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DARK MATTER FREEZE-OUT AND FREEZE-IN BEYOND KINETIC EQUILIBRIUM

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NCBJ

based on:
A.H. & M. Laletin 2204.07078
A.H. & M. Laletin 2104.05684
and **T. Binder, T. Bringmann, M. Gustafsson & A.H.** 1706.07433, 2103.01944

NuDM 2022

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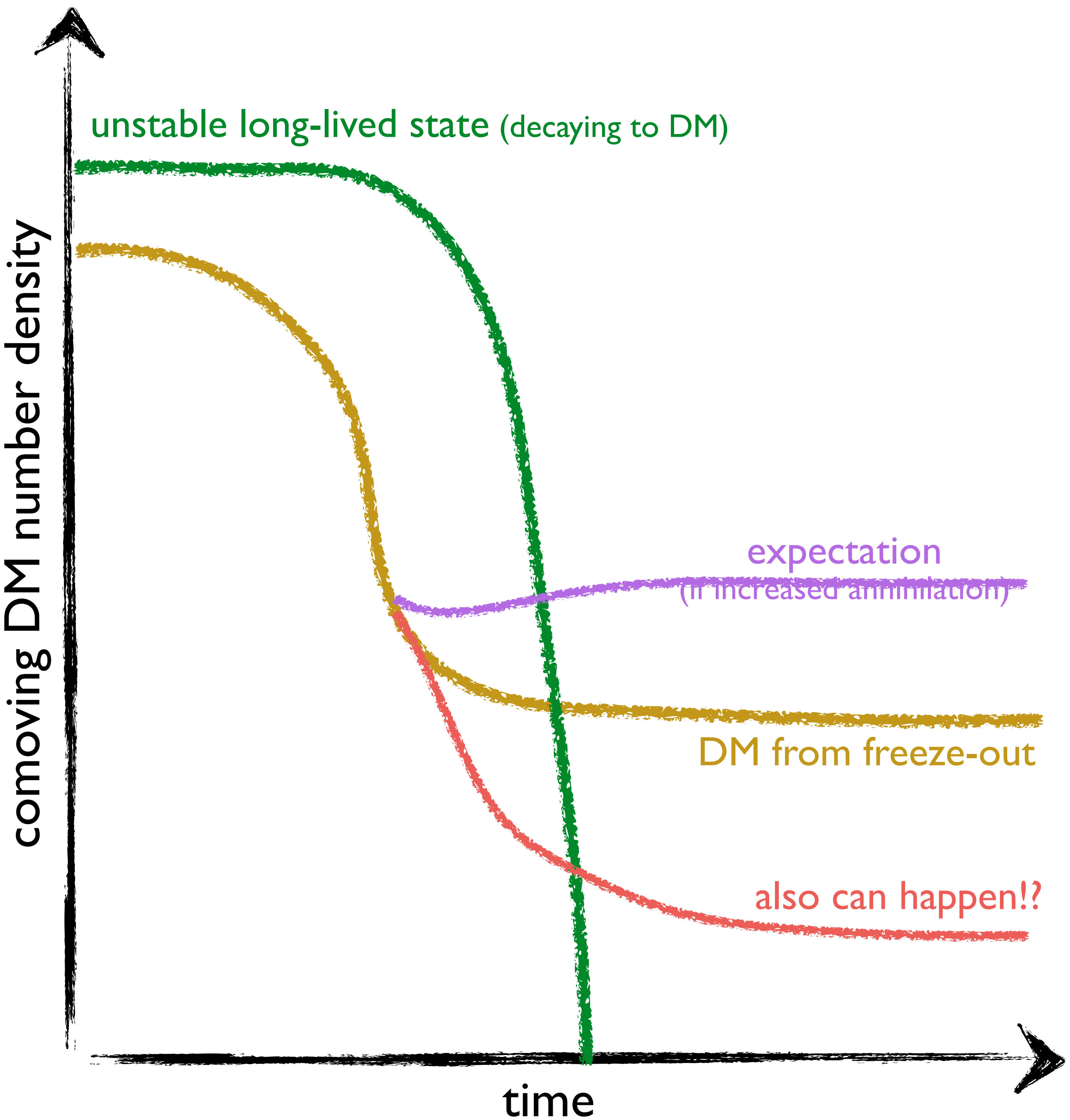
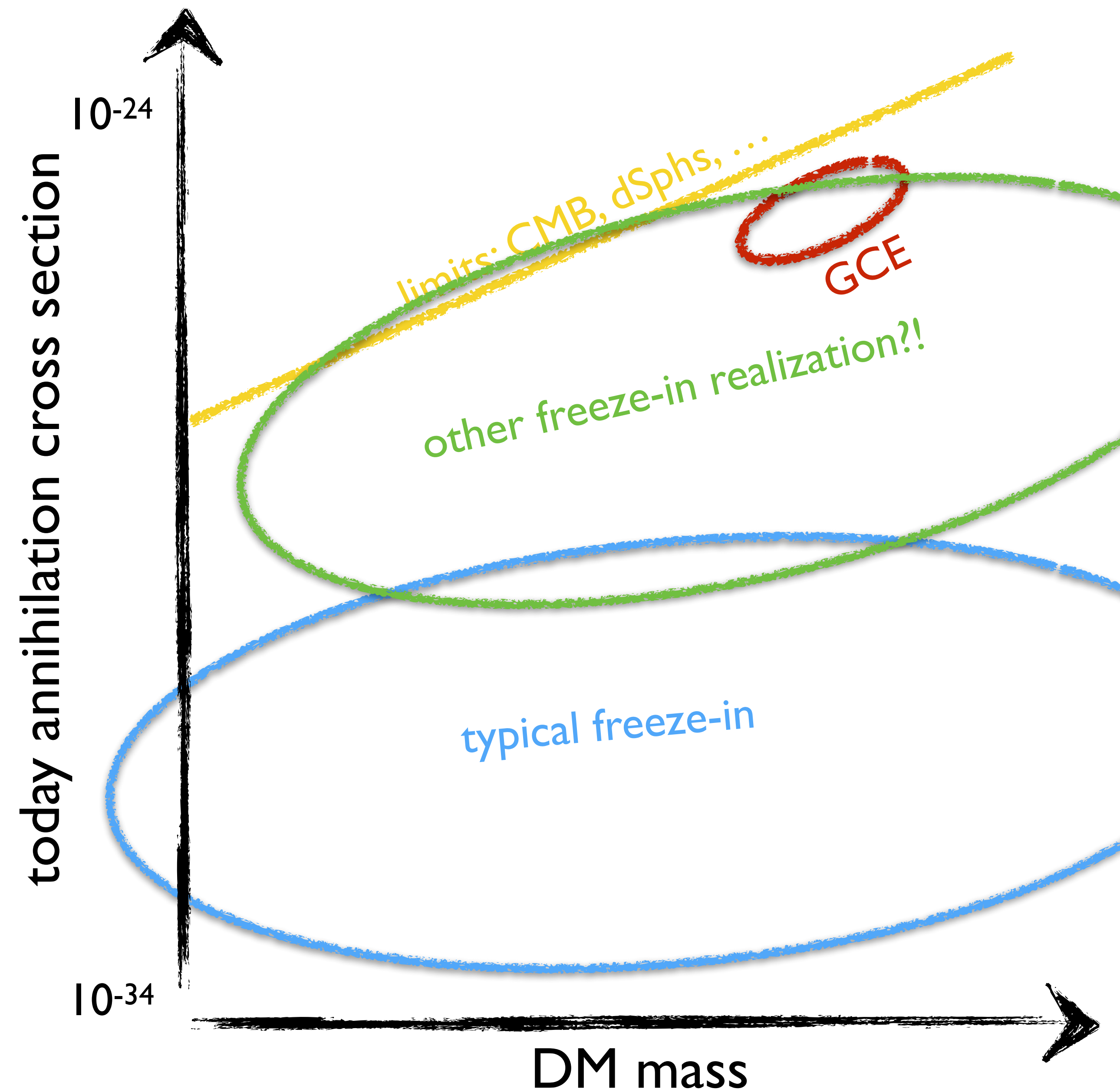
28th September 2022

