NATIONAL SCIENCE CENTRE
POLAND
grant No. 2018/31/D/ST2/00813

NON-EQUILIBRIUM EFFECTS IN THE EVOLUTION OF DARK MATTER

Andrzej Hryczuk

NATIONAL CENTRE FOR NUCLEAR RESEARCH
ŚWIERK

based on: **T. Binder, T. Bringmann, M. Gustafsson and AH** 1706.07433
A. Hektor, AH and K. Kannike 1901.08074
+ work in progress with **T. Binder, T. Bringmann, M. Gustafsson**

Dark Matter

1. We know nearly nothing at all about dark matter

- origin
- nature
- interactions
- structure
- …

---

2. We know quite a lot about dark matter

- it is there!
- particle(s) vs. sth else
- large scale structure
- limits on interactions
- mechanisms of origin
- …

MOTIVATION
THERMAL RELIC DENSITY

Theory:

Experiment:

I. Natural

Comes out automatically from the expansion of the Universe

Naturally leads to cold DM

II. Predictive

No dependence on initial conditions

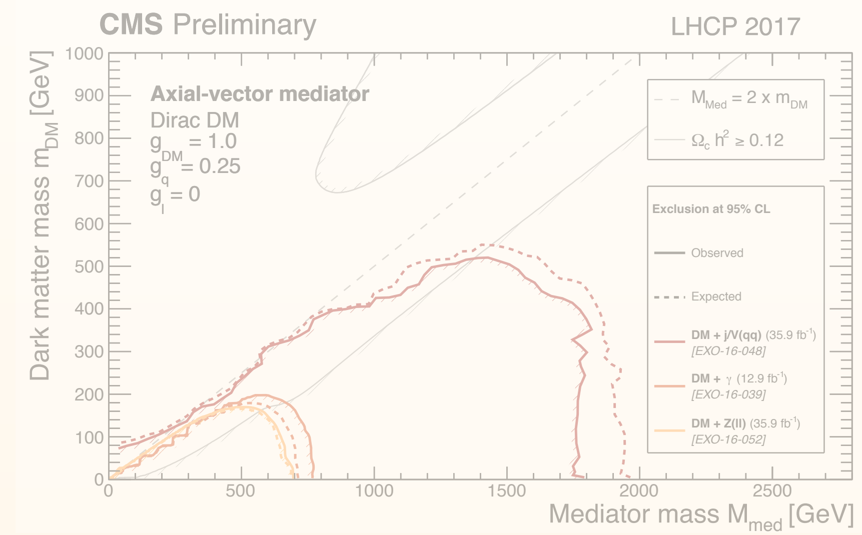
Fixes coupling(s) $\Rightarrow$ signal in DD, ID & LHC

III. It is not optional

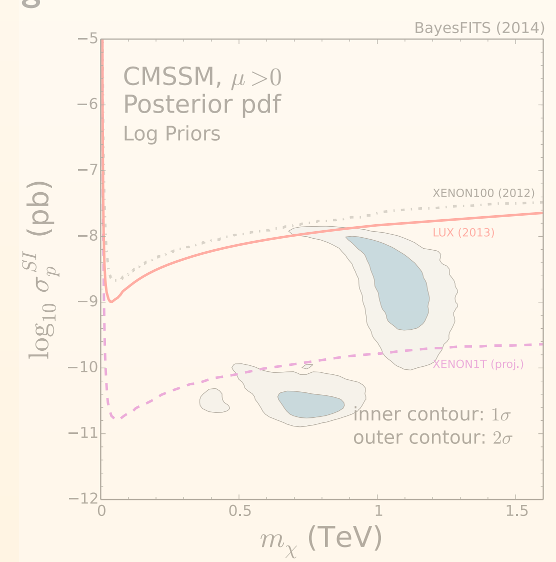
Overabundance constraint

To avoid it one needs quite significant deviations from standard cosmology

…as a constraint:



…as a target:



"(…) besides the Higgs boson mass measurement and LHC direct bounds, the constraint showing **by far the strongest impact** on the parameter space of the MSSM is the **relic density**"

Roszkowski et al. '14

…as a pin:

When a dark matter signal is (finally) found: relic abundance can pin-point the particle physics interpretation

Thermal Relic Density

STANDARD APPROACH

T ↑

$\Gamma_{\text{ann}} > H$ — DM in equilibrium

$\Gamma_{\text{ann}} \sim H$ — chemical decoupling

$\Gamma_{\text{ann}} < H$ — freeze-out

time ↓

non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

Liouville operator in FRW background

the collision term

Thermal Relic Density

standard approach

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq: classical limit, molecular chaos,...

...for derivation from thermal QFT see e.g., 1409.3049

$\Downarrow$ integrate over p (i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar\chi\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar\chi} - n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}\right)$$

where the thermally averaged cross section:

$$\langle\sigma_{\chi\bar\chi\to ij}v_{\rm rel}\rangle^{\rm eq} = -\frac{h_\chi^2}{n_\chi^{\rm eq} n_{\bar\chi}^{\rm eq}}\int\frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar\chi}}{(2\pi)^3}\,\sigma_{\chi\bar\chi\to ij}v_{\rm rel}\,f_\chi^{\rm eq} f_{\bar\chi}^{\rm eq}$$

