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DARK MATTER PRODUCTION OUT OF KINETIC EQUILIBRIUM:

LATEST DEVELOPMENTS

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NCBJ

Based on:

T. Binder, T. Bringmann, M. Gustafsson & A.H. 1706.07433, 2103.01944

A.H. & M. Laletin 2204.07078, 2104.05684

work in progress with **S. Chatterjee**

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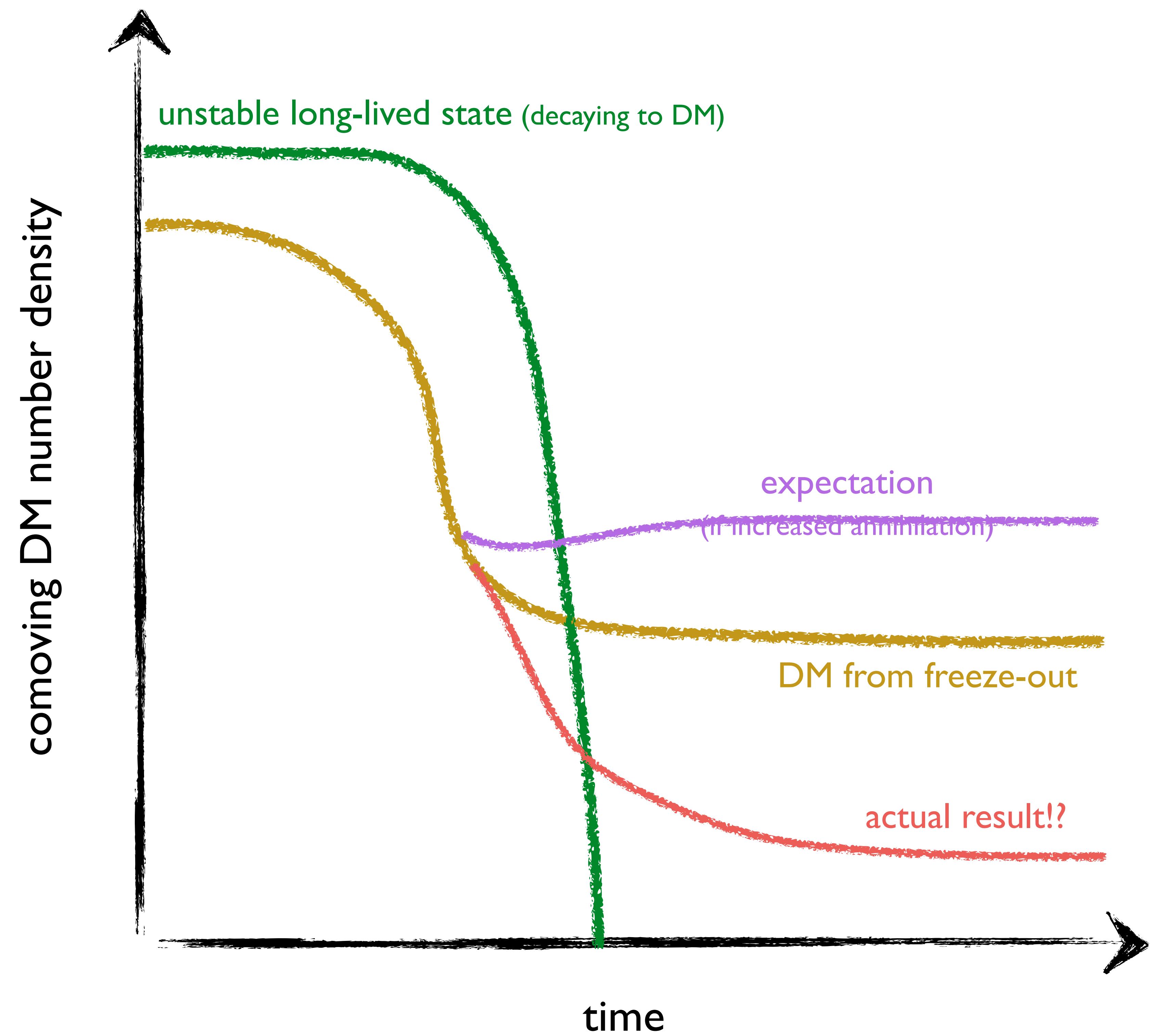
IN CASE YOU'RE NOT INTERESTED IN WHAT FOLLOWS…



TO SEE WHY AND LEARN MORE STAY TUNED :)

Thermal Relic Density

Standard Scenario

modified expansion rate

e.g., relentless DM, D'Eramo et al. '17, …

numerical codes e.g.,
DarkSUSY, micrOMEGAs, MadDM, SuperISOrelic, …

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}\to ij}\sigma_{\mathrm{rel}}\rangle^{\mathrm{eq}}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\mathrm{eq}} n_{\bar{\chi}}^{\mathrm{eq}}\right)$$

general multi-component dark sector

$$\frac{dn_\chi}{dt} + 3Hn_\chi \quad \ldots \quad n_{\bar{\chi}} - n_\chi^{\mathrm{eq}} n_{\bar{\chi}}^{\mathrm{eq}})$$

$$\frac{dn_\chi}{dt} + 3Hn_\chi \quad \ldots \quad n_{\bar{\chi}} - n_\chi^{\mathrm{eq}} n_{\bar{\chi}}^{\mathrm{eq}})$$