In case you're not interested in what follows…





to see why and learn more stay tuned :)

THERMAL RELIC DENSITY

STANDARD SCENARIO

T ↑

$\Gamma_{\mathrm{ann}} > H$ — DM in full equilibrium (chemical and kinetic)

$\Gamma_{\mathrm{ann}} \sim H$ — chemical decoupling

$\Gamma_{\mathrm{ann}} < H$ — freeze-out

$\Gamma_{\mathrm{scatt}} \sim H$ — kinetic decoupling

time ↓

non-equilibrium

time evolution of $f_\chi(p)$ in kinetic theory:

$$\underbrace{\boxed{E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right)}}_{\text{Liouville operator in FRW background}} f_\chi = \underbrace{\mathcal{C}[f_\chi]}_{\text{the collision term}}$$

Thermal Relic Density
standard approach

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

*assumptions for using Boltzmann eq: classical limit, molecular chaos,...

...for derivation from thermal QFT see e.g., 1409.3049

$\Downarrow$ integrate over p (i.e. take 0th moment)

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}\to ij}\sigma_{\text{rel}}\rangle^{\text{eq}}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\text{eq}} n_{\bar{\chi}}^{\text{eq}}\right)$$

where the thermally averaged cross section:

$$\langle\sigma_{\chi\bar{\chi}\to ij}v_{\text{rel}}\rangle^{\text{eq}} = -\frac{h_\chi^2}{n_\chi^{\text{eq}} n_{\bar{\chi}}^{\text{eq}}}\int\frac{d^3\vec{p}_\chi}{(2\pi)^3}\frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3}\,\sigma_{\chi\bar{\chi}\to ij}v_{\text{rel}}\; f_\chi^{\text{eq}} f_{\bar{\chi}}^{\text{eq}}$$

Critical assumption:
kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\text{eq}}$$

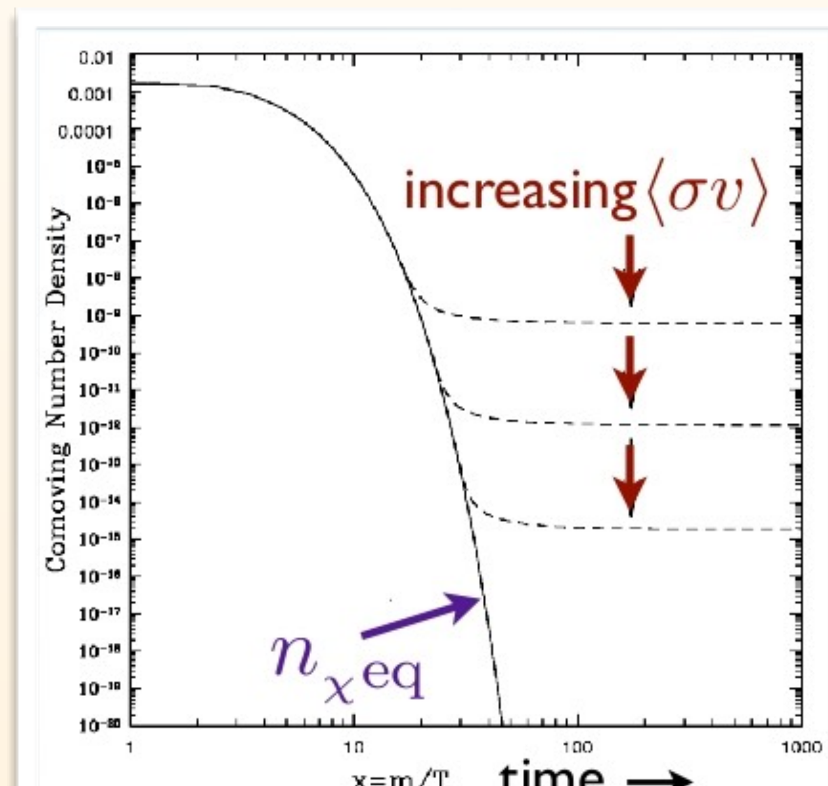
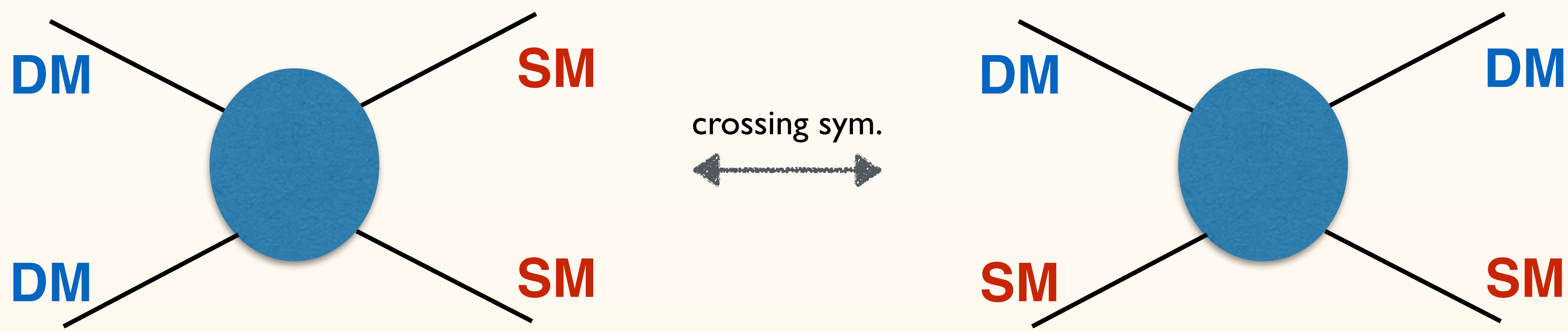


