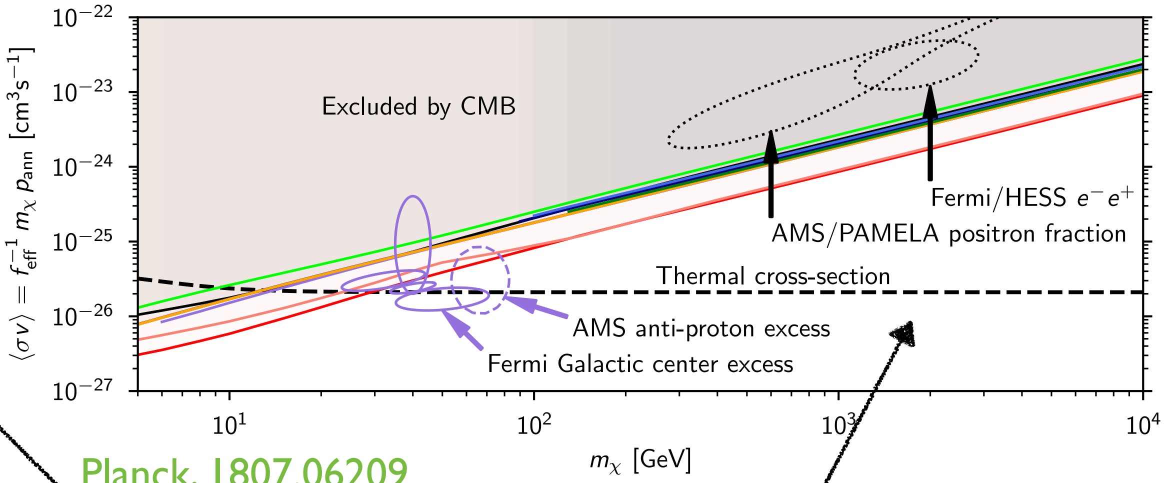
andrzej.hryczuk@ncbj.gov.pl

15th February 2023

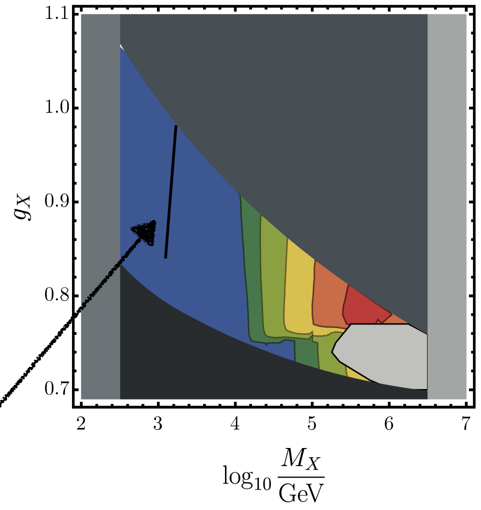
DARK MATTER IS UBIQUITOUS!



H.E.S.S. 2207.10471



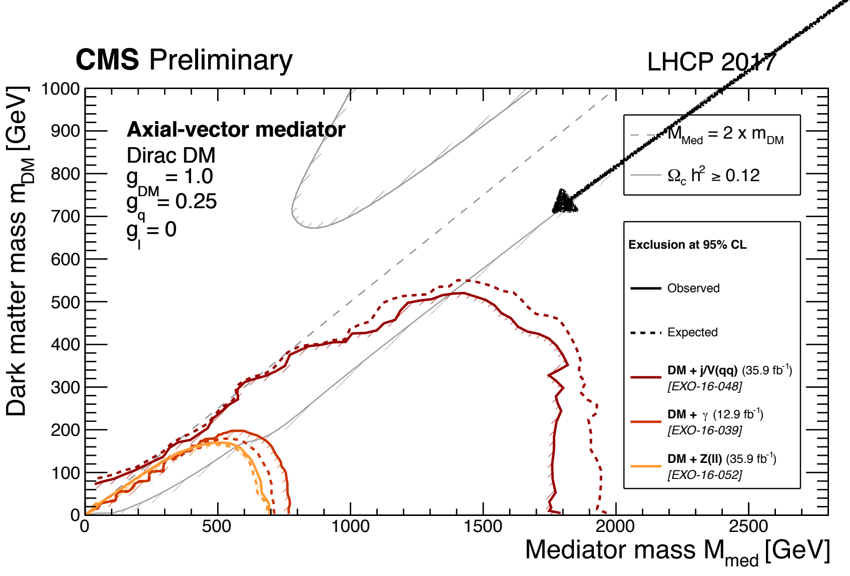
Planck, 1807.06209



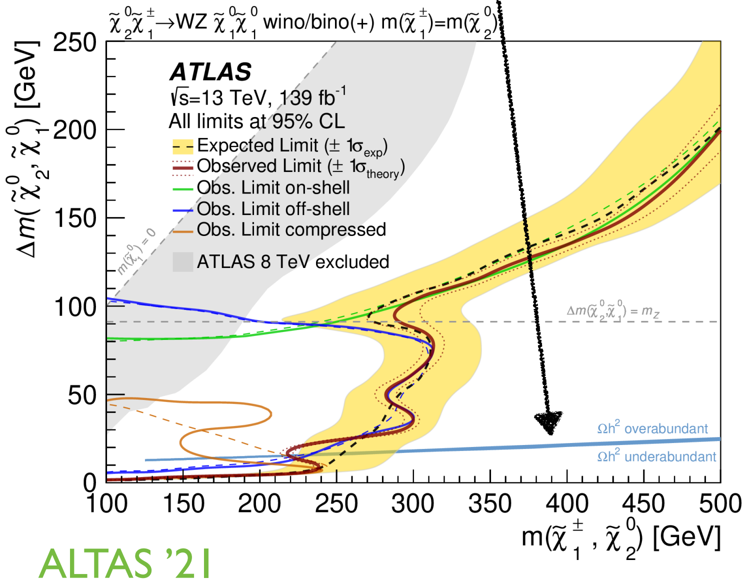
B. Świeżewska talk, 2210.07075

Thermal relic density line/region is a very common thing to highlight!

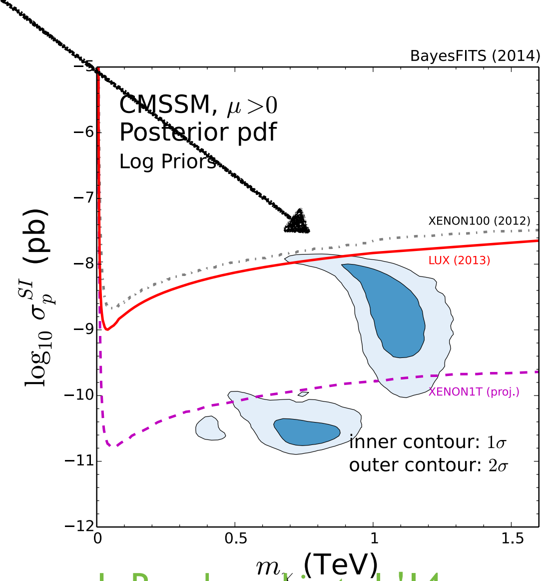
(quite possibly in your work as well)