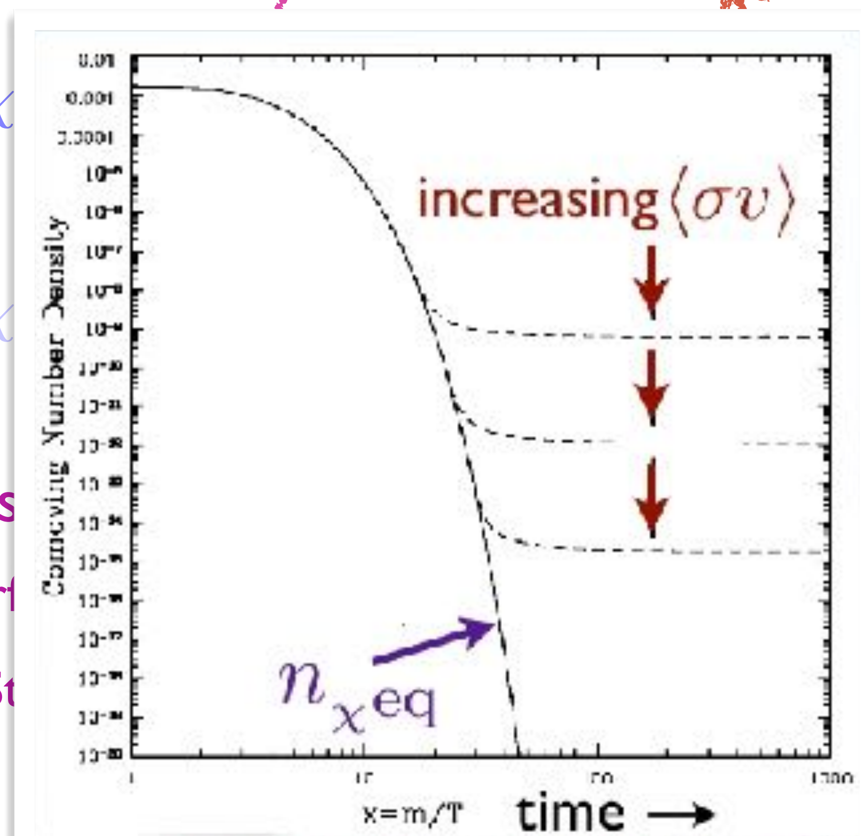


Fig.: Jungman, Kamionkowski & Griest, PR'96

modified cross

Sommerf

Bound St

NLO

finite T effects

breakdown of necessary assumptions leading to different form of the equation, <u>e.g. violation of kinetic equilibrium</u>

where the thermally averaged cross section:

$$\langle\sigma_{\chi\bar{\chi}\to ij}v_{\mathrm{rel}}\rangle^{\mathrm{eq}} = -\frac{h_\chi^2}{n_\chi^{\mathrm{eq}} n_{\bar{\chi}}^{\mathrm{eq}}}\int \frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\,\sigma_{\chi\bar{\chi}\to ij}v_{\mathrm{rel}}\; f_\chi^{\mathrm{eq}} f_{\bar{\chi}}^{\mathrm{eq}}$$

THERMAL RELIC DENSITY

STANDARD APPROACH

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq: classical limit, molecular chaos,...

...for derivation from thermal QFT see e.g., 1409.3049

integrate over p (i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar\chi\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar\chi} - n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}\right)$$

where the thermally averaged cross section:

$$\langle\sigma_{\chi\bar\chi\to ij}v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}}\int\frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\,\sigma_{\chi\bar\chi\to ij}v_{\rm rel}\, f_\chi^{\rm eq} f_{\bar\chi}^{\rm eq}$$

Critical assumption:

kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\rm eq}$$

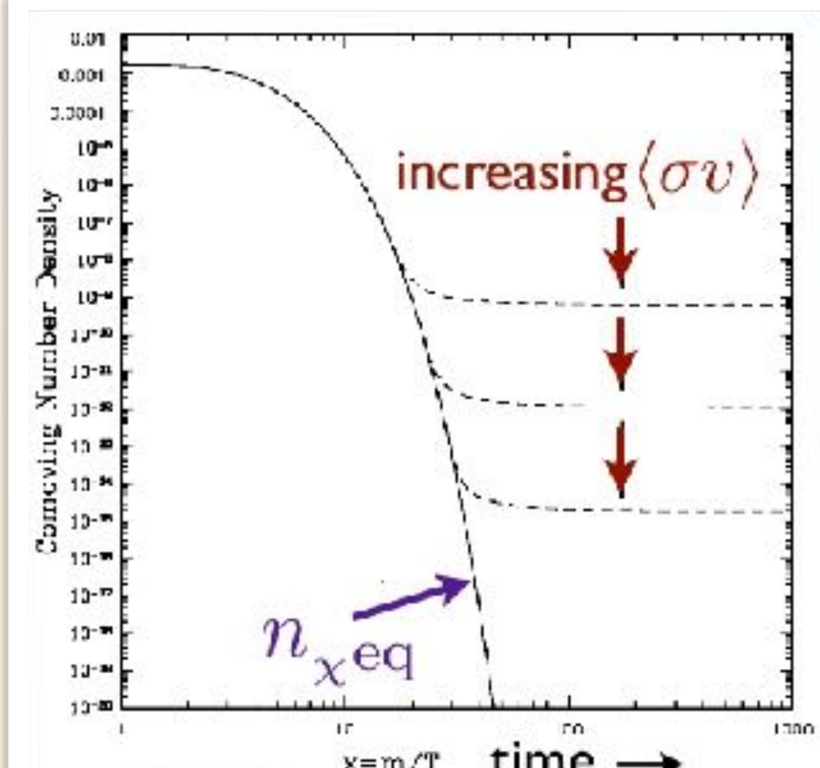
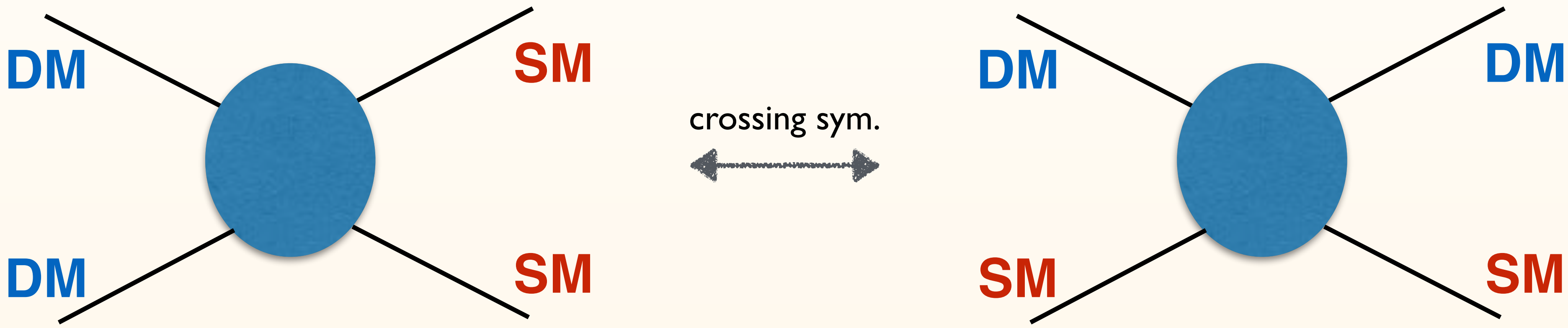