$\Downarrow$

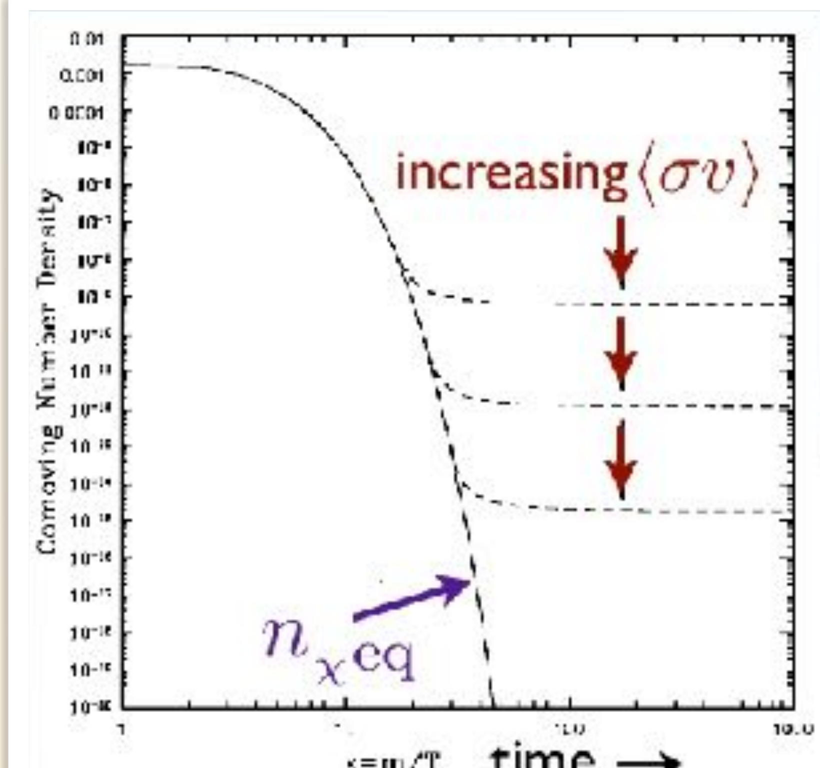


Fig.: Jungman, Kamionkowski & Griest, PR'96

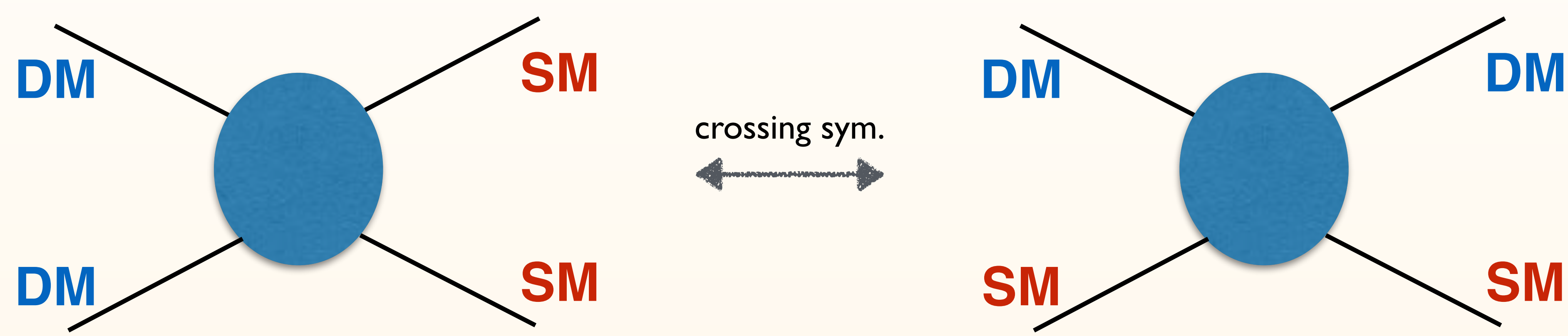
Critical assumption:

kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(\mu) f_\chi^{\rm eq}$$

FREEZE-OUT *VS*. DECOUPLING

annihilation | (elastic) scattering



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Rightarrow$ scatterings <u>typically</u> more frequent

dark matter frozen-out but <u>typically</u>
still kinetically coupled to the plasma

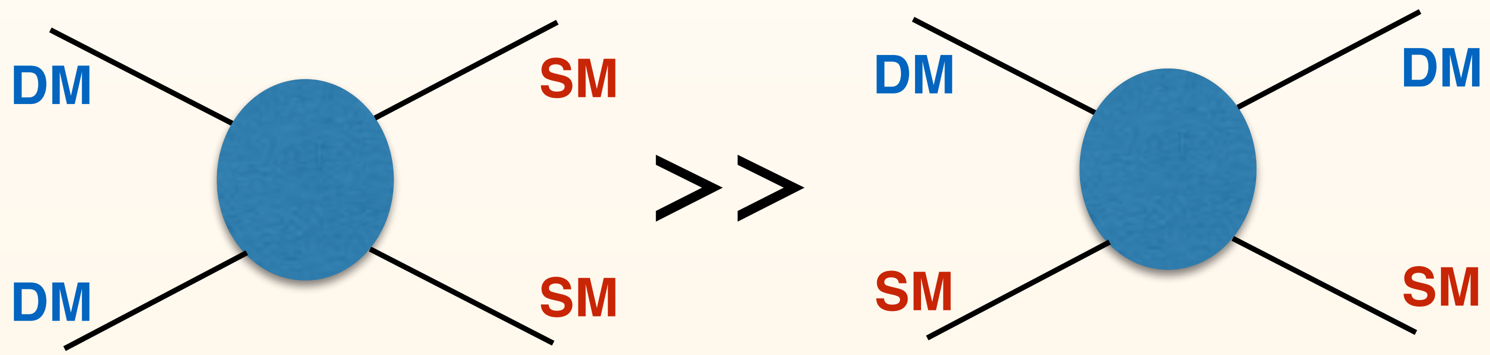
Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Early Kinetic Decoupling?

A necessary and sufficient condition: scatterings weaker than annihilation

i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:

A) 

e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

How to describe KD?

All information is in full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

→ contains both scatterings and annihilation

Two possible approaches:

0-th moment: n_χ
2-nd moment: T_χ
$\ldots$

solve numerically for full $f_\chi(p)$

- have insight on the distribution
- no constraining assumptions
- numerically challenging
- typically overkill

consider system of equations for <u>moments</u> of $f_\chi(p)$

- partially analytic/much easier numerically
- manifestly captures all of the relevant physics
- finite range of validity
- no insight on the distribution

ONE STEP FURTHER…

Now consider general KD scenario, i.e. coupled temperature and number density evolution:

annihilation and production thermal averages done at different T — feedback of modified y evolution $y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$

$$T_\chi \equiv \frac{g_\chi}{3n_\chi}\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{E} f_\chi(p)$$

$$\frac{Y'}{Y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx} sY\left(\langle\sigma v_{\rm rel}\rangle|_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\rm eq}^2}{Y^2}\langle\sigma v_{\rm rel}\rangle|_x\right)$$

$$\frac{y'}{y} = -\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{Hx}\left[2m_\chi c(T)\left(1-\frac{y_{\rm eq}}{y}\right) - sY\left(\left(\langle\sigma v_{\rm rel}\rangle - \langle\sigma v_{\rm rel}\rangle_2\right)_{x=m_\chi^2/(s^{2/3}y)} - \frac{Y_{\rm eq}^2}{Y^2}\left(\langle\sigma v_{\rm rel}\rangle - \frac{y_{\rm eq}}{y}\langle\sigma v_{\rm rel}\rangle_2\right)_x\right)\right]$$
$$+\frac{1-\frac{x}{3}\frac{g'_{*S}}{g_{*S}}}{3m_\chi}\langle p^4/E^3\rangle_{x=m_\chi^2/(s^{2/3}y)}$$

elastic scatterings term

impact of annihilation

These equations still assume the equilibrium shape of $f_\chi(p)$ — but with variant temperature

or more accurately: that the thermal averages computed with true non-equilibrium distributions don't differ much from the above ones

NUMERICAL APPROACH

… or one can just solve full phase space Boltzmann eq.

$$\begin{aligned}
\partial_x f_\chi(x,q) &= \frac{m_\chi^3}{\tilde{H}x^4}\frac{g_{\bar\chi}}{2\pi^2}\int d\tilde{q}\,\tilde{q}^2\,\frac{1}{2}\int d\cos\theta\;v_{\text{Møl}}\sigma_{\bar\chi\chi\to\bar f f} \\
&\quad\times\left[f_{\chi,\text{eq}}(q)f_{\chi,\text{eq}}(\tilde q)-f_\chi(q)f_\chi(\tilde q)\right] \\
&+ \frac{2m_\chi c(T)}{2\tilde{H}x}\left[x_q\partial_q^2+\left(q+\frac{2x_q}{q}+\frac{q}{x_q}\right)\partial_q+3\right]f_\chi \\
&+ \tilde{g}\frac{q}{x}\partial_q f_\chi,
\end{aligned}$$

fully general

expanded in NR and small momentum transfer
(semi-relativistic!)