CMS '17

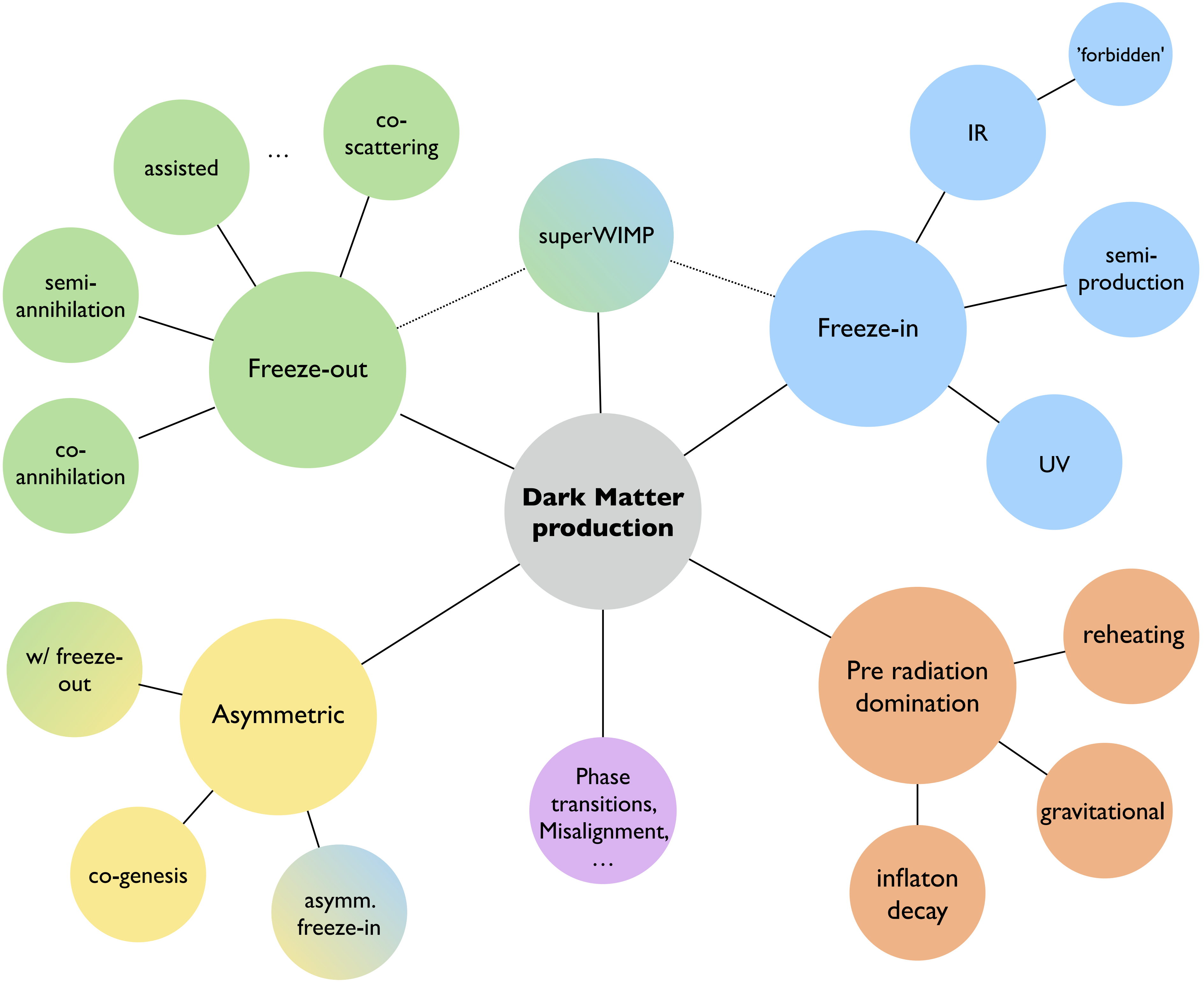


ALTAS '21

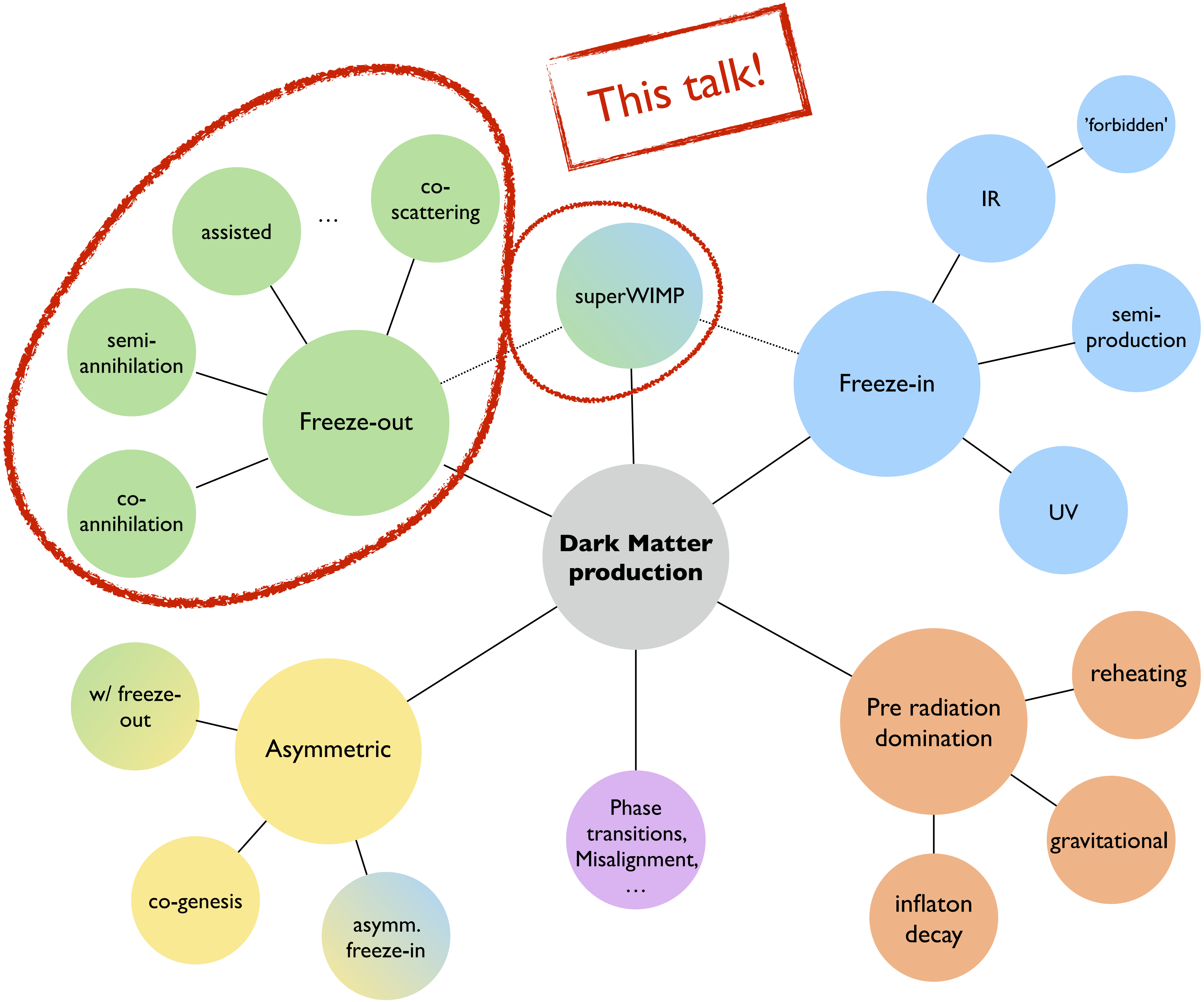


L. Roszkowski et al. '14

DARK MATTER ORIGIN



DARK MATTER ORIGIN



Thermal Relic Density

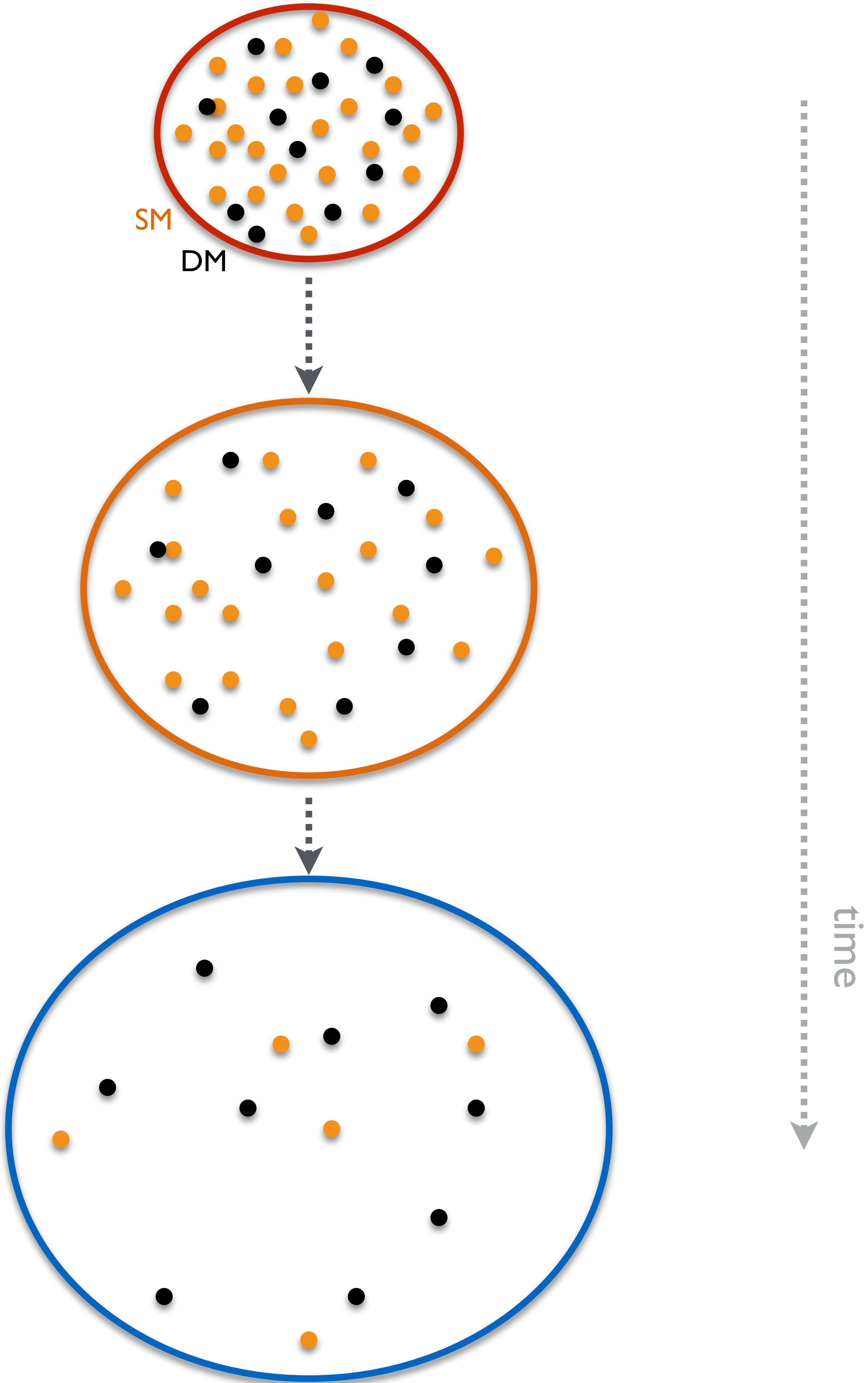
a.k.a. Freeze-Out

T

$\Gamma_{\text{ann}} > H$ DM in full equilibrium

$\Gamma_{\text{ann}} \sim H$ chemical decoupling

$\Gamma_{\text{ann}} < H$ freeze-out



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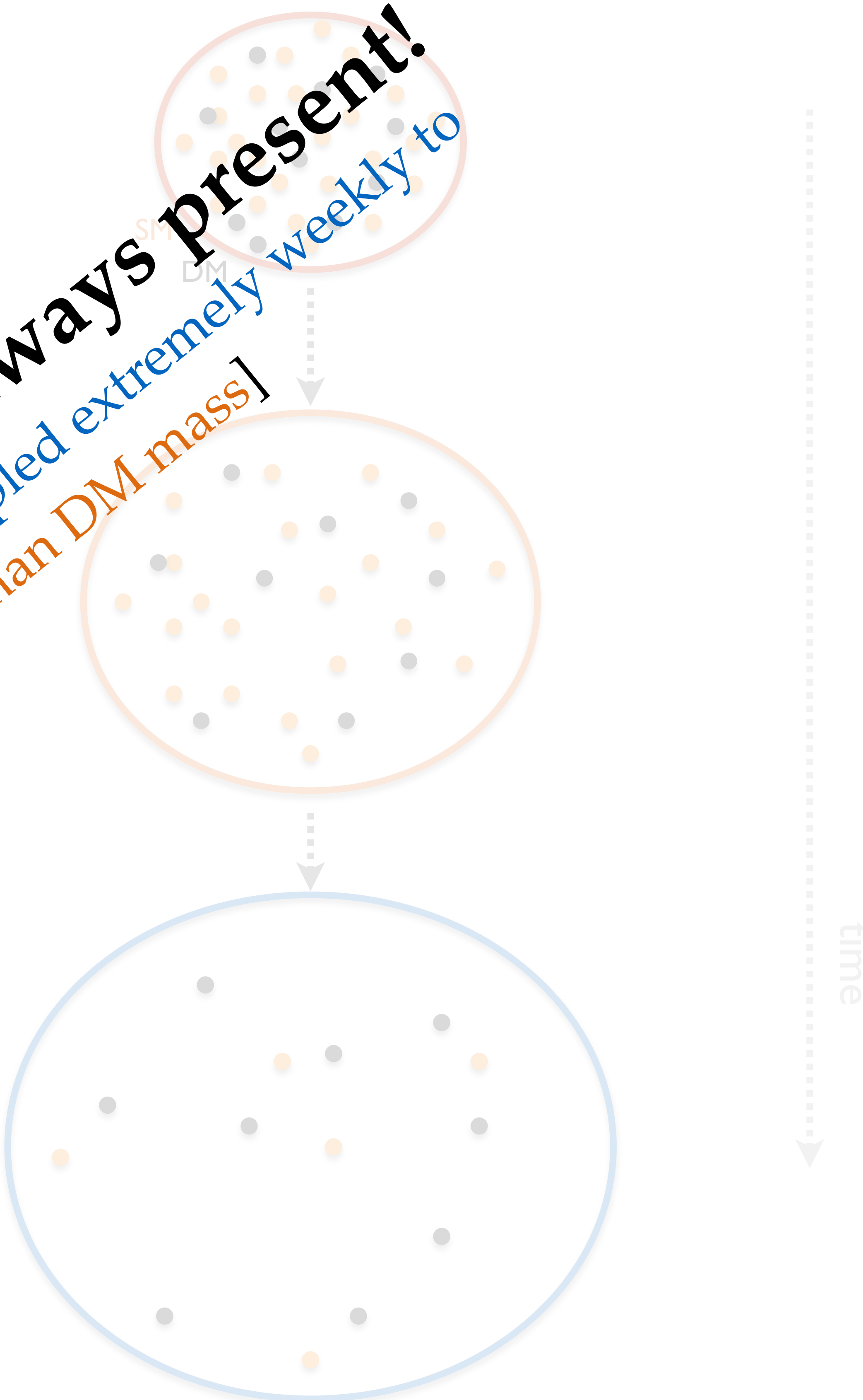

This process is (essentially) always present!

[Only exceptions: DM is not a particle or coupled extremely weekly to SM or reheating T is smaller than DM mass]

THERMAL RELIC DENSITY

STANDARD SCENARIO

numerical codes e.g.,
DarkSUSY, micrOMEGAs,
MadDM, SuperISOrelic, ...

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle \sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

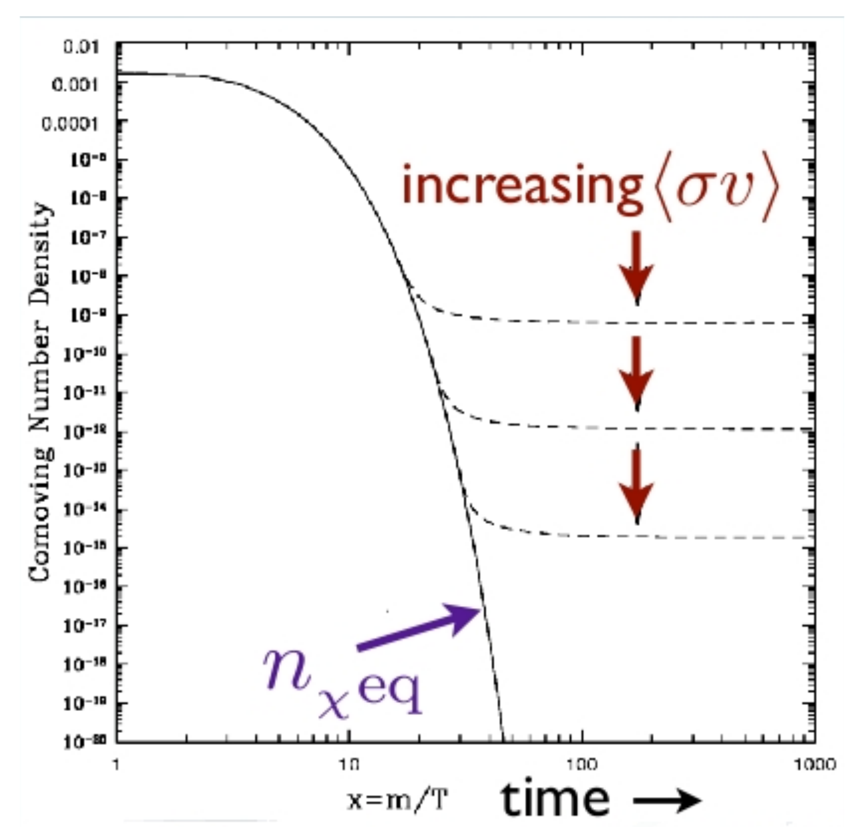

Fig.: Jungman, Kamionkowski & Griest, PR'96

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}} \int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\ \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel}\ f_\chi^{\rm eq} f_{\bar{\chi}}^{\rm eq}$$

THERMAL RELIC DENSITY

STANDARD SCENARIO

modified expansion rate

e.g., relentless DM, D'Eramo et al. '17, …

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modified cross section

- Sommerfeld enhancement
- Bound State formation
- NLO
- finite T effects

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breakdown of necessary assumptions leading to different form of the equation, e.g. violation of kinetic equilibrium

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general multi-component dark sector →

modified cross section

Sommerfeld enhancement

Bound State formation

NLO

finite T effects

breakdown of necessary assumptions leading to different form of the equation, e.g. violation of kinetic equilibrium

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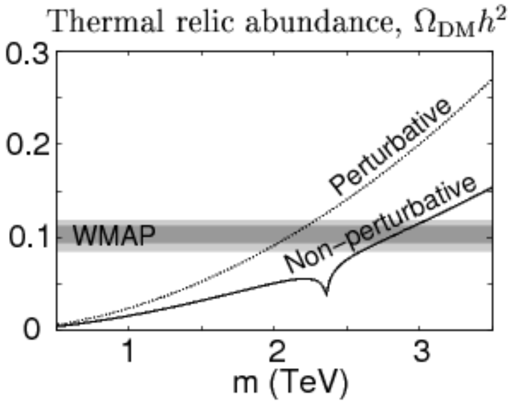
CHAPTER I:

PARTICLE PHYSICS EFFECTS

THE SOMMERFELD EFFECT

FROM EW INTERACTIONS



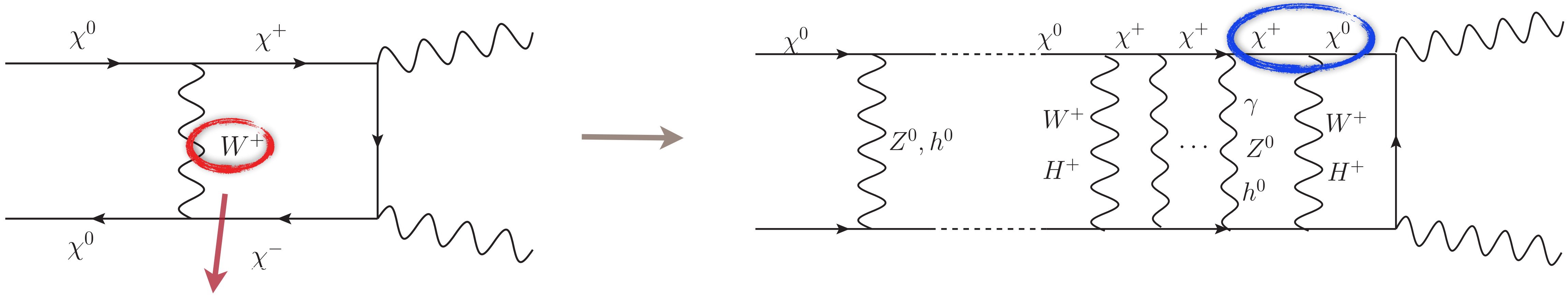
force carriers in the MSSM:

$\cancel{\gamma}, W^{\pm}, Z^0, h^0_1, h^0_2, H^{\pm}$

seminal papers

Hisano *et al.* '04,'06,...