Fig.: Jungman, Kamionkowski & Griest, PR'96

FREEZE-OUT *VS.* DECOUPLING

annihilation | (elastic) scattering



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Rightarrow$ scatterings <u>typically</u> more frequent

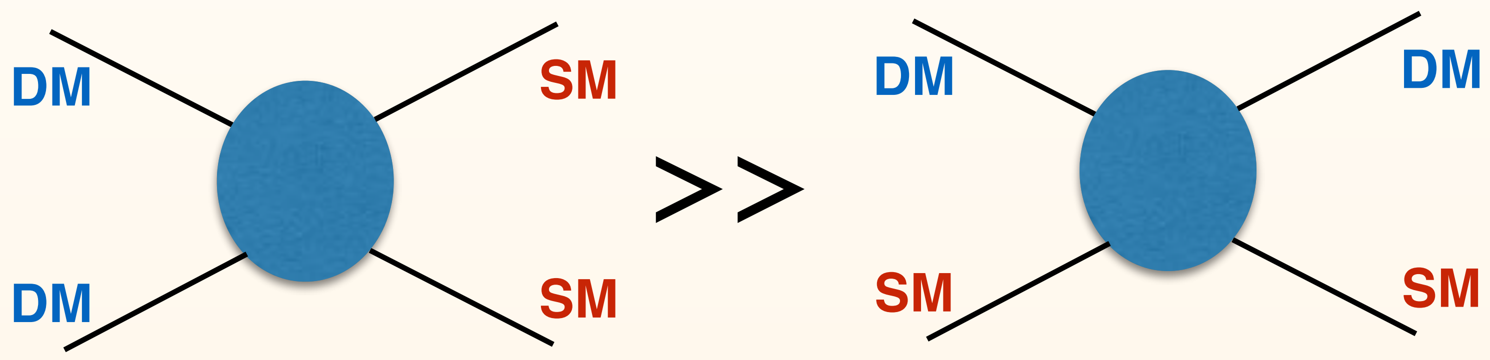
dark matter frozen-out but <u>typically</u>
still kinetically coupled to the plasma
Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

EARLY KINETIC DECOUPLING?

A necessary and sufficient condition: <u>scatterings weaker than annihilation</u>

i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:

A) 

e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

D) Multi-component dark sectors

e.g., additional sources of DM from late decays, …

How to go beyond kinetic equilibrium?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

→ contains both scatterings and annihilations

Two possible approaches:

fBE ↙ ↘ cBE

solve numerically for full $f_\chi(p)$

- have insight on the distribution
- no constraining assumptions

- numerically challenging
- often an overkill

consider system of equations for moments of $f_\chi(p)$

0-th moment: n_χ
2-nd moment: T_χ
…

- partially analytic/much easier numerically
- manifestly captures all of the relevant physics

- finite range of validity
- no insight on the distribution

T. Binder, T. Bringmann, M. Gustafsson & A.H. 2103.01944

NEW TOOL!

GOING BEYOND THE STANDARD APPROACH

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DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.
DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,**
  Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]

Currently, an user guide can be found in the Appendix A of this reference.
Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for
any (user defined) model*

Interplay between chemical and kinetic decoupling

Prediction for the DM
phase space distribution

Late kinetic decoupling
and impact on cosmology

see e.g., 1202.5456

…

(only) prerequisite:
Wolfram Language (or *Mathematica*)

*at the moment for a single DM species and w/o co-annihlations… but stay tuned for extensions!