discretization,
~1000 steps

$$\begin{aligned}
\partial_x f_i &= \\
&\frac{m_\chi^3}{\tilde{H}x^4}\frac{g_{\bar\chi}}{2\pi^2}\sum_{j=1}^{N-1}\frac{\Delta\tilde{q}_j}{2}\Big[\tilde{q}_j^2\,\langle v_{\text{Møl}}\sigma_{\bar\chi\chi\to\bar f f}\rangle_{i,j}^{\theta}\left(f_i^{\text{eq}}f_j^{\text{eq}}-f_if_j\right) \\
&+ \tilde{q}_{j+1}^2\,\langle v_{\text{Møl}}\sigma_{\bar\chi\chi\to\bar f f}\rangle_{i,j+1}^{\theta}\left(f_i^{\text{eq}}f_{j+1}^{\text{eq}}-f_if_{j+1}\right)\Big] \\
&+ \frac{2m_\chi c(T)}{2\tilde{H}x}\left[x_{q,i}\partial_q^2+\left(q_i+\frac{2x_{q,i}}{q_i}+\frac{q_i}{x_{q,i}}\right)\partial_q+3\right]f_i \\
&+ \tilde{g}\frac{q_i}{x}\partial_q f_i,
\end{aligned}$$

Solved numerically with MatLab

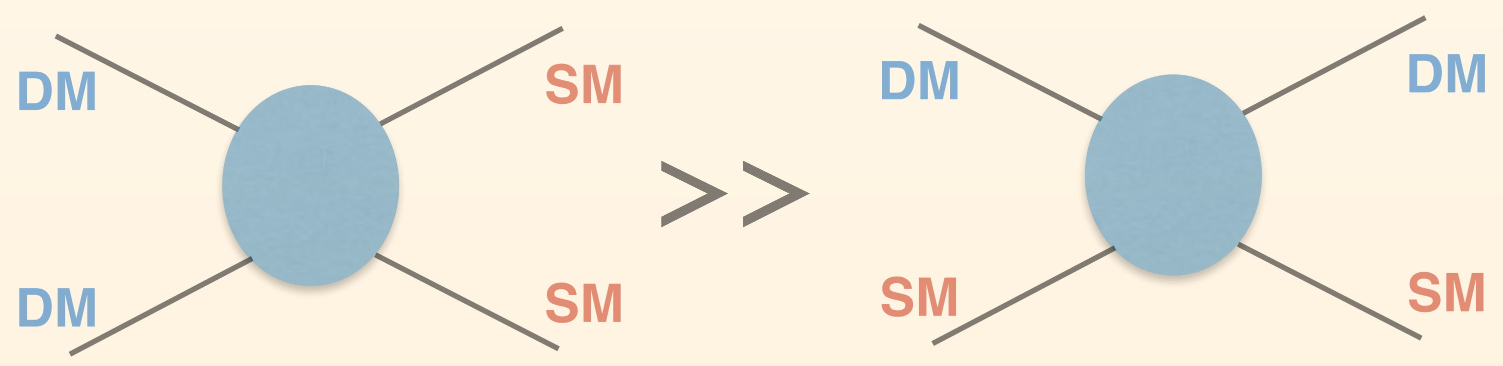
Note:

can be extended to e.g. self-scatterings

very stiff, care needed with numerics

EXAMPLE A:
SCALAR SINGLET DM

A)

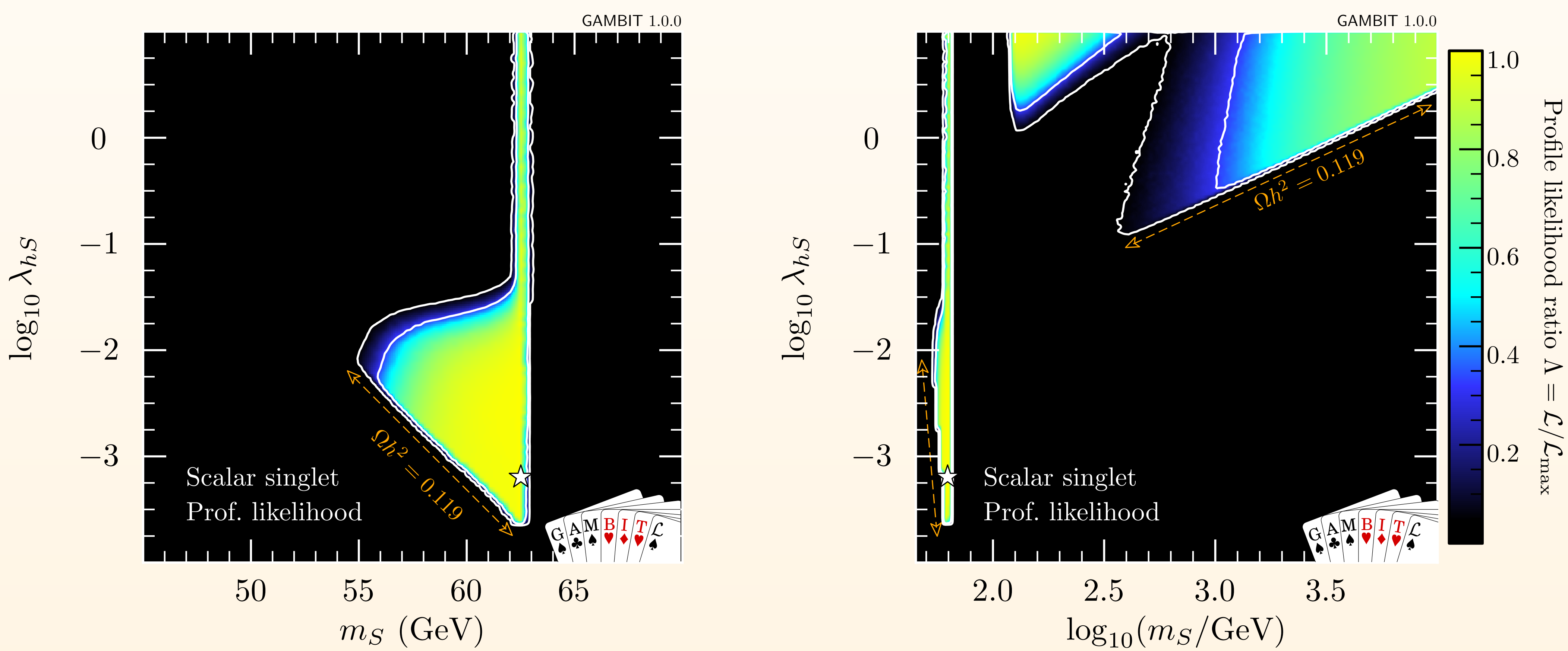


SCALAR SINGLET DM

VERY SHORT INTRODUCTION

To the SM Lagrangian add one singlet scalar field *S* with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2}\mu_S^2 S^2 - \frac{1}{2}\lambda_s S^2 |H|^2 \qquad m_s = \sqrt{\mu_S^2 + \frac{1}{2}\lambda_s v_0^2}$$