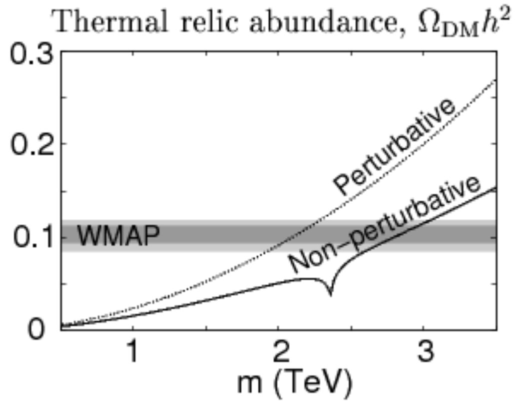
$\delta m \ll m_\chi$



$m_\chi \gg m_W$

THE SOMMERFELD EFFECT

FROM EW INTERACTIONS



force carriers in the MSSM:

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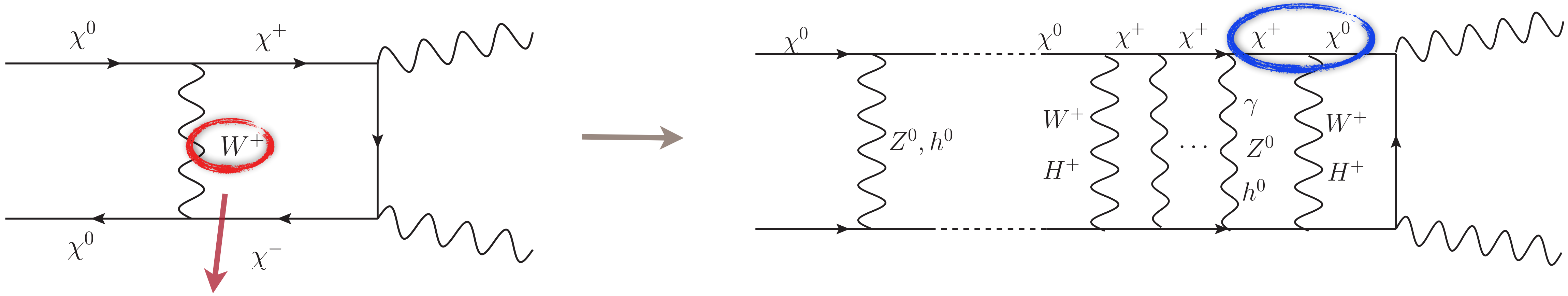


$m_\chi \gg m_W$

at TeV scale $\Rightarrow$ generically effect of $\mathcal{O}(1-100\%)$

on top of that resonance structure

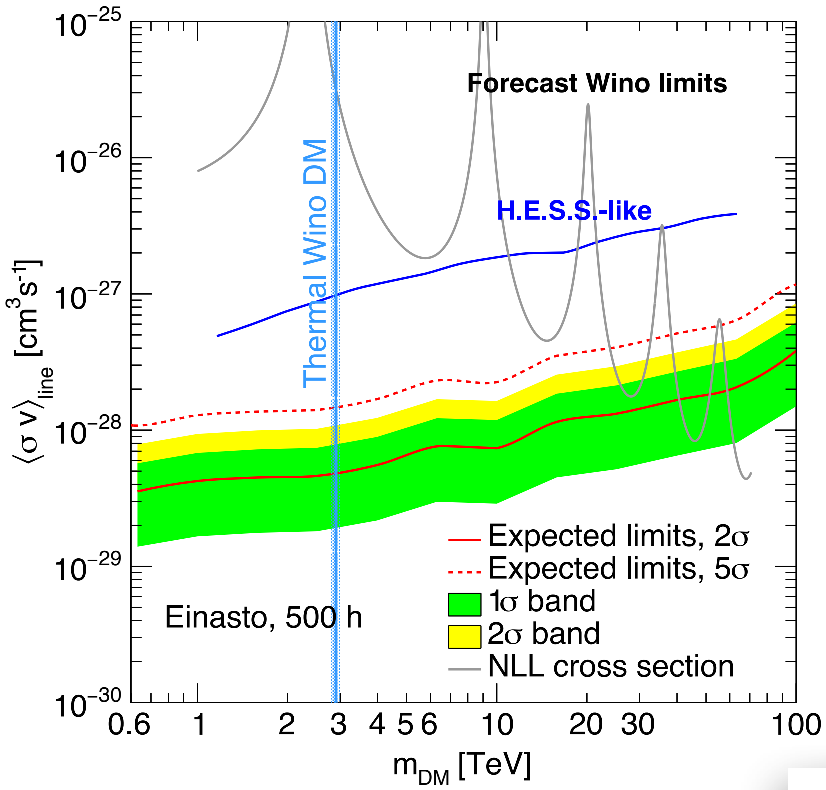
$\rightarrow$ effect of $\mathcal{O}(\text{few})$
for the relic density

AH, R. Iengo, P. Ullio. '10
AH '11
AH *et al.* '17, M. Beneke *et al.*; '16

can be understood as being close to
a threshold of lowest bound state

THE SOMMERFELD EFFECT

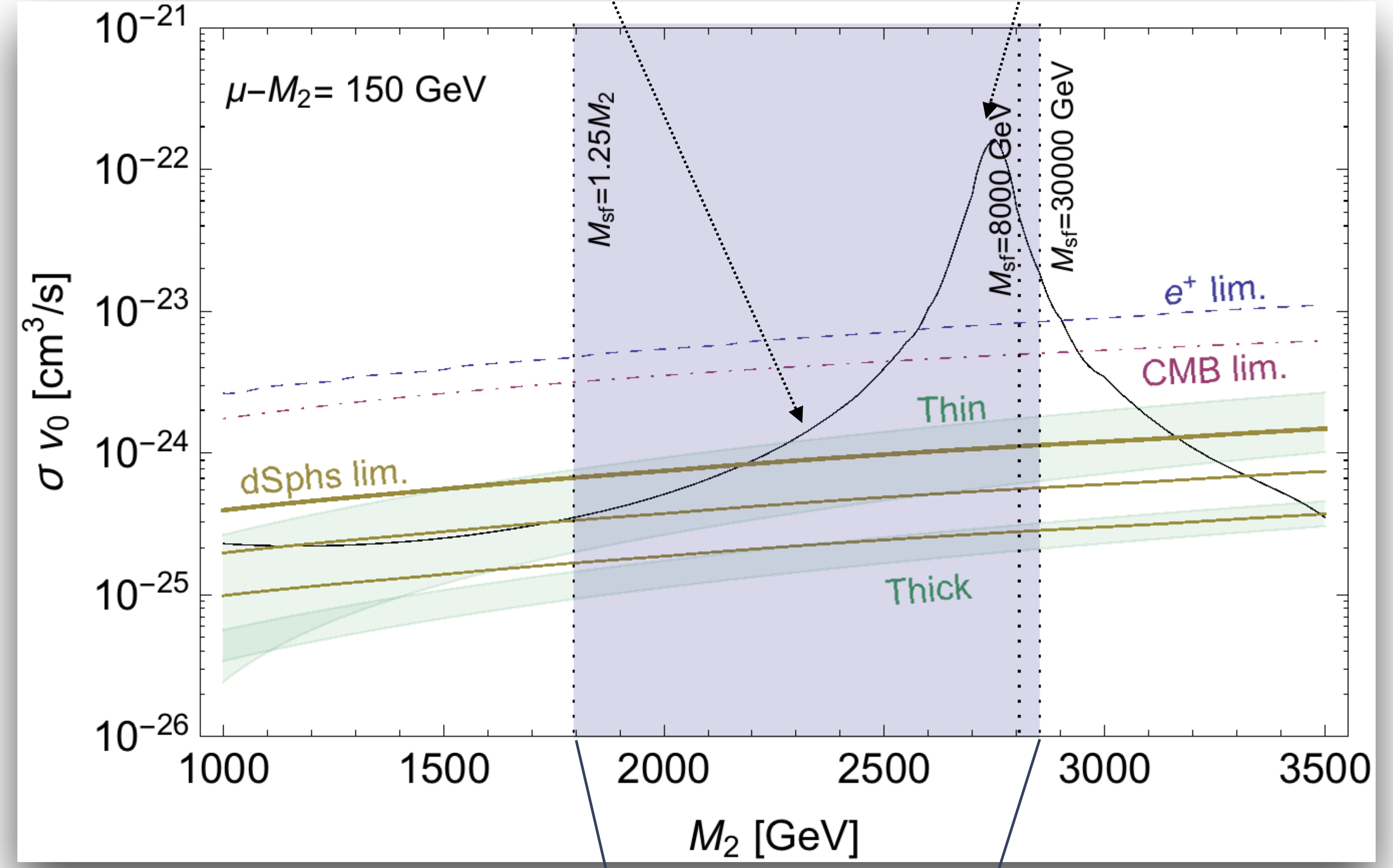
INDIRECT DETECTION



Slatyer *et al.*, **'21**

actual
cross section

resonance moves
to the right
w.r.t. pure wino



Beneke, ...AH, ... *et al.*, **'16**

correct RD can be achieved:
when varying sfermion masses

similar study, pure Wino case: Ibe ***et al.*** **'15**

BOUND STATE FORMATION

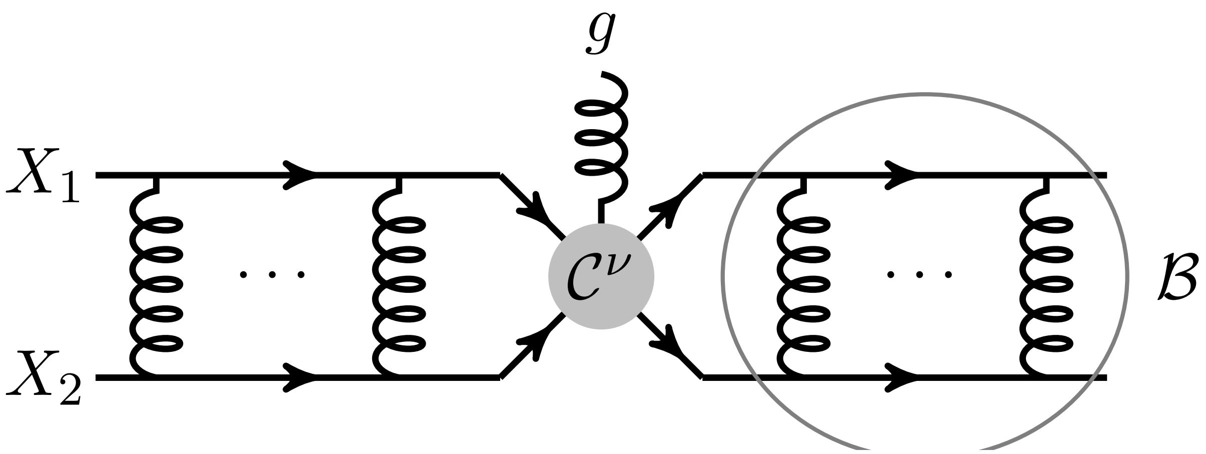
As noticed before Sommerfeld effect has resonances when Bohr radius ~ potential range, i.e. when close to a bound state threshold →

Can DM form actual bound states from such long range interactions?

↓

Yes, it can!

Q: How to describe such bound states and their formation?



free DM states

DM bound state

*the effect was first studied in simplified models with light mediators, then gradually extended to non-Abelian interactions, double emissions, co-annihilations, etc.

see papers by **K. Petraki** *et al.* '14-19

vide also "WIMPonium" **March-Russel, West '10

EXAMPLE:
IMPACT ON THE UNITARITY BOUND

Conservation of probability (for any partial wave) $\Longrightarrow$ $(\sigma v_{\rm rel})^J_{\rm total} < (\sigma v)^J_{\rm max} = \frac{4\pi(2\,J+1)}{M^2_{\rm DM} v_{\rm rel}}$

$\Longrightarrow$ upper limit on DM mass if thermally produced: "$M_{\rm DM} < 340\,{\rm TeV}$" (for a Majorana fermion and $\Omega h^2 = 1$)

$M_{\rm DM} < 200\,{\rm TeV}$ (updated)

Griest and **Kamionkowski** ’89

With the bound state annihilation taken into account:

$$(\sigma v_{\rm rel})_{\rm total} \;=\; (\sigma v_{\rm rel})_{\rm ann} + \underline{\textstyle\sum_I (\sigma_I v_{\rm rel})_{\rm BSF}}$$

but some of the bound states dissociate before they are able to annihilate!

$\Downarrow$

$(\sigma v_{\rm rel})_{\rm total}$ overestimates the cross section in the Boltzmann eq.

$\Lleftarrow$

maximal attainable mass for thermal DM is lower

$\Lleftarrow$

$M_{\rm DM} < 144\,{\rm TeV}$
(for a Majorana fermion coupled vis $SU(2)_L$)

Smirnov, Beacom ’19

(see also **von Harling, Petraki** ’14, **Cirelli** *et al.* ’16, …)

CHAPTER II:

NON-EQUILIBRIUM EFFECTS

THERMAL RELIC DENSITY

STANDARD SCENARIO

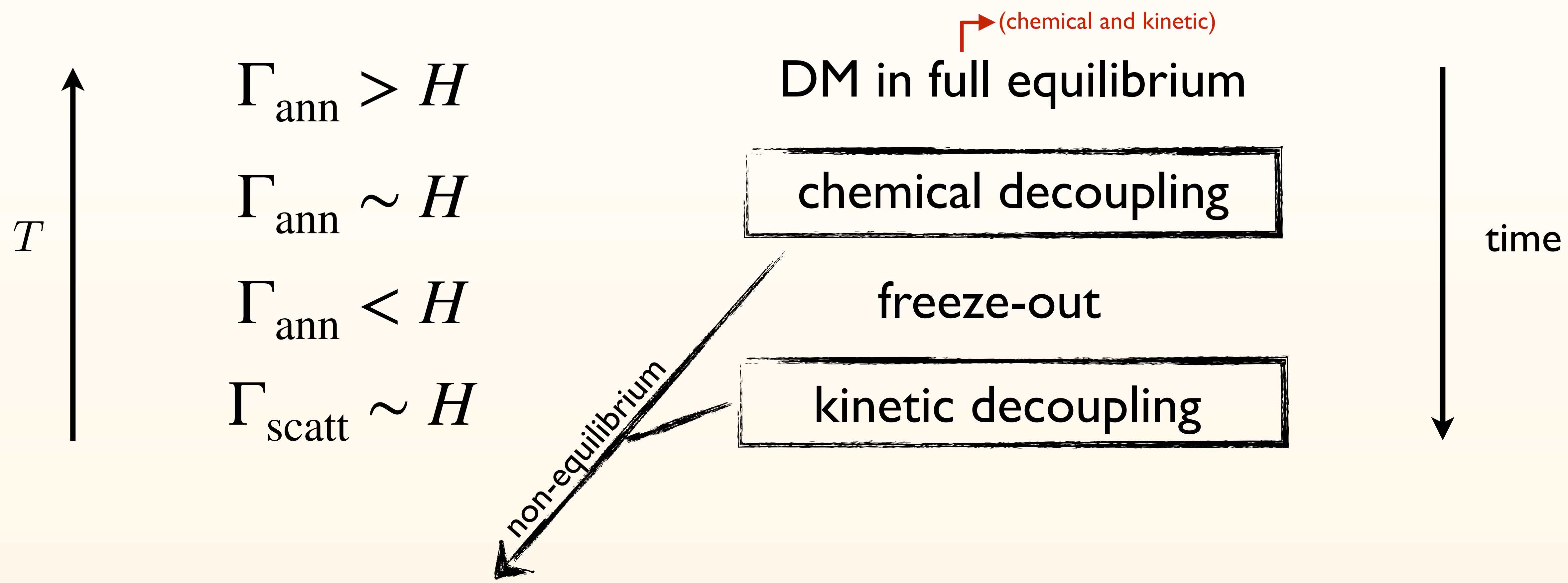
T

$\Gamma_{\text{ann}} > H$	DM in full equilibrium (chemical and kinetic)
$\Gamma_{\text{ann}} \sim H$	chemical decoupling
$\Gamma_{\text{ann}} < H$	freeze-out
$\Gamma_{\text{scatt}} \sim H$	kinetic decoupling

time

THERMAL RELIC DENSITY

STANDARD SCENARIO



(chemical and kinetic)

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time evolution of $f_\chi(p)$ in kinetic theory:

$$\boxed{E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right)} f_\chi = \mathcal{C}[f_\chi]$$

Liouville operator in FRW background

the collision term

THERMAL RELIC DENSITY

STANDARD APPROACH

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq:
classical limit, molecular chaos,...