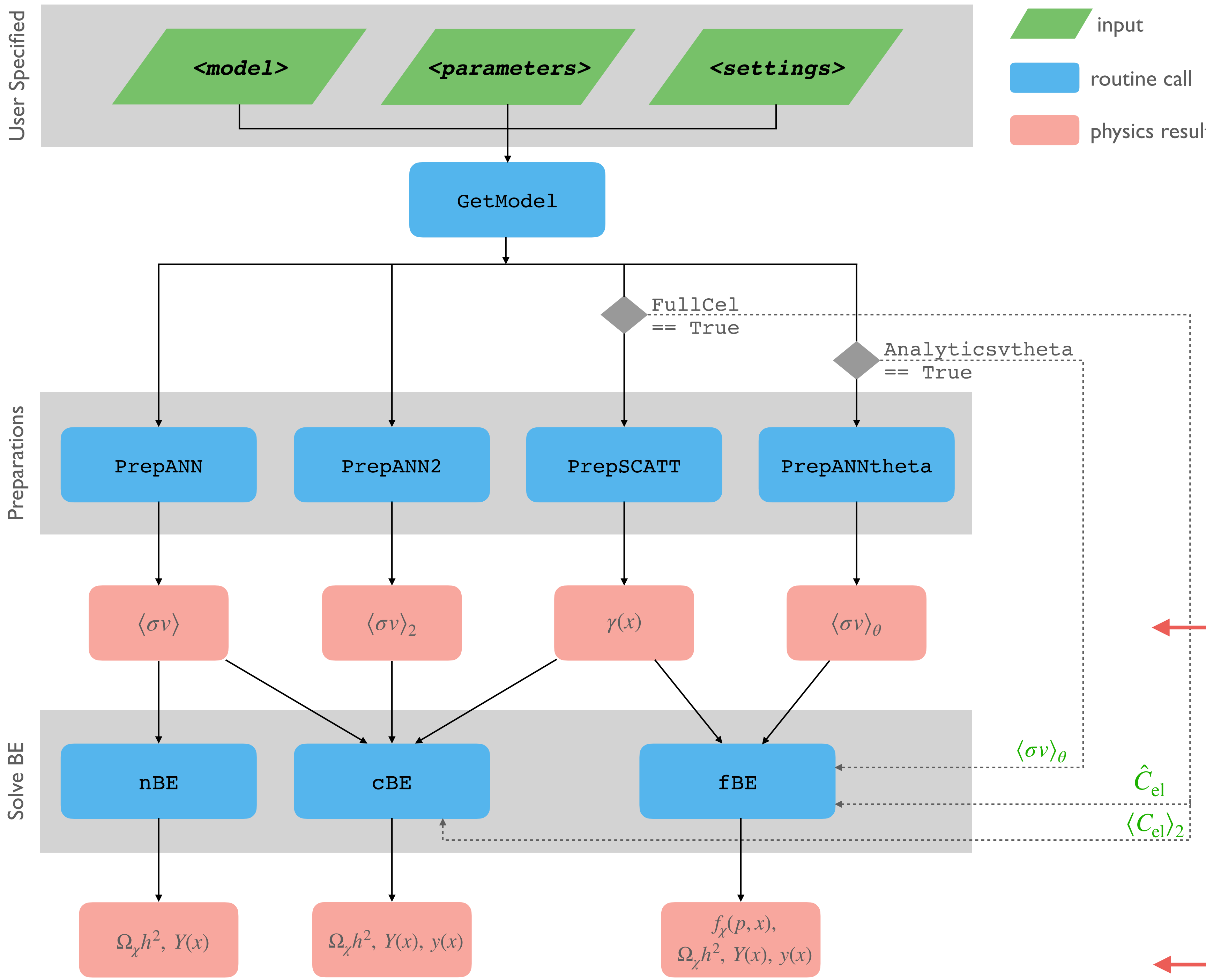
DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

https://drake.hepforge.org

Few words about the Code

written in *Wolfram Language*, lightweight, modular
and simple to use both via script and front end usage



<u>Input</u>: model file (few models implemented, user extendable), parameter values and settings choices

<u>Preparation phase</u>: all needed thermal averages etc.

[slowest part (~ few sec - some min), can be bypassed by user own routines/results…]

← (intermediate) results for rates

<u>Final phase</u>: tailored solvers for Boltzmann eq.

[~ few seconds]

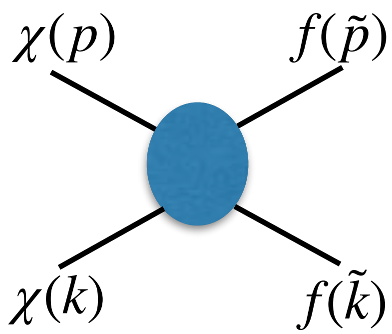
← (final) results for $\Omega h^2, f_{DM}(p)$

Collision term

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

→ contains both scatterings and annihilations

Annihilation:



$$C_{\rm ann} \sim \int d\tilde{\Pi}\, |\mathscr{M}|^2_{\chi\chi\leftrightarrow f\bar{f}} \left(f_f^{\rm eq}(\tilde{p}) f_f^{\rm eq}(\tilde{k}) - f_\chi(p) f_\chi(k)\right)$$

easy — easy

easy: no unknown f_χ under integral

$\Rightarrow$ 1D integration

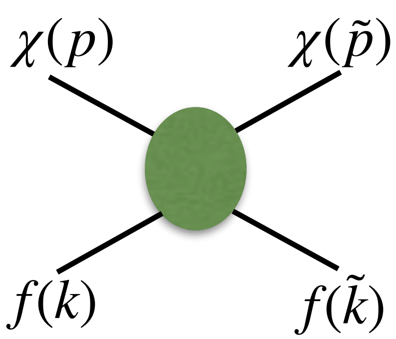
medium: no unknown f_χ under integral

$\Rightarrow$ 2-3D integration

hard: unknown f_χ under integral

$\Rightarrow$ 2-4D integration

El. scattering (on SM particles):

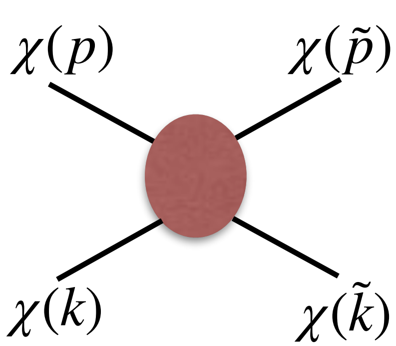


$$C_{\rm el} \sim \int d\tilde{\Pi}\, |\mathscr{M}|^2_{\chi f\leftrightarrow \chi f} \left(f_\chi(\tilde{p}) f_f^{\rm eq}(\tilde{k})(1 \pm f_f^{\rm eq}(k)) - f_\chi(p) f_f^{\rm eq}(k)(1 \pm f_f^{\rm eq}(\tilde{k}))\right)$$

hard — medium

An approximate method needed!