GAMBIT collaboration
1705.07931

Most of the parameter space excluded, but… even such a simple model is hard to kill

SCALAR SINGLET DM

ANNIHILATION VS. SCATTERINGS

$$\sigma v_{\rm rel} = \frac{2\lambda_s^2 v_0^2}{\sqrt{s}} |D_h(s)|^2 \Gamma_h(\sqrt{s})$$

tabulated Higgs width

with:

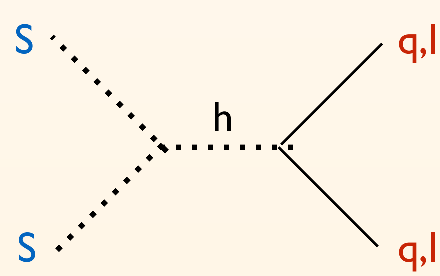
$$|D_h(s)|^2 \equiv \frac{1}{(s-m_h^2)^2 + m_h^2 \Gamma_h^2(m_h)}\,.$$

$$\left\langle |\mathcal{M}|^2 \right\rangle_t = \sum_f \frac{N_f \lambda_S^2 m_f^2}{8k^4} \Big[\frac{2k_{\rm cm}^2 - 2m_f^2 + m_h^2}{1 + m_h^2/(4k_{\rm cm}^2)} - \left(m_h^2 - 2m_f^2\right) \log\left(1 + 4k_{\rm cm}^2/m_h^2\right) \Big]\,.$$

Hierarchical Yukawa couplings: strongest coupling to more Boltzmann suppressed quarks/leptons

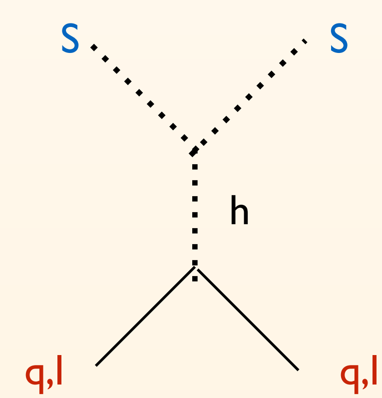
Annihilation processes:

resonant



El. scattering processes:

non-resonant



Freeze-out at few GeV ⟶ what is the abundance of heavy quarks in QCD plasma?

two scenarios:

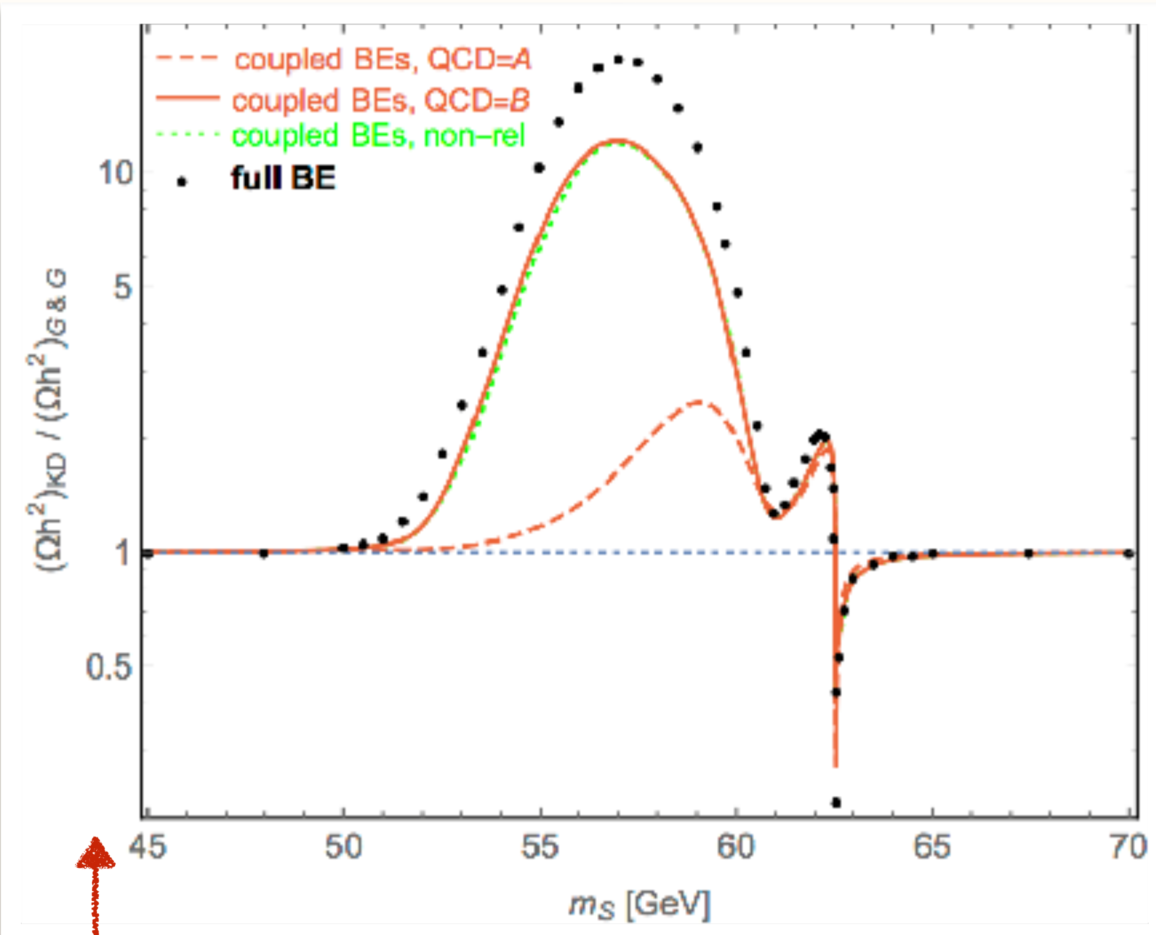
QCD = A - all quarks are free and present in the plasma down to T_c =154 MeV