…for derivation from thermal QFT
see e.g., 1409.3049

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…for derivation from thermal QFT see e.g., 1409.3049

$\Downarrow$ integrate over p (i.e. take 0th moment)

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$\Downarrow$

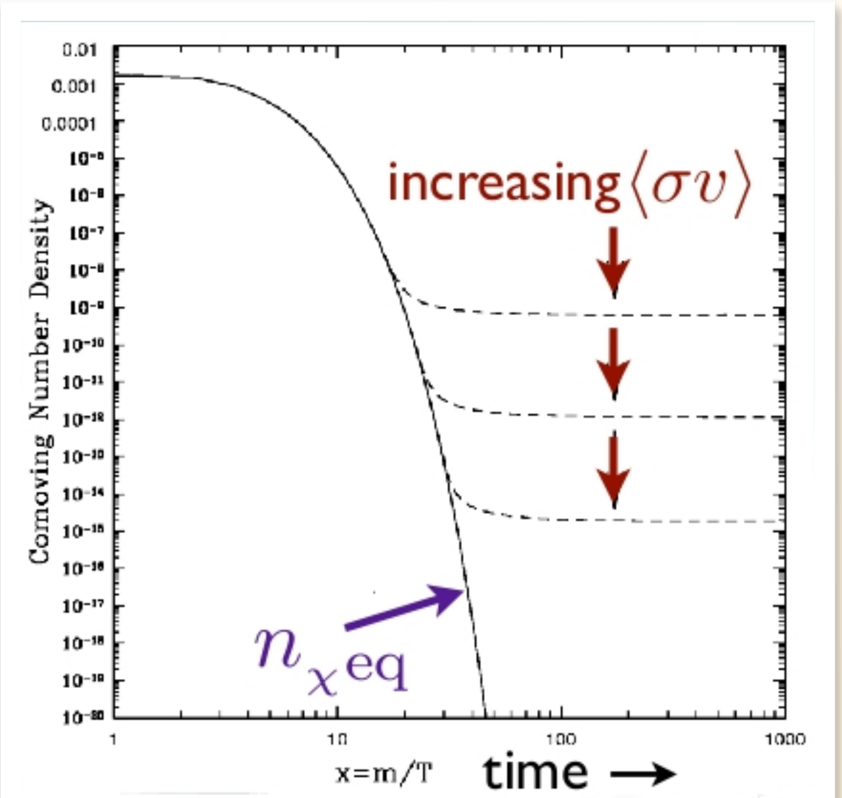

Fig.: Jungman, Kamionkowski & Griest, PR'96

THERMAL RELIC DENSITY

STANDARD APPROACH

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⇓ integrate over *p*
(i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar\chi\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar\chi} - n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}\right)$$

⇓

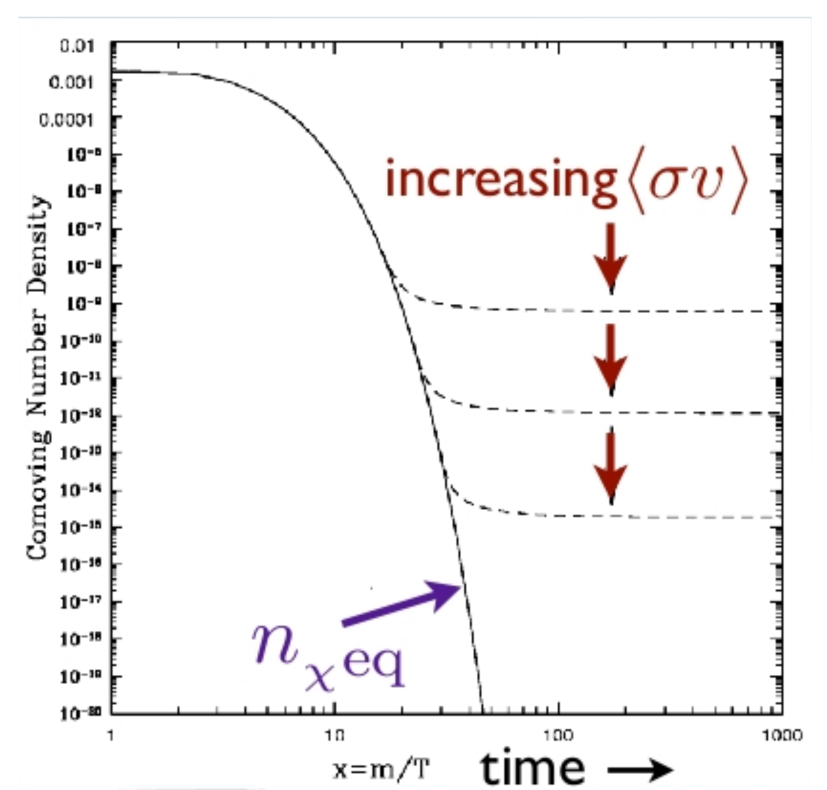


Fig.: Jungman, Kamionkowski & Griest, PR'96

Critical assumption:
kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\rm eq}$$

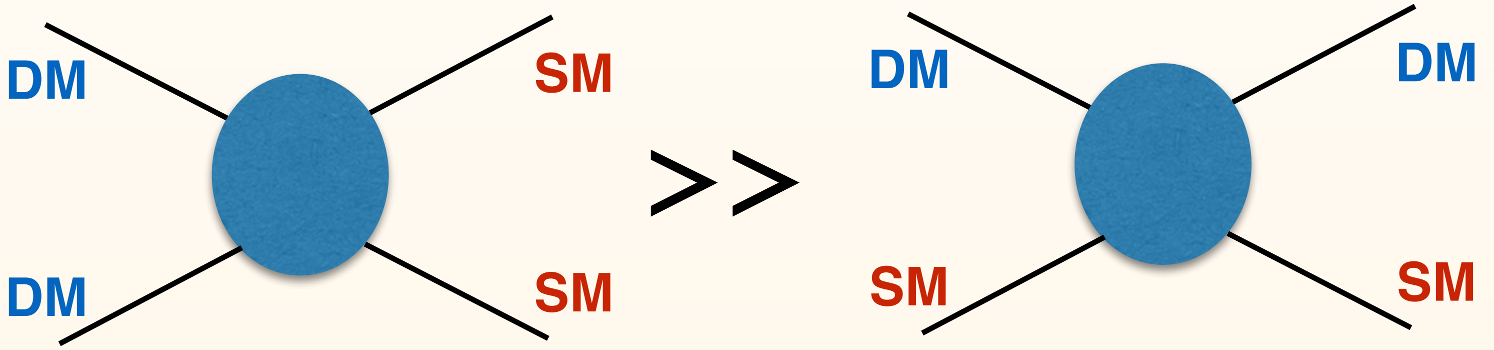
Early Kinetic Decoupling?

A necessary and sufficient condition: scatterings weaker than annihilation

i.e. rates around freeze-out: $H \sim \Gamma_{\mathrm{ann}} \gtrsim \Gamma_{\mathrm{el}}$

Possibilities:

A)



e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

D) Multi-component dark sectors

e.g., additional sources of DM from late decays, …

HOW TO GO BEYOND KINETIC EQUILIBRIUM?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

contains both scatterings and annihilations

Two possible approaches:

fBE

cBE

0-th moment: n_χ
2-nd moment: T_χ
$\dots$

solve numerically for full $f_\chi(p)$

have insight on the distribution

no constraining assumptions

numerically challenging

often an overkill

consider system of equations for moments of $f_\chi(p)$

partially analytic/much easier numerically

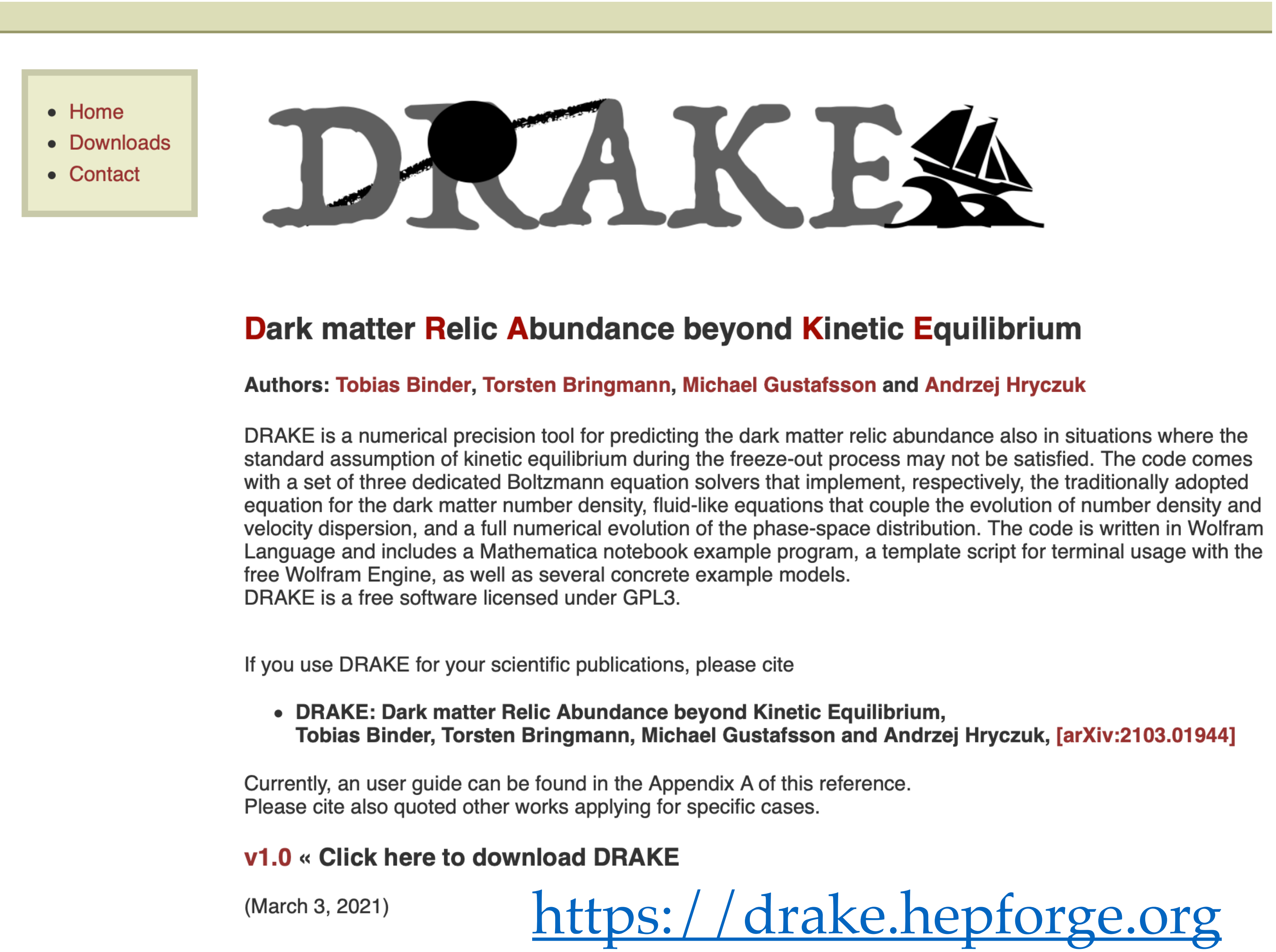
manifestly captures all of the relevant physics

finite range of validity

no insight on the distribution

NEW TOOL!

GOING BEYOND THE STANDARD APPROACH



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DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.
DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,
  Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]**

Currently, an user guide can be found in the Appendix A of this reference.
Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for
any (user defined) model*

Interplay between chemical and
kinetic decoupling

Prediction for the DM
phase space distribution

Late kinetic decoupling
and impact on cosmology

see e.g., 1202.5456

…

(only) prerequisite:
Wolfram Language (or *Mathematica*)

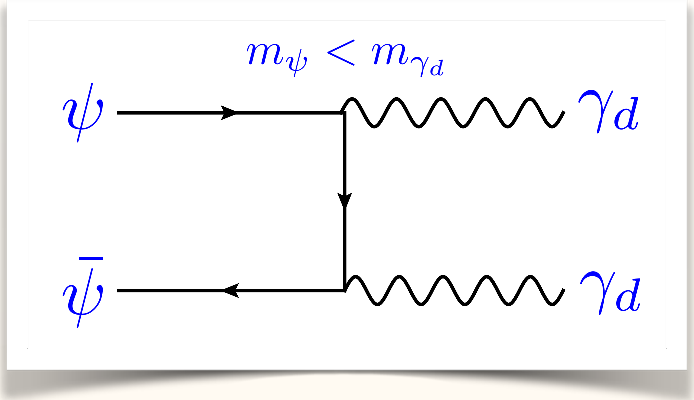
*at the moment for a single DM species and w/o co-annihlations… but stay tuned for extensions!

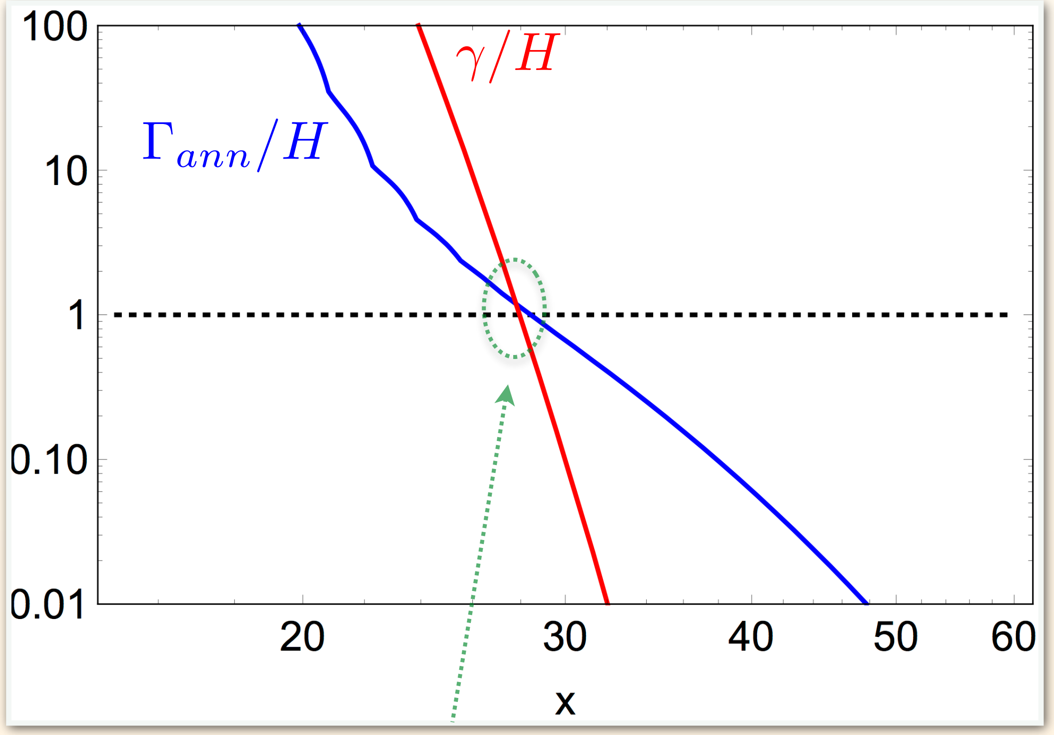
EXAMPLE:

FORBIDDEN DARK MATTER

DM is a thermal relic that annihilates only to heavier states
(forbidden in zero temperature)