El. self-scattering (DM on DM):



$$C_{\rm self} \sim \int d\tilde{\Pi}\, |\mathscr{M}|^2_{\chi\chi\leftrightarrow \chi\chi} \left(f_\chi(\tilde{p}) f_\chi(\tilde{k}) - f_\chi(p) f_\chi(k)\right)$$

hard

$$d\tilde{\Pi} = d\Pi_{\tilde{p}}\, d\Pi_k\, d\Pi_{\tilde{k}}\, \delta^{(4)}(\tilde{p} + p - \tilde{k} - k)$$

APPROACHES

I) Expand in „small momentum transfer"

$M_{\rm DM} \gg |\vec{q}\,| \sim T \gg m_{\rm SM}$

typical momentum transfer

Bringmann, Hofmann '06

$$\delta^{(3)}(\tilde{\mathbf{p}} + \tilde{\mathbf{k}} - \mathbf{p} - \mathbf{k}) \approx \sum_n \frac{1}{n!}(\mathbf{q}\nabla_{\tilde{\mathbf{p}}})^n \delta^{(3)}(\tilde{\mathbf{p}} - \mathbf{p})$$

Kasahara '09; Binder, Covi, Kamada, Murayama, Takahashi, Yoshida '16

$$f_3 \simeq f_1 + \tilde{\mathbf{q}}_i \frac{\partial f_1}{\partial \mathbf{p}_{1i}} + \frac{1}{2}\tilde{\mathbf{q}}_i \tilde{\mathbf{q}}_j \frac{\partial^2 f_1}{\partial \mathbf{p}_{1i} \partial \mathbf{p}_{1j}}$$

A.H. & S. Chatterjee, *work in progress…*
(on different expansion schemes)

$\Longrightarrow$ all lead to Fokker-Planck type eq.

II) Replace the backward term with a simpler one (i.e. a relaxation-like approximation)

$\Longrightarrow$ simpler, but generally incorrect

Ala-Mattinen, Kainulainen '19
Ala-Mattinen, Heikinheimo, Kainulainen, Tuominen '22

$$\hat{C}_{{\rm E},m}(p_1,t) \to -\delta f(p_1,t)\,\Gamma^m_{\rm E}(p_1,t) = \left(g_m(t) f_{\rm eq}(p_1,t) - f(p_1,t)\right)\Gamma^m_{\rm E}(p_1,t)$$

III) Langevin simulations

$$(\hat{p}^i)' = -\hat{\eta}\,\hat{p}^i + \hat{f}^i\,, \quad \langle \hat{f}^i(x_1)\,\hat{f}^j(x_2)\rangle = \hat{\zeta}\,\delta^{ij}\,\delta(x_1 - x_2)$$

$\Longrightarrow$ perhaps promising…

Kim, Laine '23

stochastic term, taking care of detailed balance

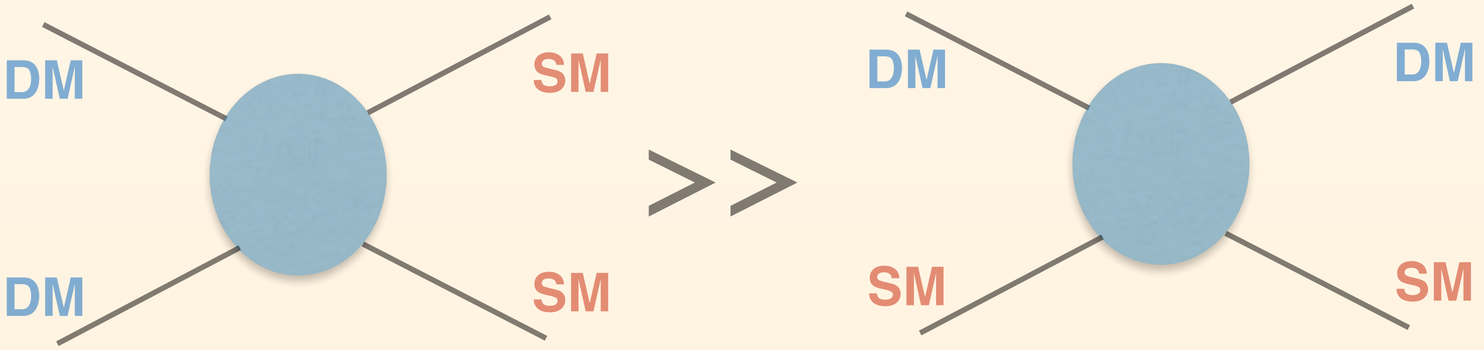
IV) Fully numerical implementation

A.H. & M. Laletin 2204.07078 (focus on DM self-scatterings)
Ala-Mattinen, Heikinheimo, Kainulainen, Tuominen '22
Du, Huang, Li, Li, Yu '21
Aboubrahim, Klasen, Wiggering '23

$\Longrightarrow$ doable, but (very) CPU expensive

Example A:
Scalar Singlet DM

A)