QCD = B - only light quarks contribute to scattering and only down to $4T_c$

Results

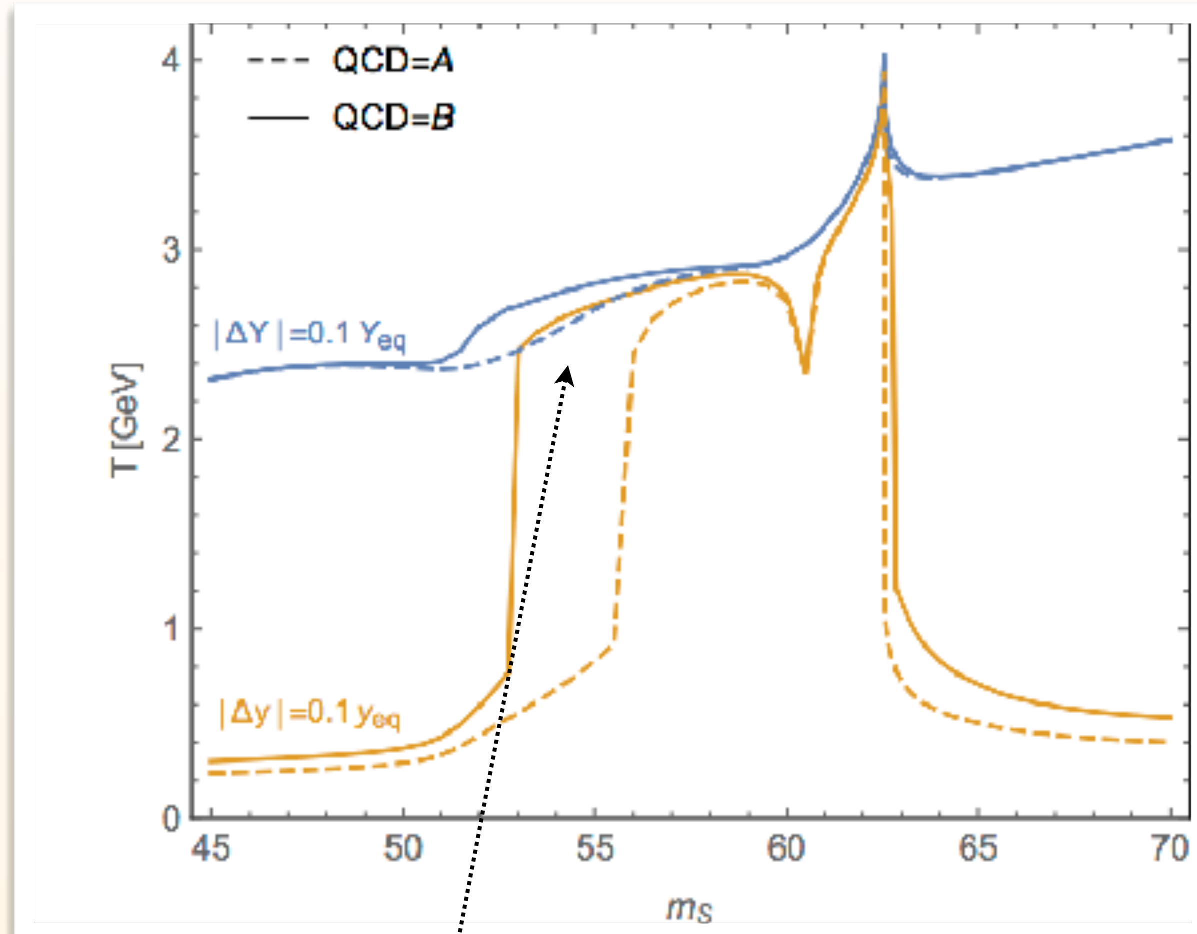
Effect

effect on relic density:



effect on relic density:
up to O(~10)

kinetic and chemical decoupling:



ratio approaches 1,
but does not reach it!

Why such non-trivial shape of the effect of early kinetic decoupling?