…, D'Agnolo, Ruderman '15, …





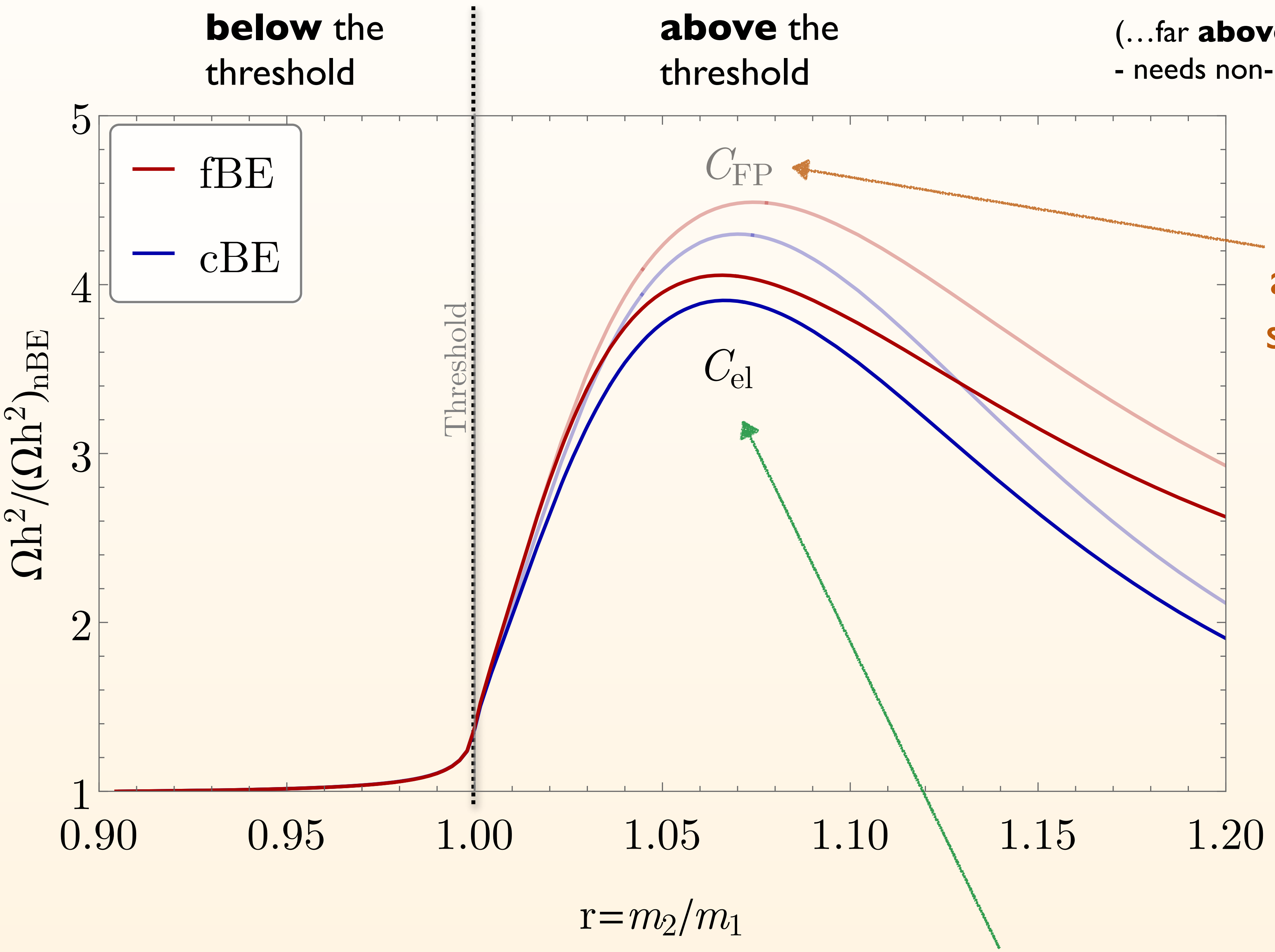
kinetic and chemical
decoupling close

Annihilation threshold → velocity dependence

"heavy" SM particle → scattering rate low

FORBIDDEN DARK MATTER

EXAMPLE EFFECT OF EARLY KD ON RELIC DENSITY



Fokker-Planck approx. for the scattering term

effect on relic density: up to O(~few)

full (numerical) scattering term

DM ELASTIC SCATTERINGS

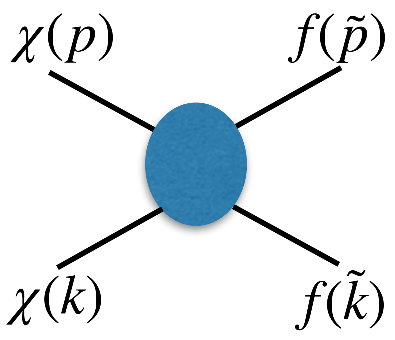
(WITH ITSELF AND WITH PLASMA PARTICLES)

ELASTIC SCATTERING COLLISION TERM

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

→ contains both scatterings and annihilations

Annihilation:



$$C_{\rm ann} \sim \int d\tilde{\Pi}\, |\mathscr{M}|^2_{\chi\chi\leftrightarrow f\bar{f}} \left(\underbrace{f_f^{\rm eq}(\tilde{p}) f_f^{\rm eq}(\tilde{k})}_{\text{easy}} - \underbrace{f_\chi(p) f_\chi(k)}_{\text{easy}}\right)$$

easy: no unknown f_χ under integral

⇒ 1D integration

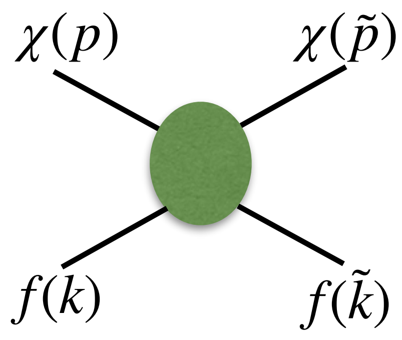
medium: no unknown f_χ under integral

⇒ 2-3D integration

hard: unknown f_χ under integral

⇒ 2-4D integration

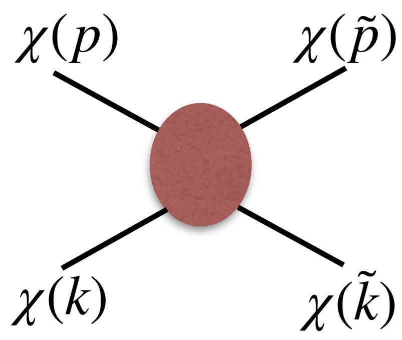
El. scattering (on SM particles):



$$C_{\rm el} \sim \int d\tilde{\Pi}\, |\mathscr{M}|^2_{\chi f\leftrightarrow \chi f} \left(\underbrace{f_\chi(\tilde{p}) f_f^{\rm eq}(\tilde{k})(1 \pm f_f^{\rm eq}(k))}_{\text{hard}} - \underbrace{f_\chi(p) f_f^{\rm eq}(k)(1 \pm f_f^{\rm eq}(\tilde{k}))}_{\text{medium}}\right)$$

An approximate method needed!

El. self-scattering (DM on DM):



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$$d\tilde{\Pi} = d\Pi_{\tilde{p}} d\Pi_k d\Pi_{\tilde{k}} \delta^{(4)}(\tilde{p} + p - \tilde{k} - k)$$

APPROACHES

I) Expand in „small momentum transfer”

$$M_{\rm DM} \gg |\overrightarrow{q}| \sim T \gg m_{\rm SM}$$

typical momentum transfer

Bringmann, Hofmann ’06

$$\delta^{(3)}(\tilde{\mathbf{p}} + \tilde{\mathbf{k}} - \mathbf{p} - \mathbf{k}) \approx \sum_n \frac{1}{n!}(\mathbf{q}\nabla_{\tilde{\mathbf{p}}})^n \delta^{(3)}(\tilde{\mathbf{p}} - \mathbf{p})$$

Kasahara ’09; Binder, Covi, Kamada, Murayama, Takahashi, Yoshida ’16

$$f_3 \simeq f_1 + \tilde{\mathbf{q}}_i \frac{\partial f_1}{\partial \mathbf{p}_{1i}} + \frac{1}{2}\tilde{\mathbf{q}}_i \tilde{\mathbf{q}}_j \frac{\partial^2 f_1}{\partial \mathbf{p}_{1i} \partial \mathbf{p}_{1j}}$$

A.H. & S. Chatterjee, *work in progress…*

(on different expansion schemes)

$\Longrightarrow$ all lead to Fokker-Planck type eq.

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$\Longrightarrow$ simpler, but generally incorrect

Ala-Mattinen, Kainulainen ’19

Ala-Mattinen, Heikinheimo, Kainulainen, Tuominen ’22

$$\begin{aligned}\hat{C}_{\mathrm{E},m}(p_1,t) &\to -\delta f(p_1,t)\,\Gamma^m_{\mathrm{E}}(p_1,t)\\ &= (\,g_m(t) f_{\mathrm{eq}}(p_1,t) - f(p_1,t)\,)\;\Gamma^m_{\mathrm{E}}(p_1,t)\end{aligned}$$

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$$(\hat{p}^i)' = -\hat{\eta}\,\hat{p}^i + \hat{f}^i \,, \quad \langle\, \hat{f}^i(x_1)\,\hat{f}^j(x_2)\,\rangle = \hat{\zeta}\,\delta^{ij}\,\delta(x_1 - x_2)$$

⇒ very new, promising…

Kim, Laine '23

stochastic term, taking care of detailed balance

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stochastic term, taking care of detailed balance

IV) Fully numerical implementation

A.H. & M. Laletin 2204.07078 (focus on DM self-scatterings)
Ala-Mattinen, Heikinheimo, Kainulainen, Tuominen ’22
Du, Huang, Li, Li, Yu ’21

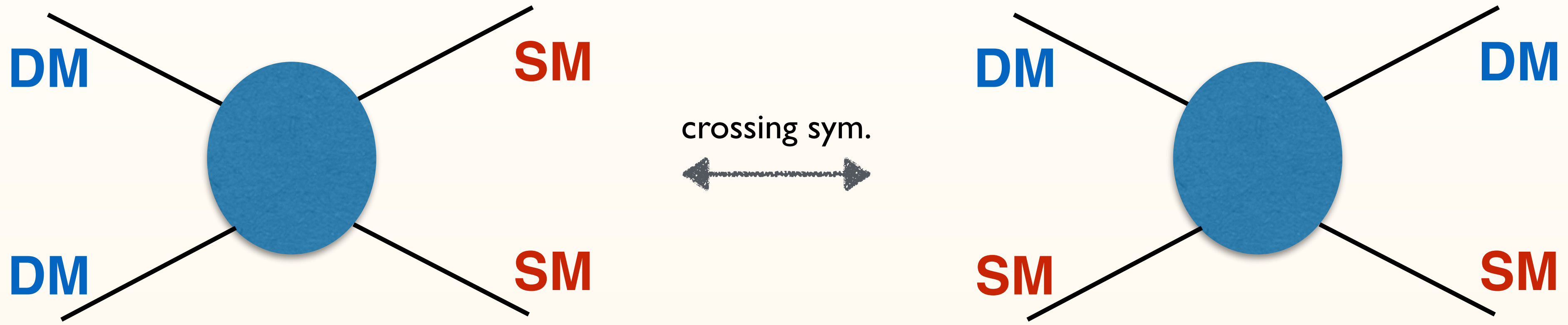
$\Longrightarrow$ doable, but very CPU expensive

CHAPTER III:

MULTI-COMPONENT DARK MATTER

WHAT IF A NON-MINIMAL SCENARIO?

In a minimal WIMP case only two types of processes are relevant:



annihilation

(elastic) scattering

drives number density evolution

scatterings typically more frequent
(keeping the distribution to be in local thermal eq.)

Schmid, Schwarz, Widern ’99; Green, Hofmann, Schwarz

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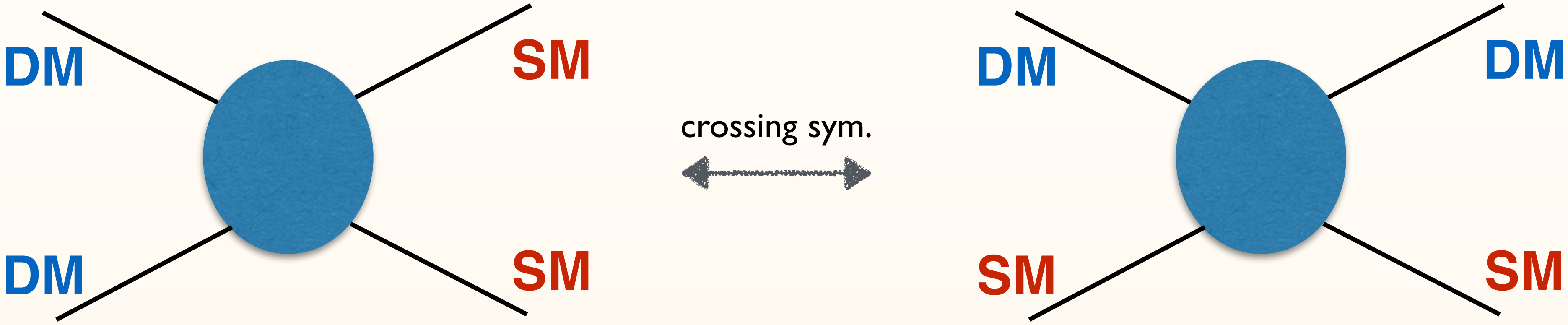

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Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz

Recall: in *standard* thermal relic density calculation:

Critical assumption:
kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(\mu) f_\chi^{\mathrm{eq}}$$