EXAMPLE A
SCALAR SINGLET DM

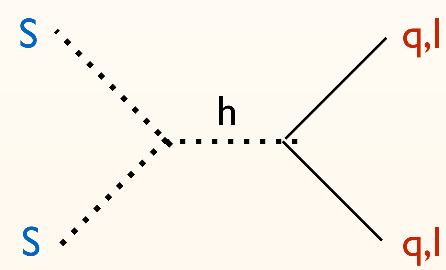
To the SM Lagrangian add one singlet scalar field *S* with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_s S^2 |H|^2$$

$$m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$

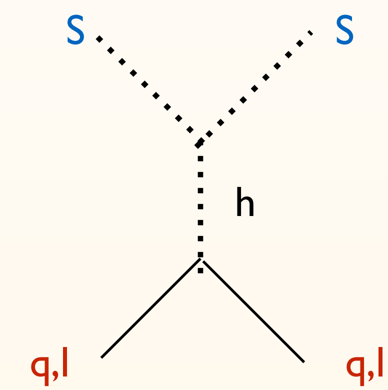
Annihilation processes:

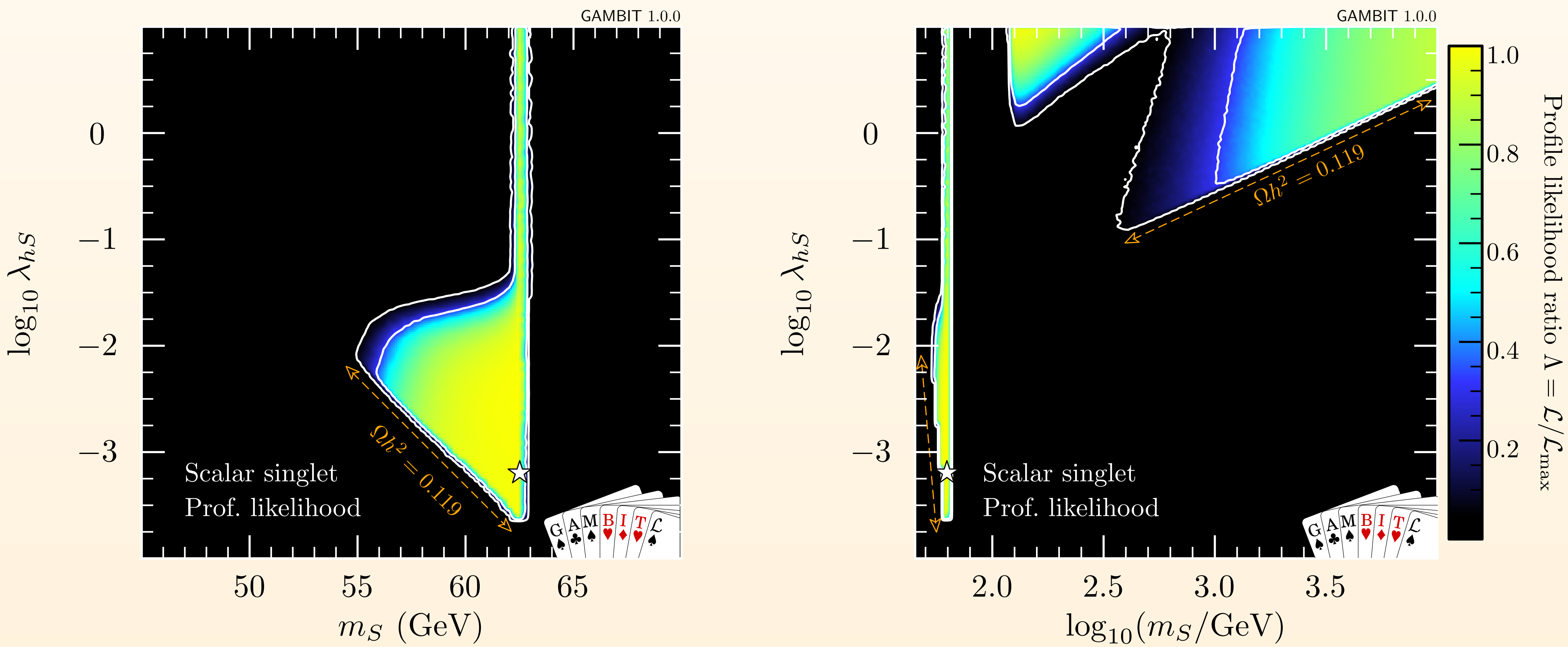
resonant



El. scattering processes:

non-resonant



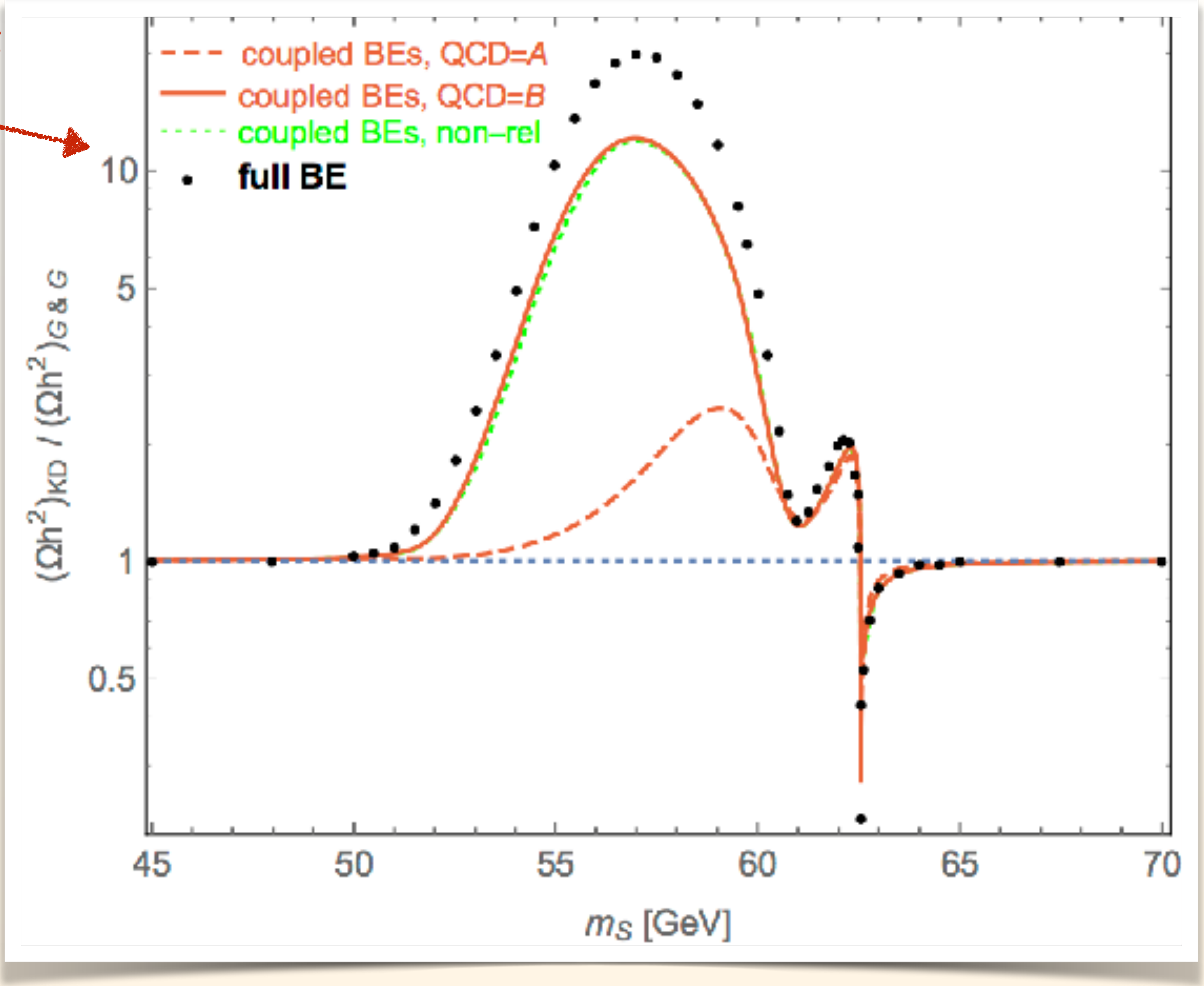


GAMBIT collaboration
1705.07931

RESULTS

EFFECT ON THE Ωh^2

effect on relic density:
up to O(~10)



[… Freeze-out at few GeV ⟶ what is the abundance of heavy quarks in QCD plasma?

two scenarios: QCD = A - all quarks are free and present in the plasma down to T_c =154 MeV
QCD = B - only light quarks contribute to scattering and only down to $4T_c$ …]

Example D:
When additional influx of DM arrives

D) Multi-component dark sectors

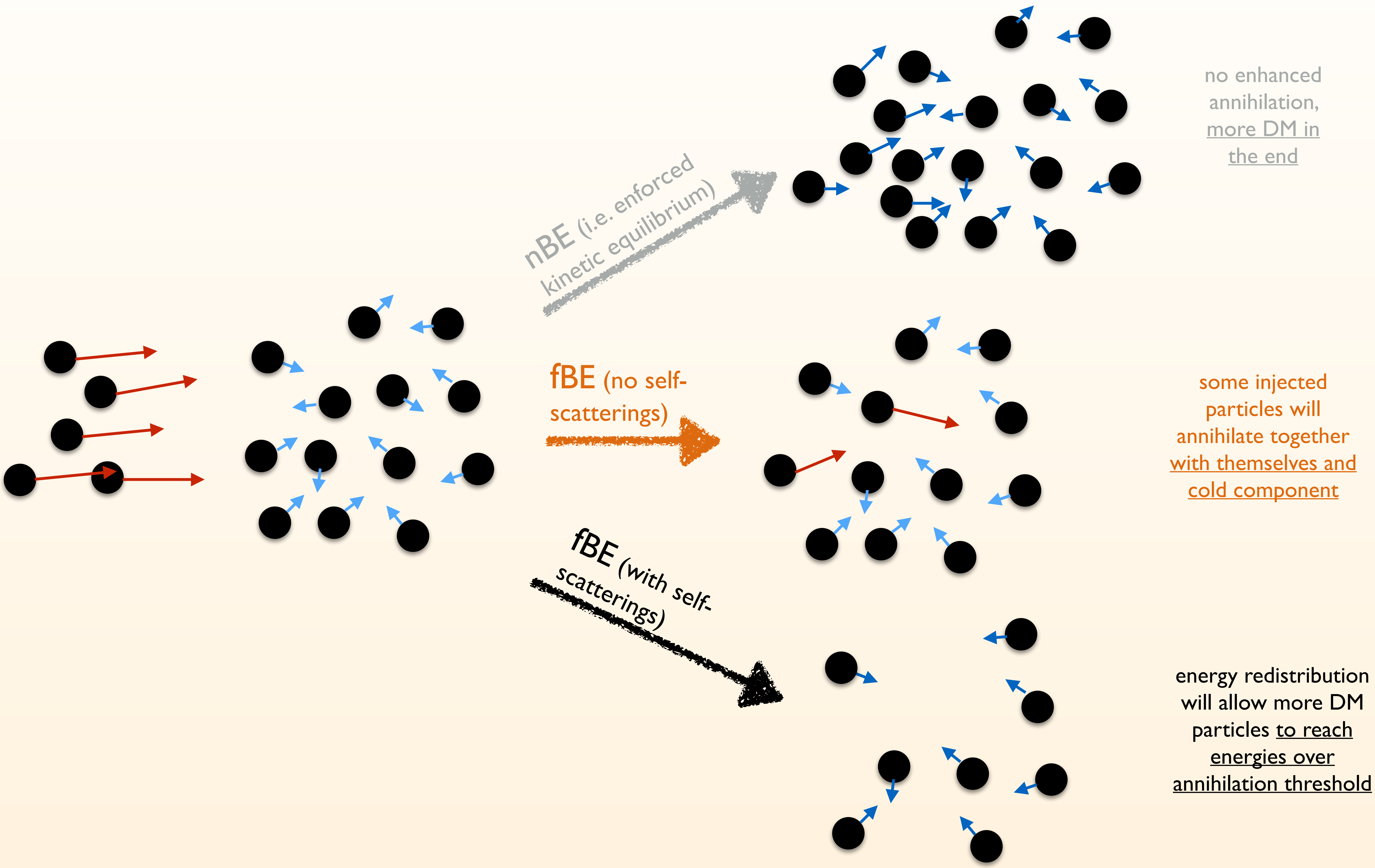
Sudden injection of more DM particles distorts $f_\chi(p)$
(e.g. from a decay or annihilation of other states)