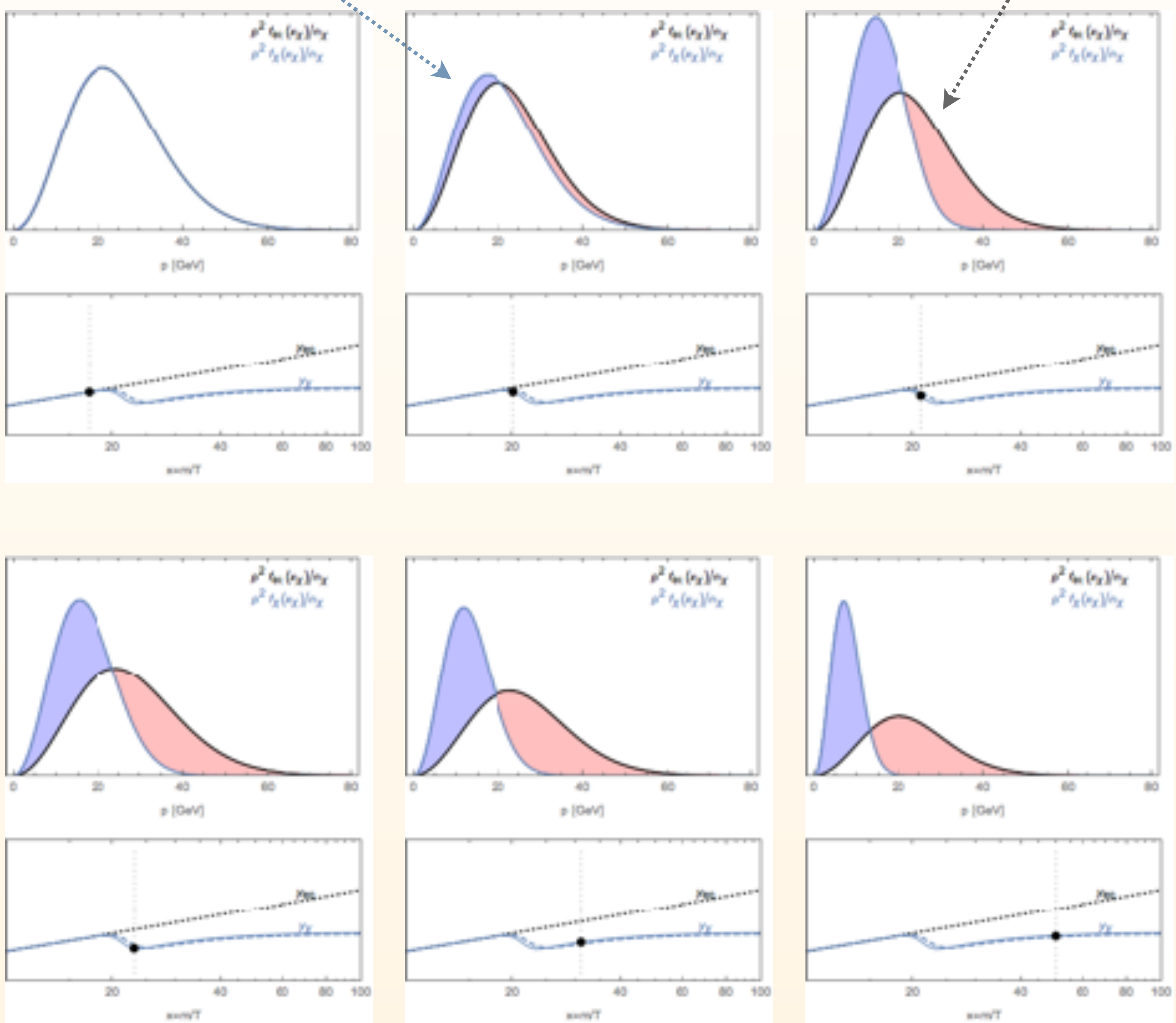
→ we'll inspect the y and Y evolution…

Full phase-space Evolution

blue - full solution for f_{DM} at T_{DM}

m_{DM} = 58 GeV

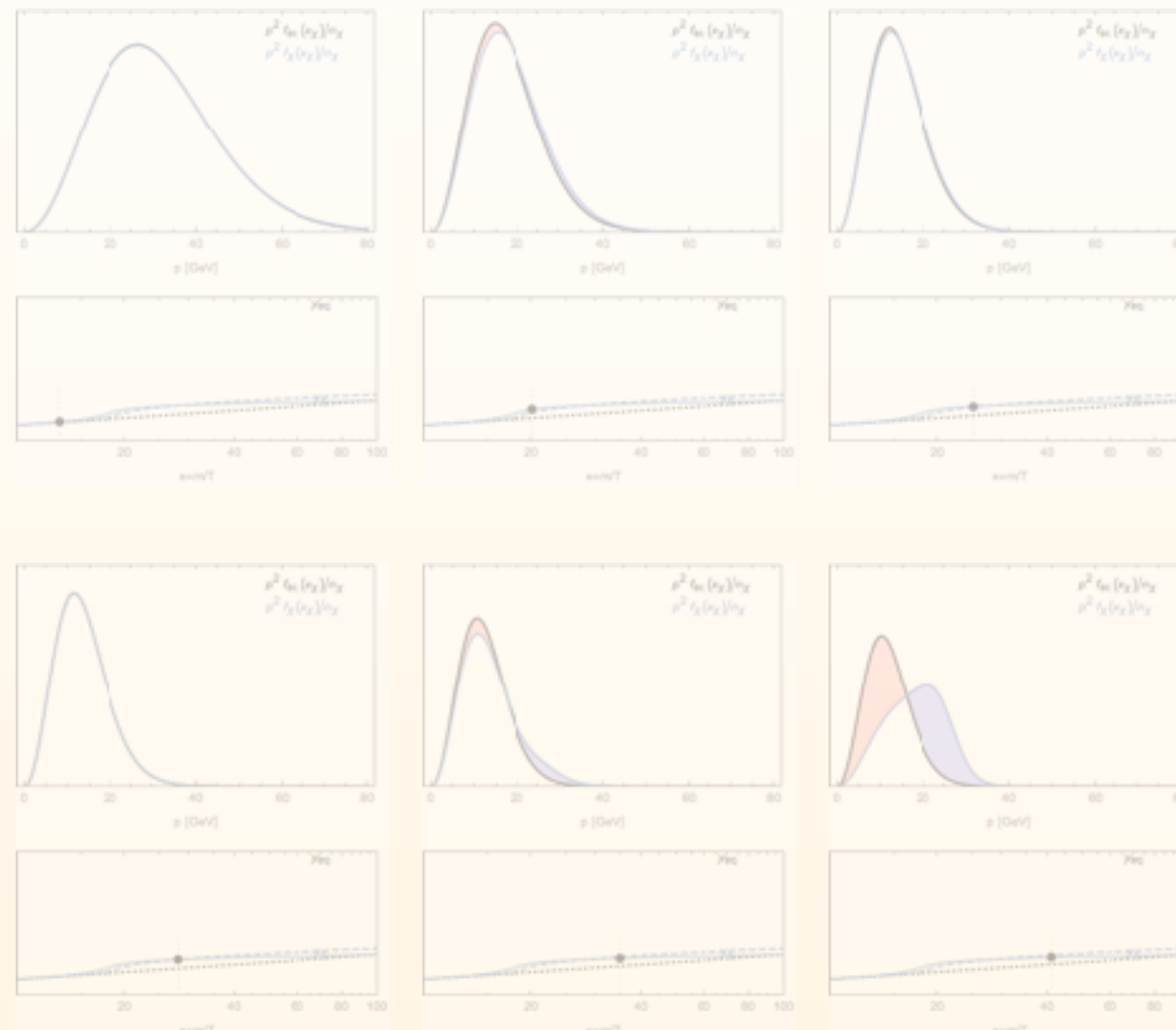
black - equilibrium at T_{DM}



significant deviation from equilibrium shape already around freeze-out

⟶ effect on relic density largest, both from different T and f_{DM}

m_{DM} = 62.5 GeV

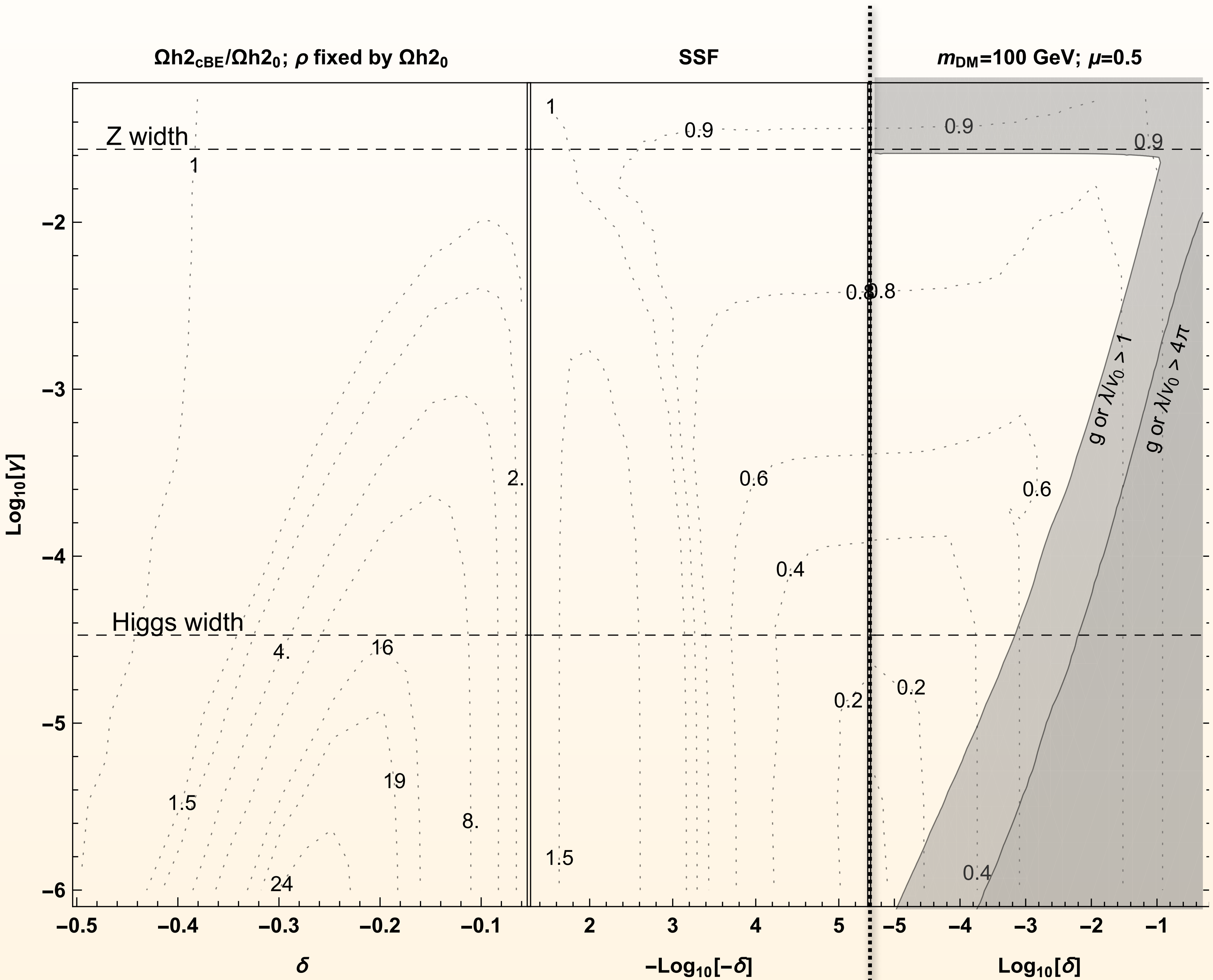


large deviations only at later times, around freeze-out not far from eq. shape

⟶ effect on relic density ~only from different T

Generic Resonant Annihilation

Example effect on early KD on relic density