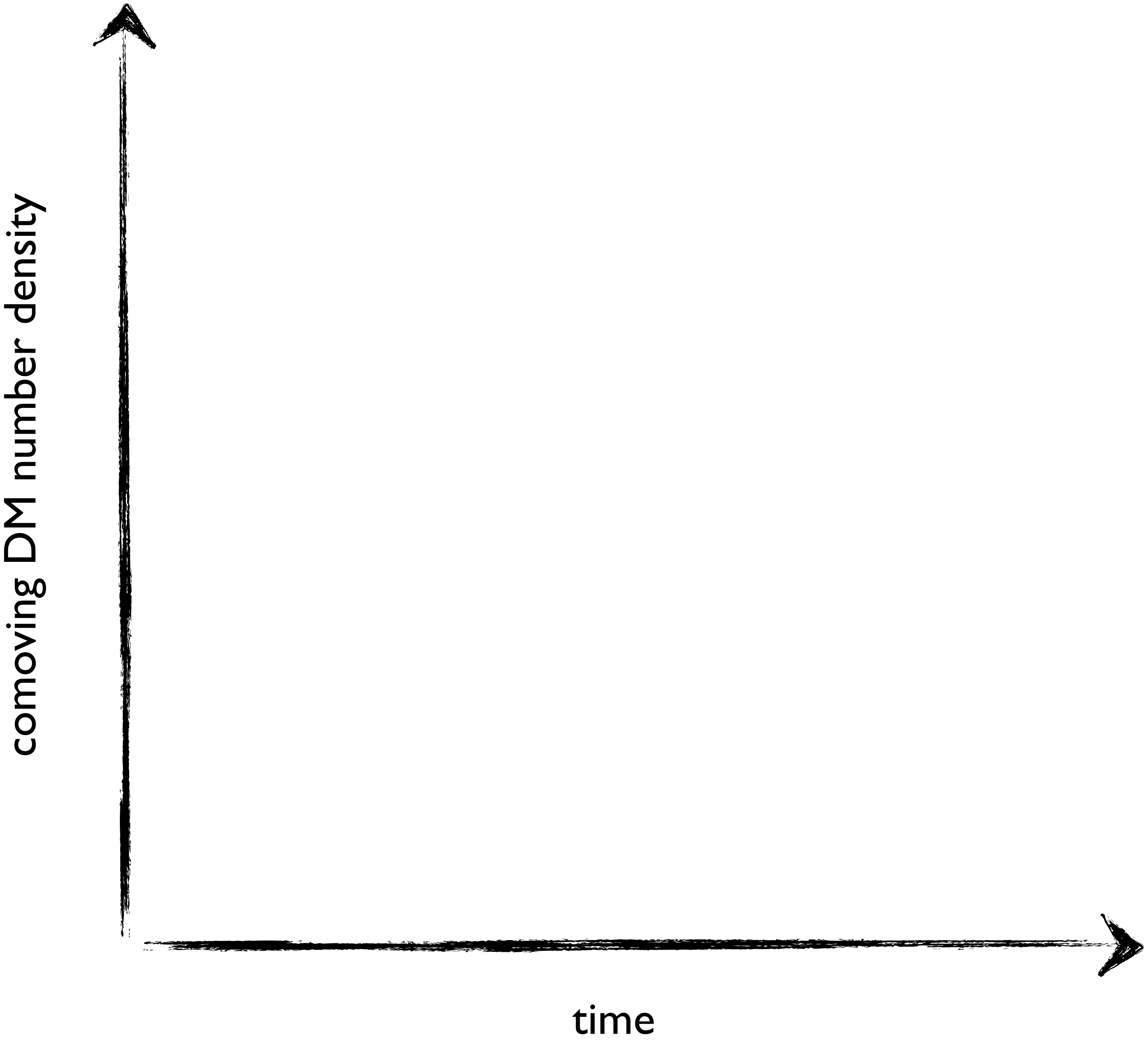
EXAMPLE D:

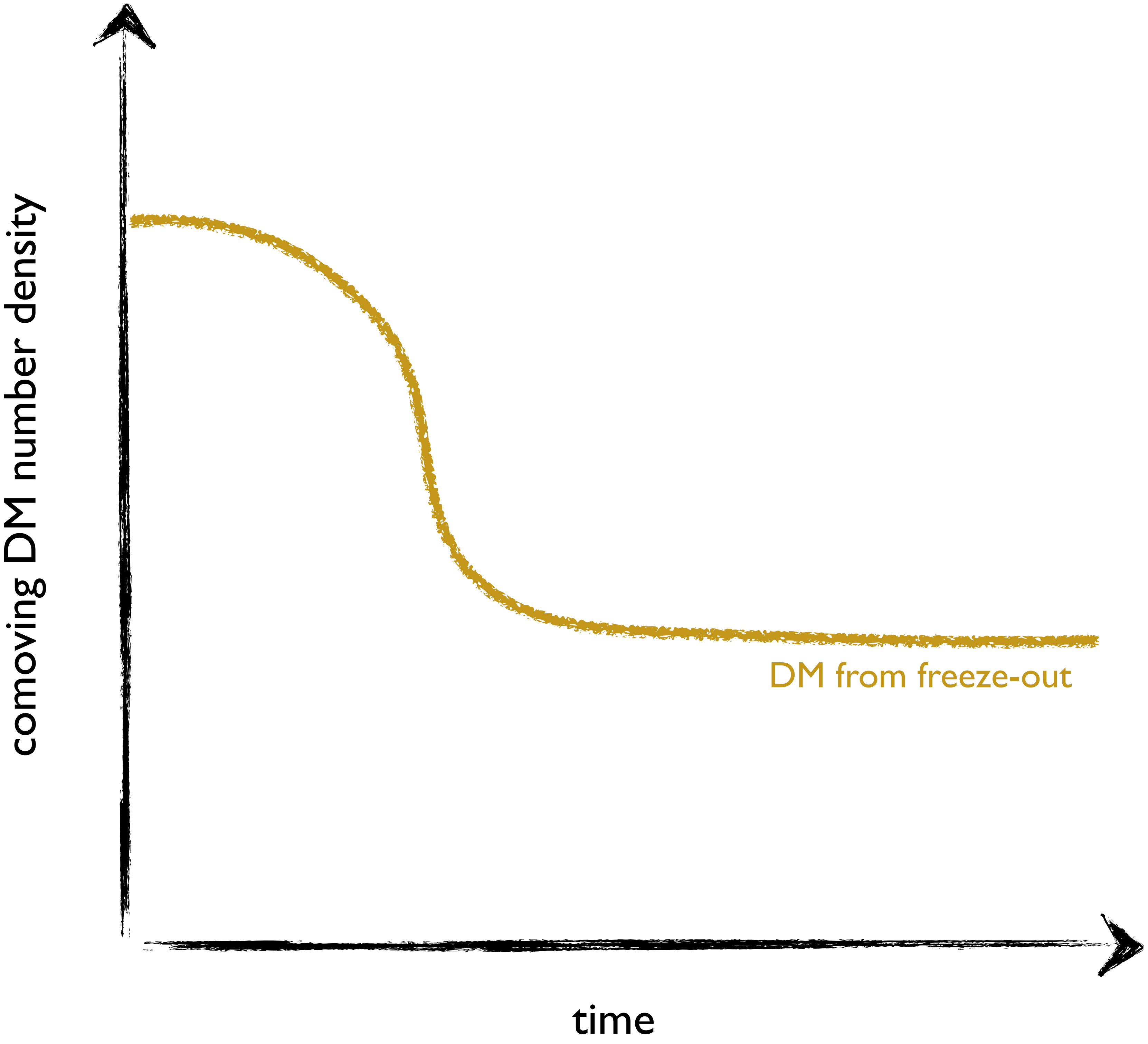
WHEN ADDITIONAL INFLUX OF DM ARRIVES

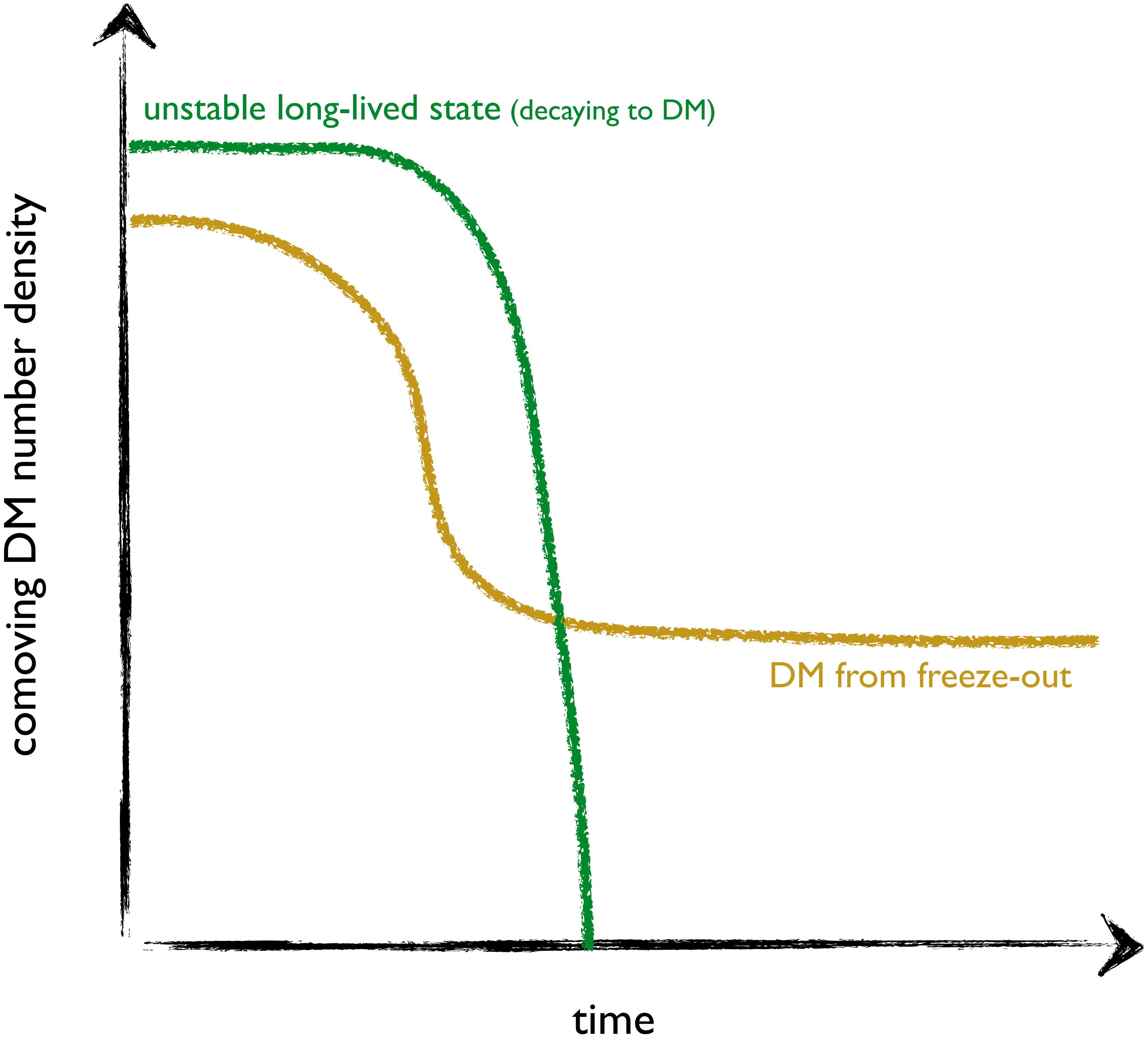
D) Multi-component dark sectors

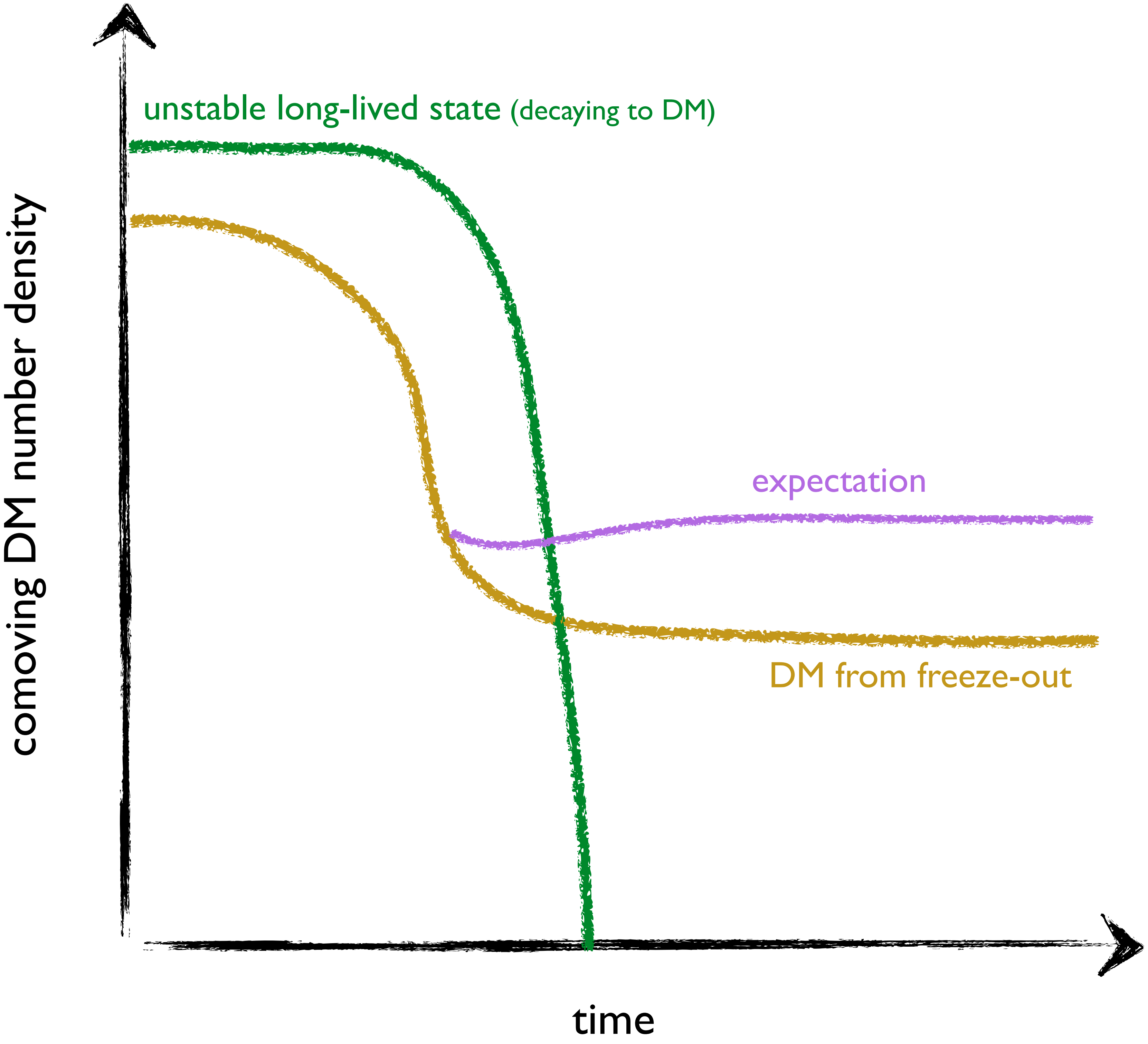
Sudden injection of more DM particles distorts $f_\chi(p)$
(e.g. from a decay or annihilation of other states)

- this can modify the annihilation rate (if still active)
- how does the thermalization due to elastic scatterings happen?