- this can modify the annihilation rate (if still active)
- how does the thermalization due to elastic scatterings happen?

1) DM produced via:
- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

2) DM annihilation has a threshold

e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$



AH, Laletin 2204.07078

EXAMPLE EVOLUTION

1) DM produced via:
- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

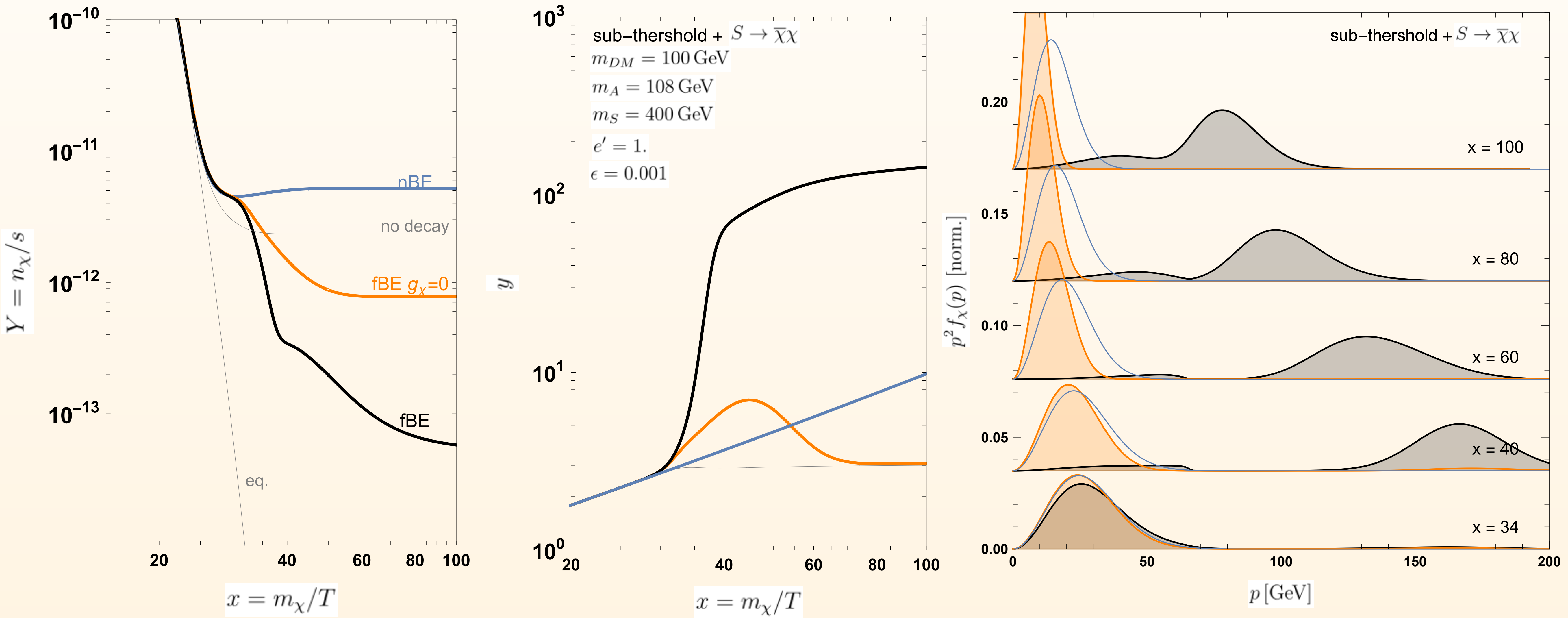
2) DM annihilation has a threshold

e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$

$Y \sim$ number density

$y \sim$ temperature

$p^2 f(p) \sim$ momentum distribution



SUMMARY

1. In recent years a significant progress in refining the relic density calculations (not yet fully implemented in public codes!)

2. Kinetic equilibrium is a necessary (often implicit) assumption for standard relic density calculations in all the numerical tools…
…while it is not always warranted!

3. Introduced coupled system of Boltzmann eqs. for 0th and 2nd moments (cBE) allows for much more accurate treatment while the full phase space Boltzmann equation (fBE) can be also successfully solved for higher precision and/or to obtain result for $f_{\rm DM}(p)$

(we also introduced DRAKE a new tool to extend the current capabilities to the regimes beyond kinetic equilibrium)

4. Multi-component sectors, when studied at the fBE level, can reveal quite unexpected behavior