can reach O(10%) even for *Z* resonance

very large effect for Higgs resonance

resonance **width** normalized to m_{DM}

distance in mass from the exact condition $2m_{DM}=m_R$

*calculated with the coupled BEs method

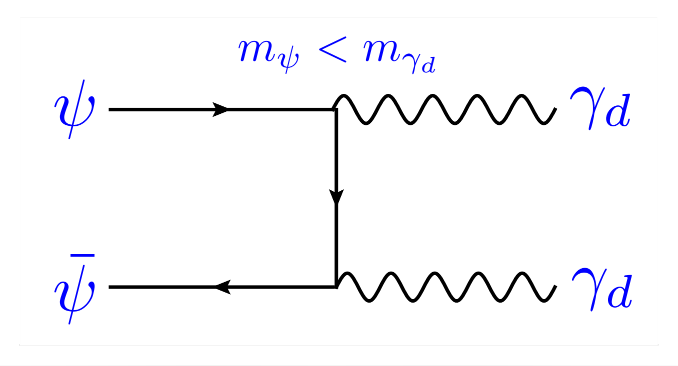
EXAMPLE B:
FORBIDDEN DM

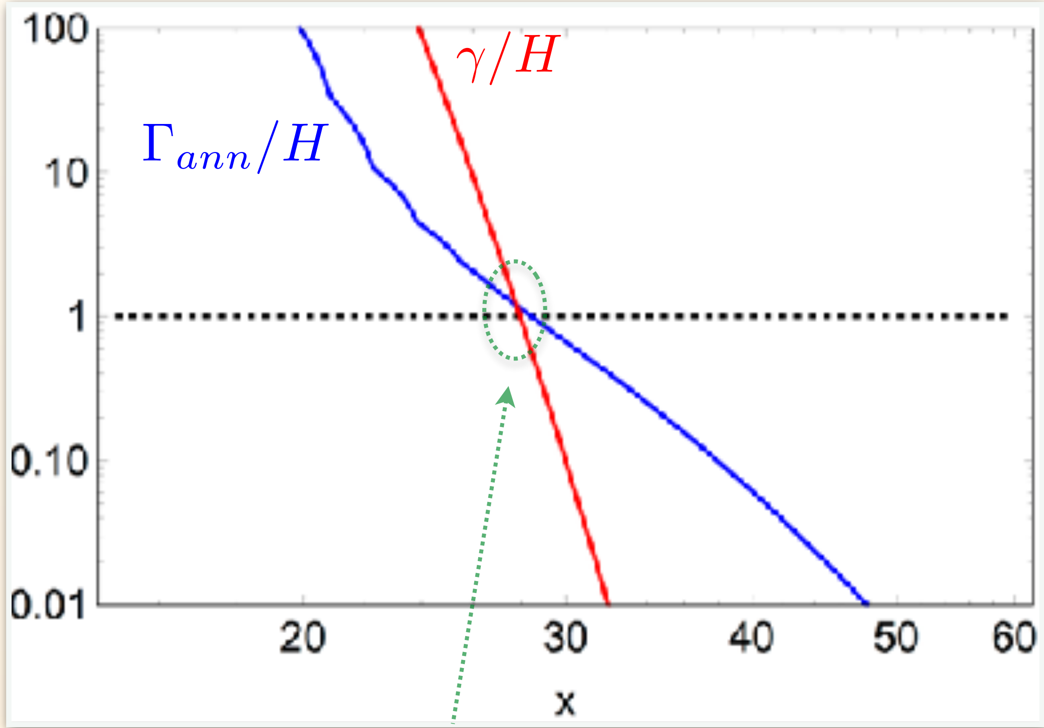
B) Boltzmann suppression of SM as strong as for DM

FORBIDDEN DARK MATTER

DM is a thermal relic that annihilates <u>only</u> to heavier states
(forbidden in zero temperature)

…, D'Agnolo, Ruderman '15, …





kinetic and chemical
decoupling close

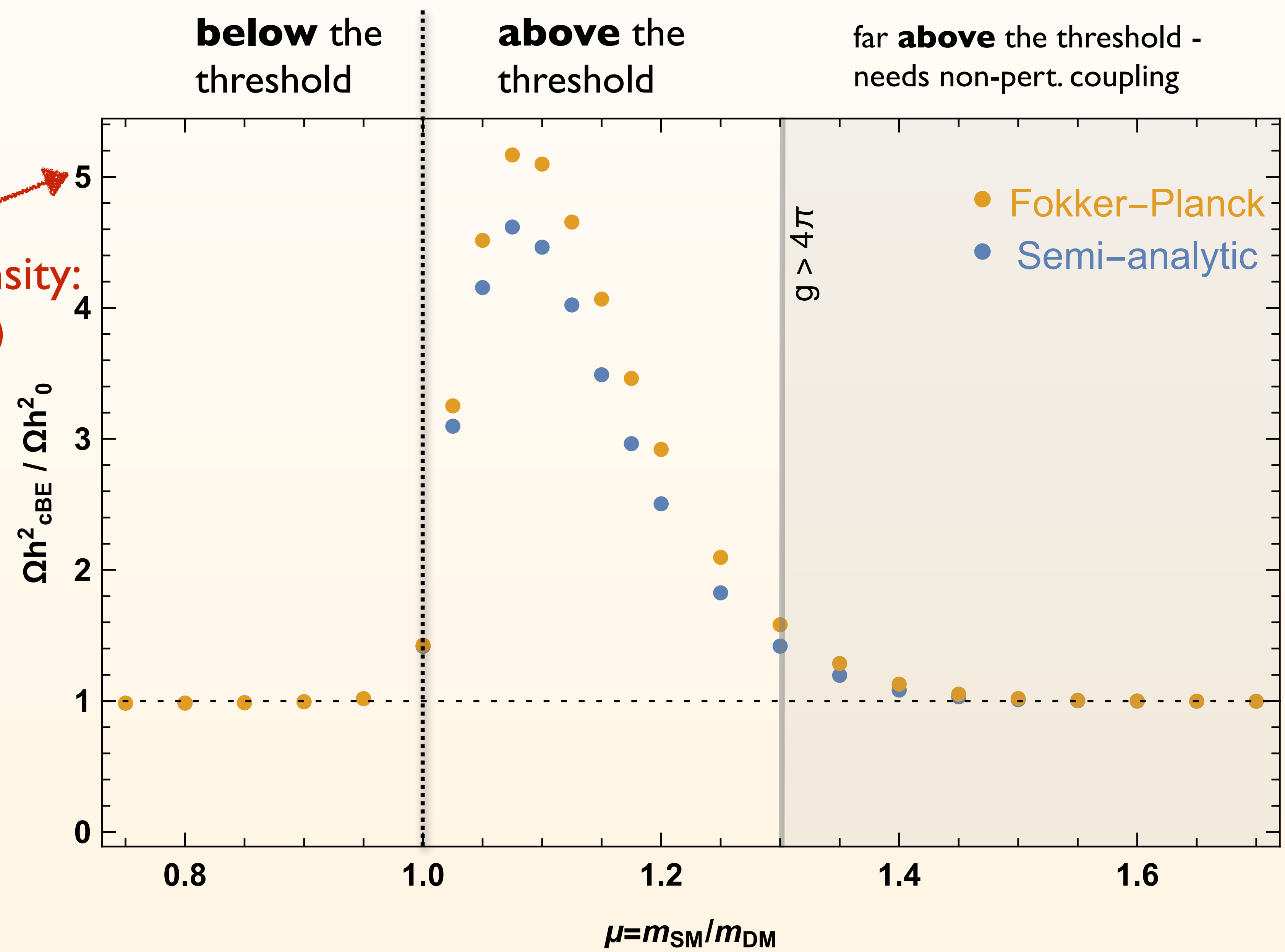
Annihilation threshold → velocity dependence