comoving DM number density
time


comoving DM number density
DM from freeze-out
time


unstable long-lived state (decaying to DM)
comoving DM number density
DM from freeze-out
time


unstable long-lived state (decaying to DM)
expectation
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comoving DM number density
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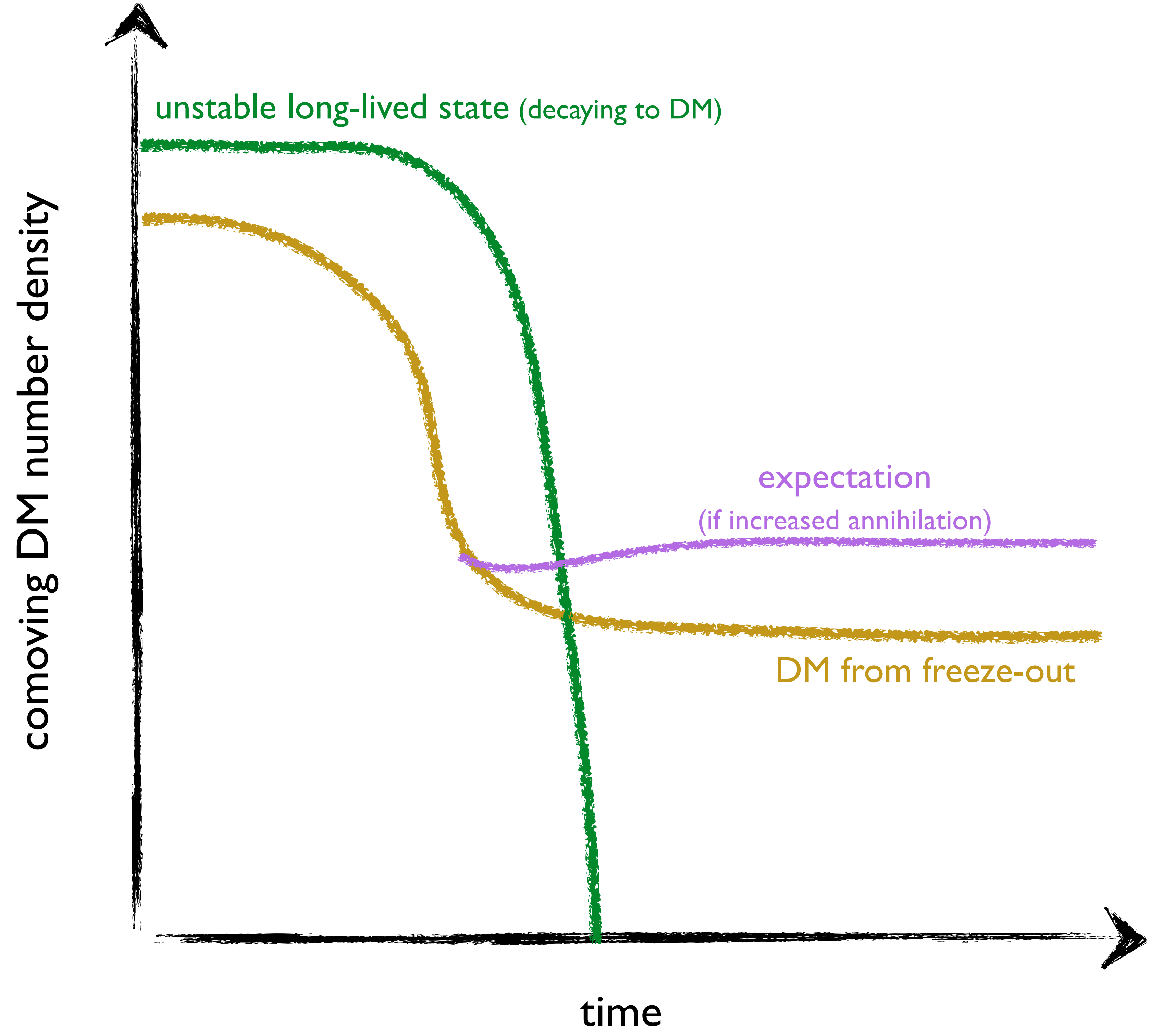
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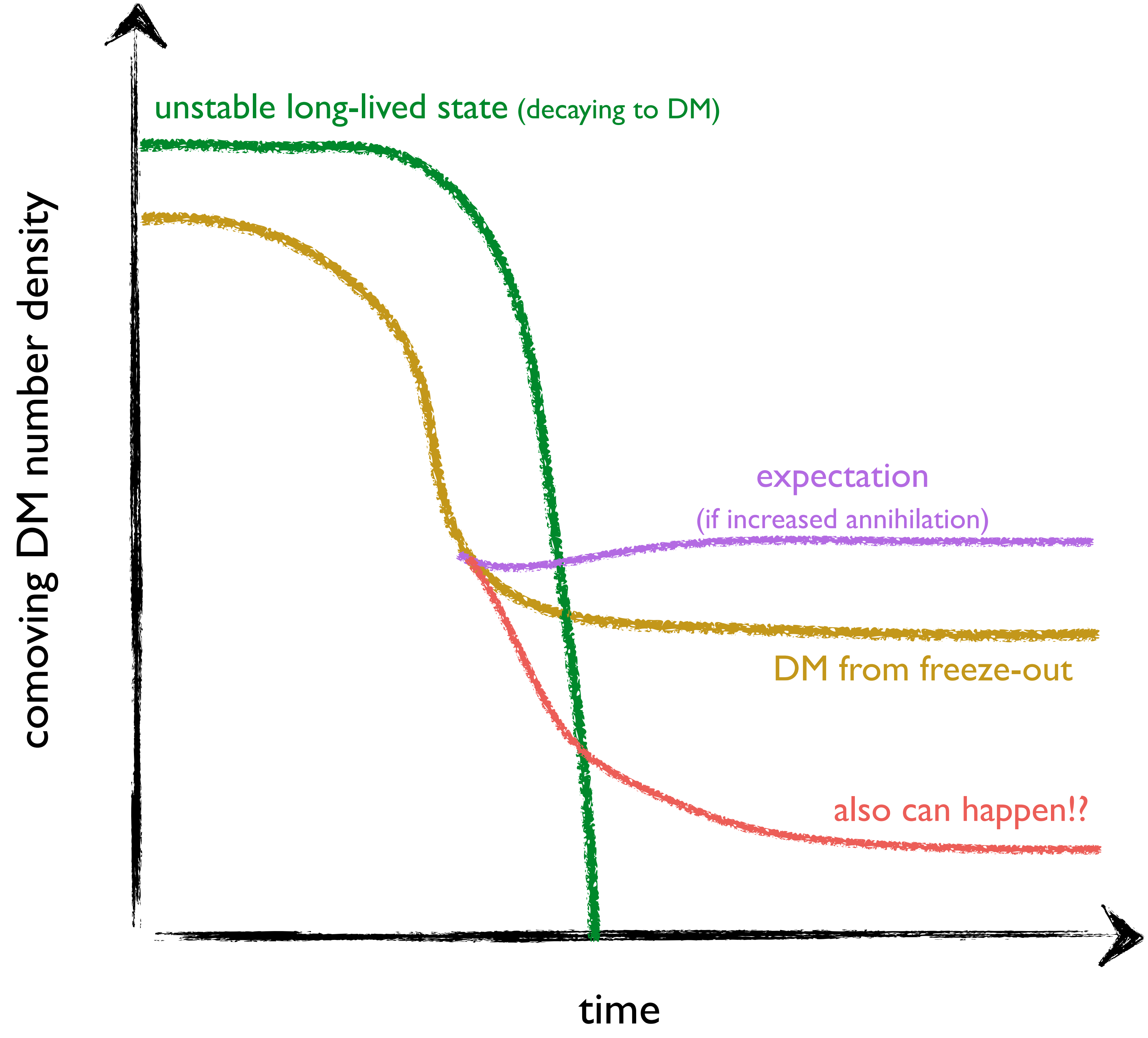
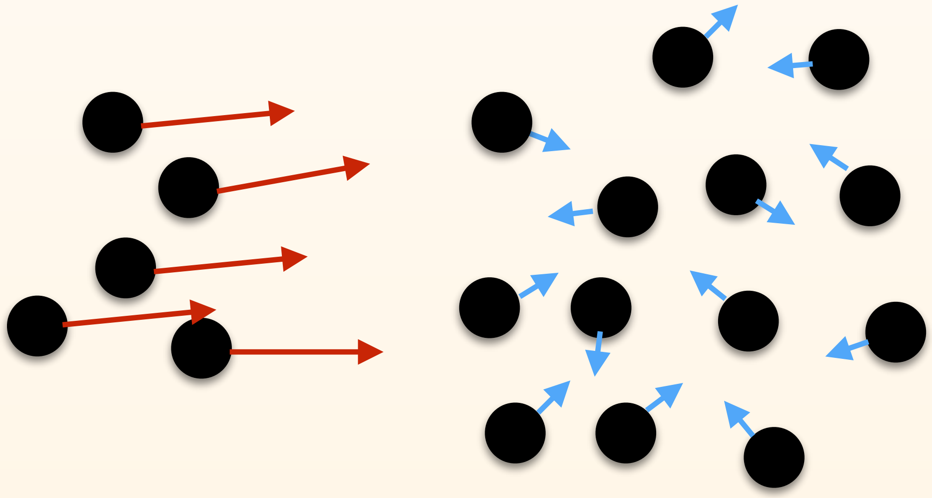

unstable long-lived state (decaying to DM)
expectation
(if increased annihilation)
DM from freeze-out
also can happen!?
comoving DM number density
time

1) DM produced via:

- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

2) DM annihilation has a threshold

e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$



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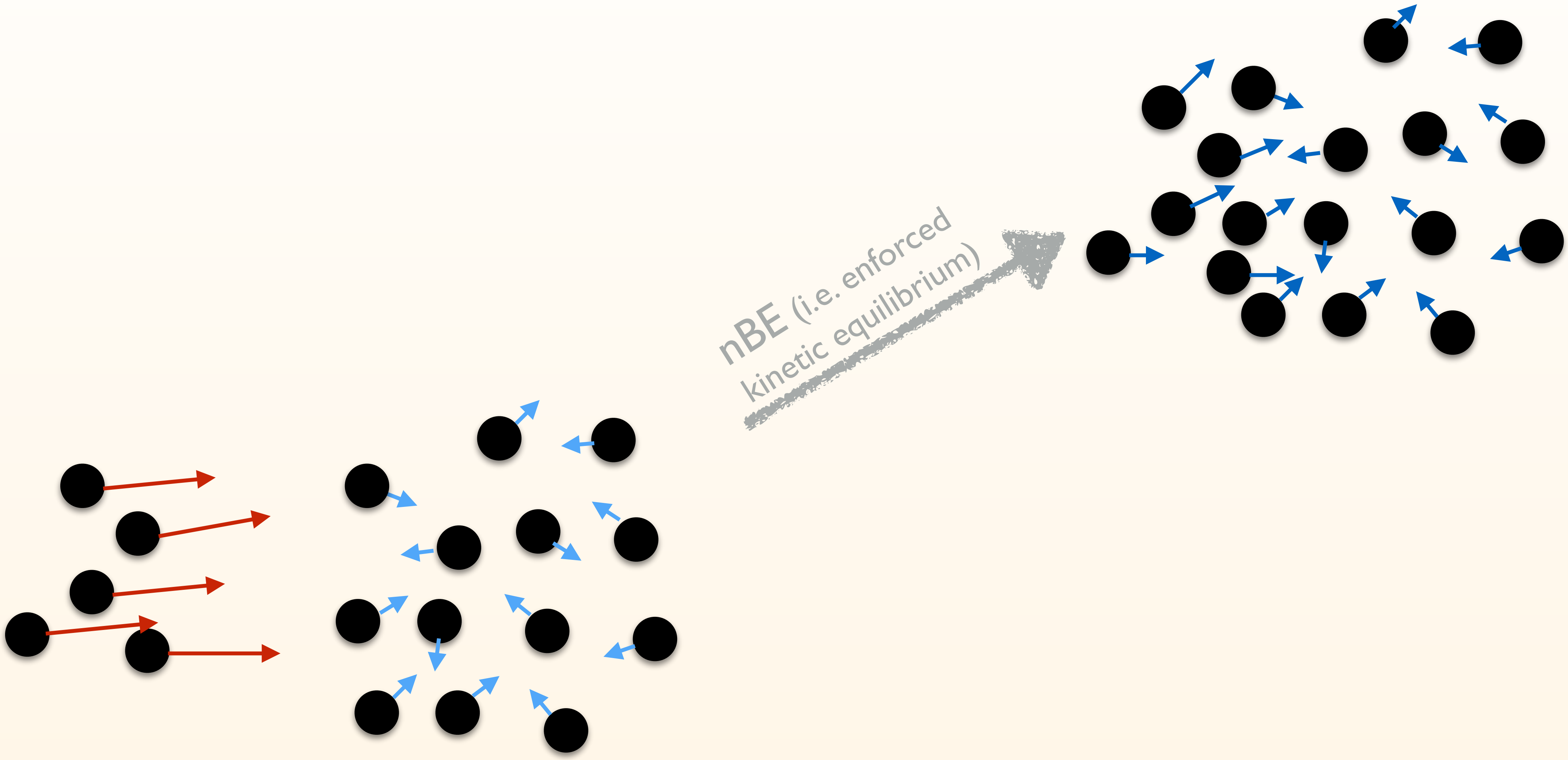

no enhanced annihilation, more DM in the end

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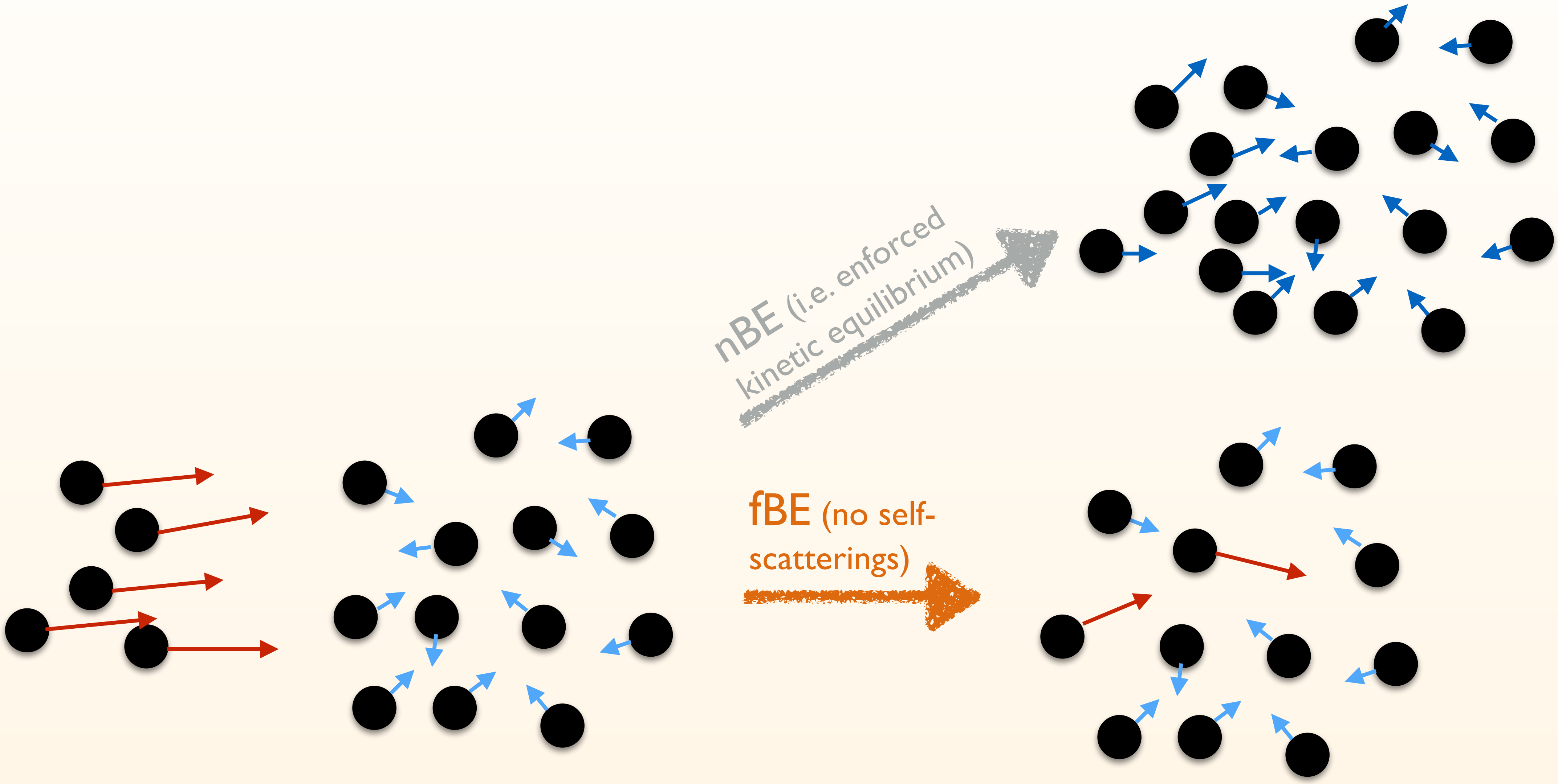