"heavy" SM particle → scattering rate low

Forbidden Dark Matter

Example effect on early KD on relic density

effect on relic density: up to O(~few)



*calculated with the coupled BEs method

EXAMPLE C:
SEMI-ANNIHILATION

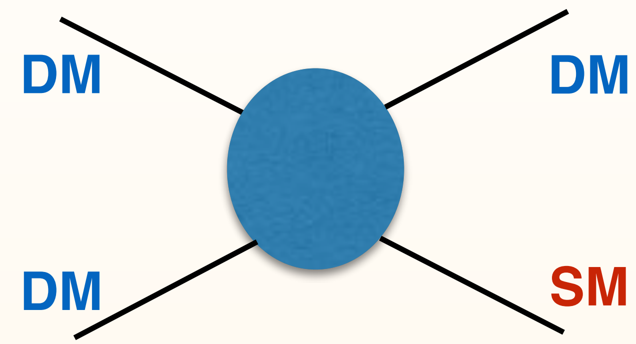
C) Scatterings and annihilation have different structure

Dark Matter Semi-Annihilation

and its simplest realization

DM is a thermal relic but with freeze-out governed by the semi-annihilation process

D'Eramo, Thaler '10; …



Z_3 complex scalar singlet:

$$V = \mu_H^2 |H|^2 + \lambda_H |H|^4 + \mu_S^2 |S|^2 + \lambda_S |S|^4 + \lambda_{SH} |S|^2 |H|^2 + \frac{\mu_3}{2}(S^3 + S^{\dagger 3}).$$

just above the Higgs threshold semi-annihilation dominant!

Belanger, Kannike, Pukhov, Raidal '13

very weak elastic scatterings → semi-annihilation by itself does not equilibrate DM

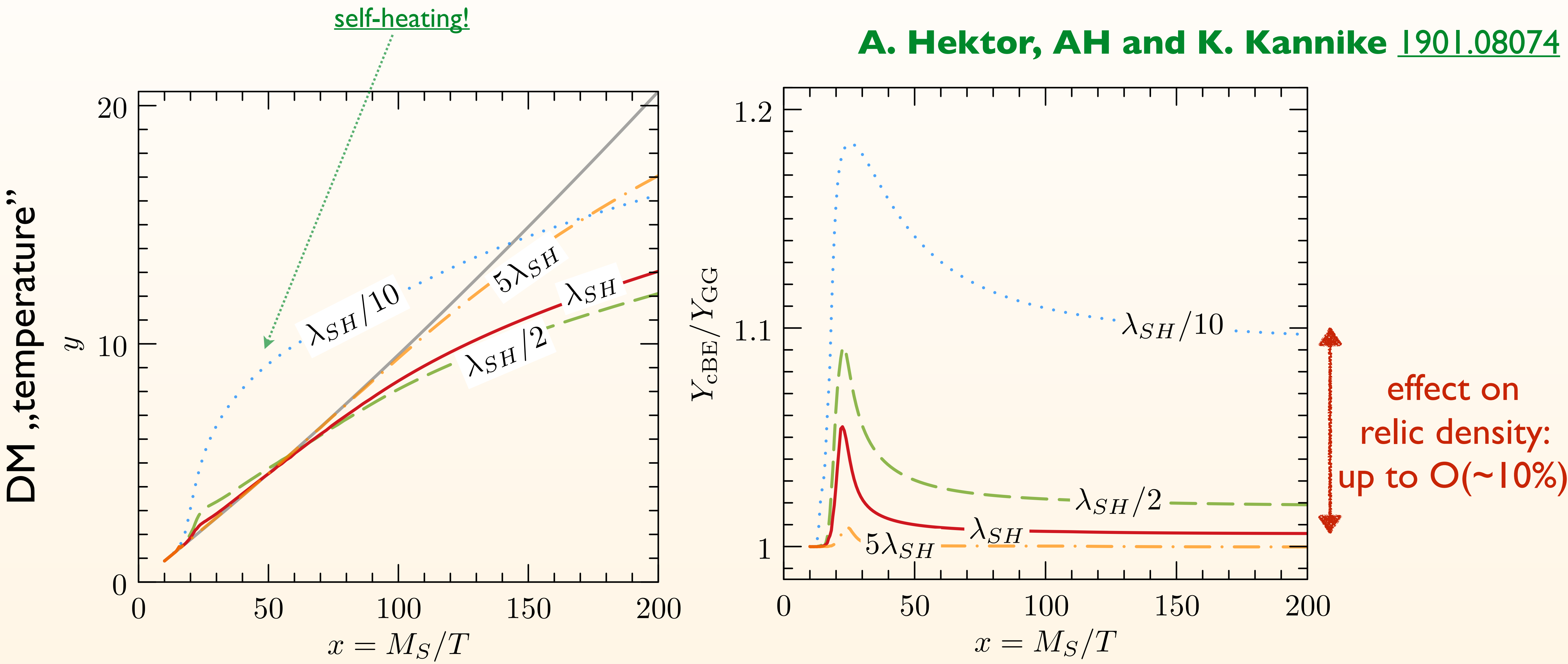
↓

but rather leads to self-heating! → **implications for ID**

see also Cai, Spray 1807.00832

SEMI-ANNIHILATION
EXAMPLE EFFECT ON EARLY KD ON RELIC DENSITY

A. Hektor, AH and K. Kannike 1901.08074



Note: here the final effect is relatively mild (though still larger than the observational error), but only because in the simplest model the velocity dependence of annihilation is mild as well…

*calculated with the (modified) coupled BEs method

Conclusions

1. One needs to remember that kinetic equilibrium is a necessary assumption for standard relic density calculations

2. Coupled system of Boltzmann equations for 0th and 2nd moments allow for a very accurate treatment of the kinetic decoupling and its effect on relic density

3. In special cases the full phase space Boltzmann equation can be necessary — especially if one wants to trace DM temperature as well

…a step towards more fundamental and reliable relic density determination