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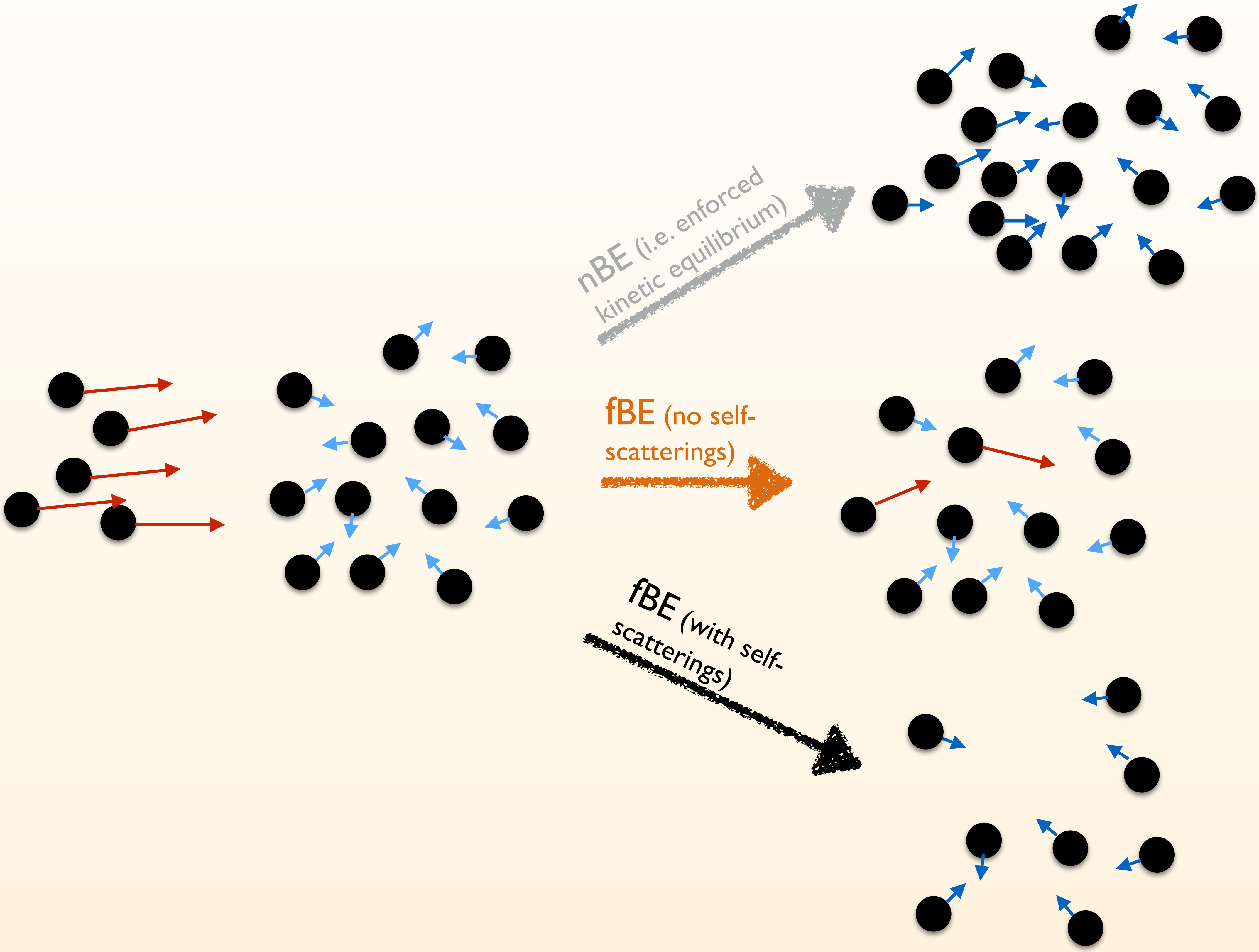

no enhanced annihilation, more DM in the end

some injected particles will annihilate together with themselves and cold component

energy redistribution will allow more DM particles to reach energies over annihilation threshold

AH, Laletin 2204.07078

EXAMPLE EVOLUTION

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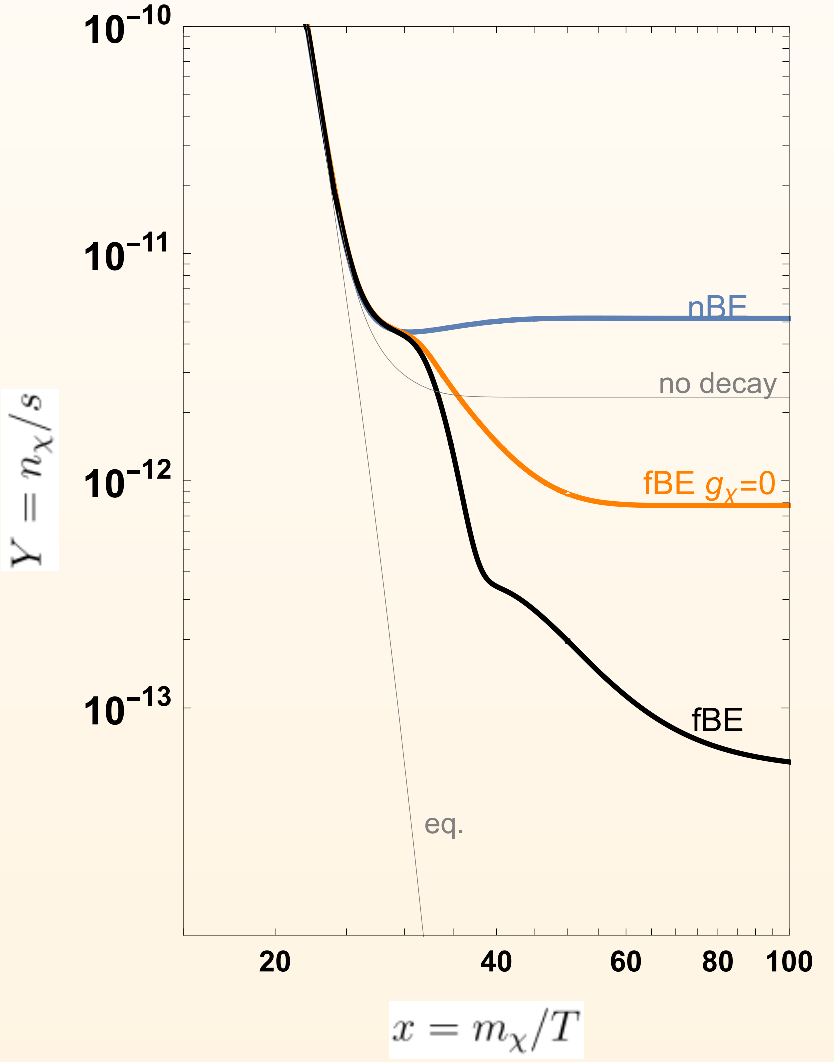
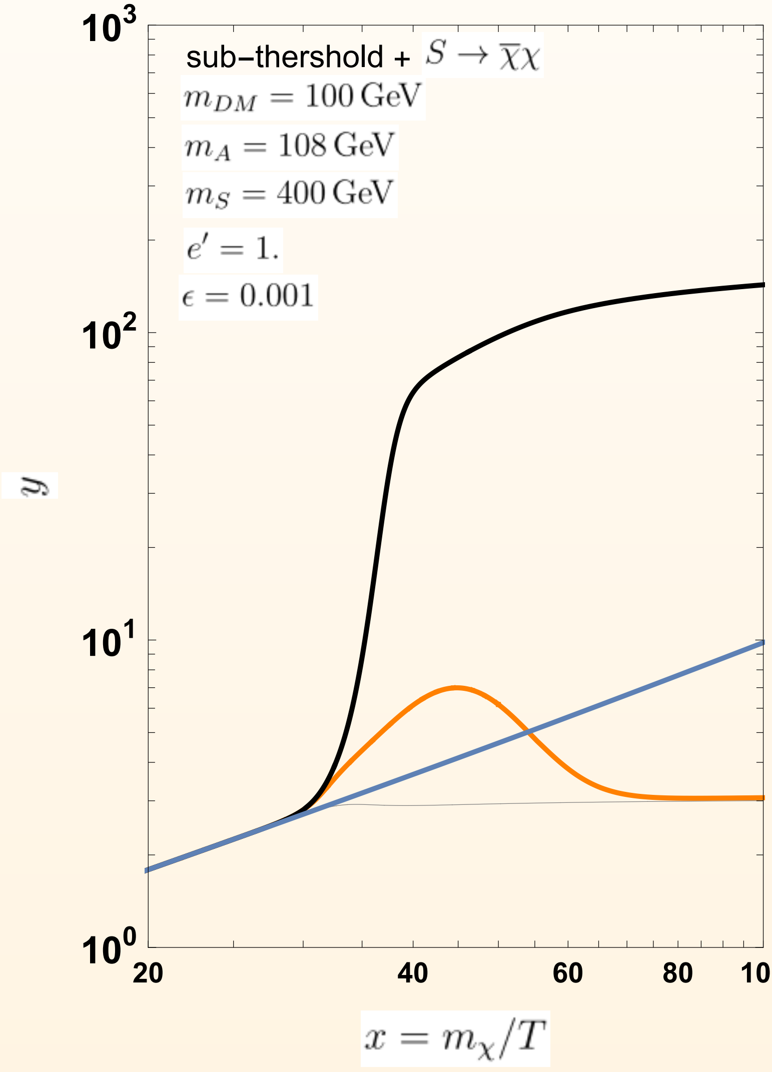
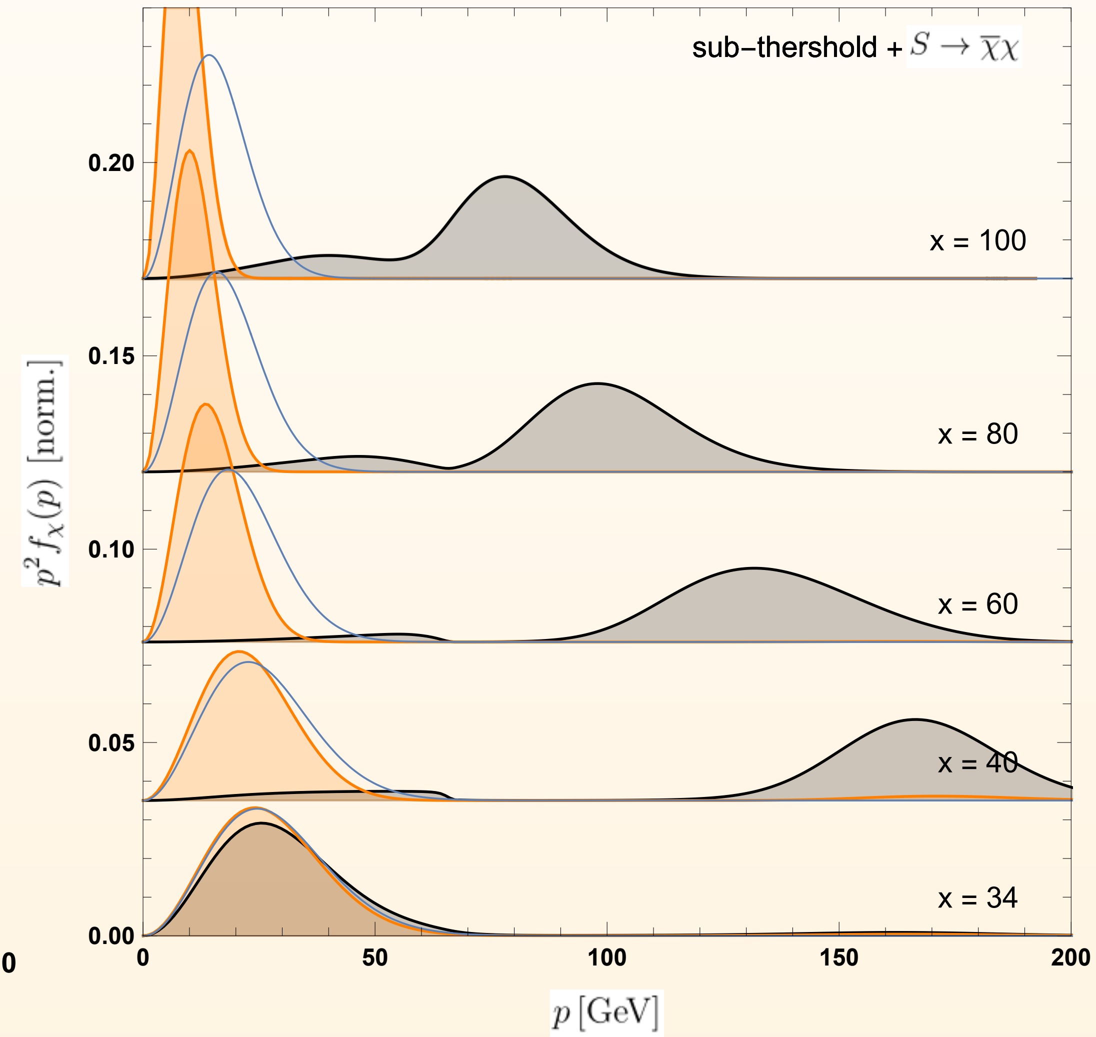
$Y \sim$ number density



$y \sim$ temperature



$p^2 f(p) \sim$ momentum distribution



SUMMARY

1. In recent years a significant progress in refining the relic density calculations (not yet fully implemented in public codes!)

2. Kinetic equilibrium is a necessary (often implicit) assumption for standard relic density calculations in all the numerical tools…

…while it is not always warranted!

3. Introduced coupled system of Boltzmann eqs. for 0th and 2nd moments (cBE) allows for much more accurate treatment while the full phase space Boltzmann equation (fBE) can be also successfully solved for higher precision and/or to obtain result for $f_{\rm DM}(p)$

(we also introduced DRAKE a new tool to extend the current capabilities to the regimes beyond kinetic equilibrium)

TAKEAWAY MESSAGE

When computing relic density of dark matter one needs carefully to check if the standard treatment is sufficient for the case at hand

”Everything should be made as simple as possible, but no simpler.”

attributed to* **Albert Einstein**

*The published quote reads:

”It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience.”

„On the Method of Theoretical Physics" ,The Herbert Spencer Lecture, delivered at Oxford (10 June 1933); also published in *Philosophy of Science*, Vol. 1, No. 2 (April 1934), pp. 163-169., p. 165