Fig.: Jungman, Kamionkowski & Griest, PR'96

Freeze-out vs. Decoupling

annihilation

(elastic) scattering



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Rightarrow$ scatterings typically more frequent

dark matter frozen-out but typically still kinetically coupled to the plasma

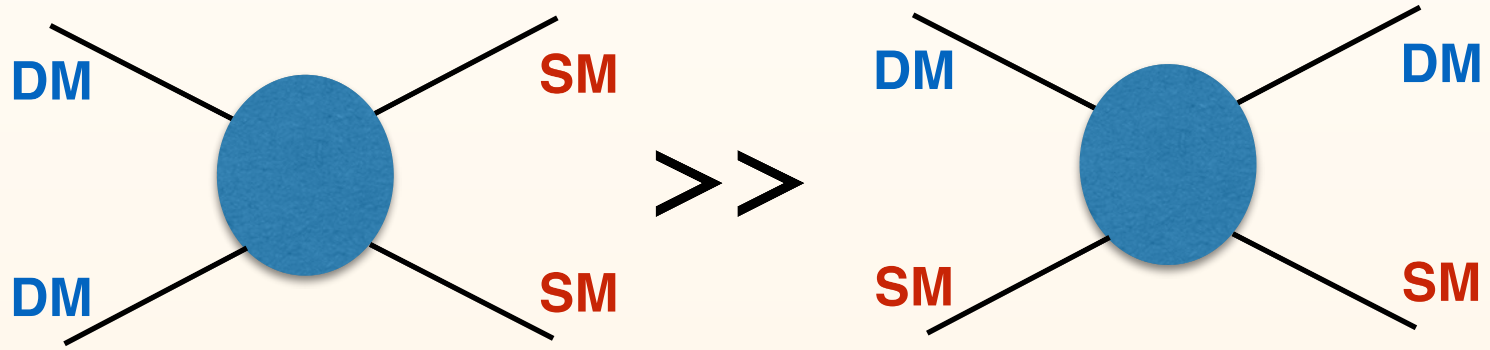
Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Early Kinetic Decoupling?

A necessary and sufficient condition: scatterings weaker than annihilation

i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:

A) 

e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

D) Multi-component dark sectors

e.g., additional sources of DM from late decays, …

How to go beyond kinetic equilibrium?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

contains both scatterings and annihilations

Two possible approaches:

fBE

cBE

0-th moment: n_χ
2-nd moment: T_χ
…

solve numerically for full $f_\chi(p)$

have insight on the distribution
no constraining assumptions

numerically challenging
often an overkill

consider system of equations for <u>moments</u> of $f_\chi(p)$

partially analytic/much easier numerically
manifestly captures all of the relevant physics

finite range of validity
no insight on the distribution

NEW TOOL!

GOING BEYOND THE STANDARD APPROACH

- Home
- Downloads
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DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.
DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,**
  Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]

Currently, an user guide can be found in the Appendix A of this reference.
Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for
any (user defined) model*

Interplay between chemical and kinetic decoupling

Prediction for the DM
phase space distribution

Late kinetic decoupling
and impact on cosmology

see e.g., 1202.5456

…

(only) prerequisite:
Wolfram Language (or *Mathematica*)

*at the moment for a single DM species and w/o co-annihlations… but stay tuned for extensions!

Example D:
When additional influx of DM arrives

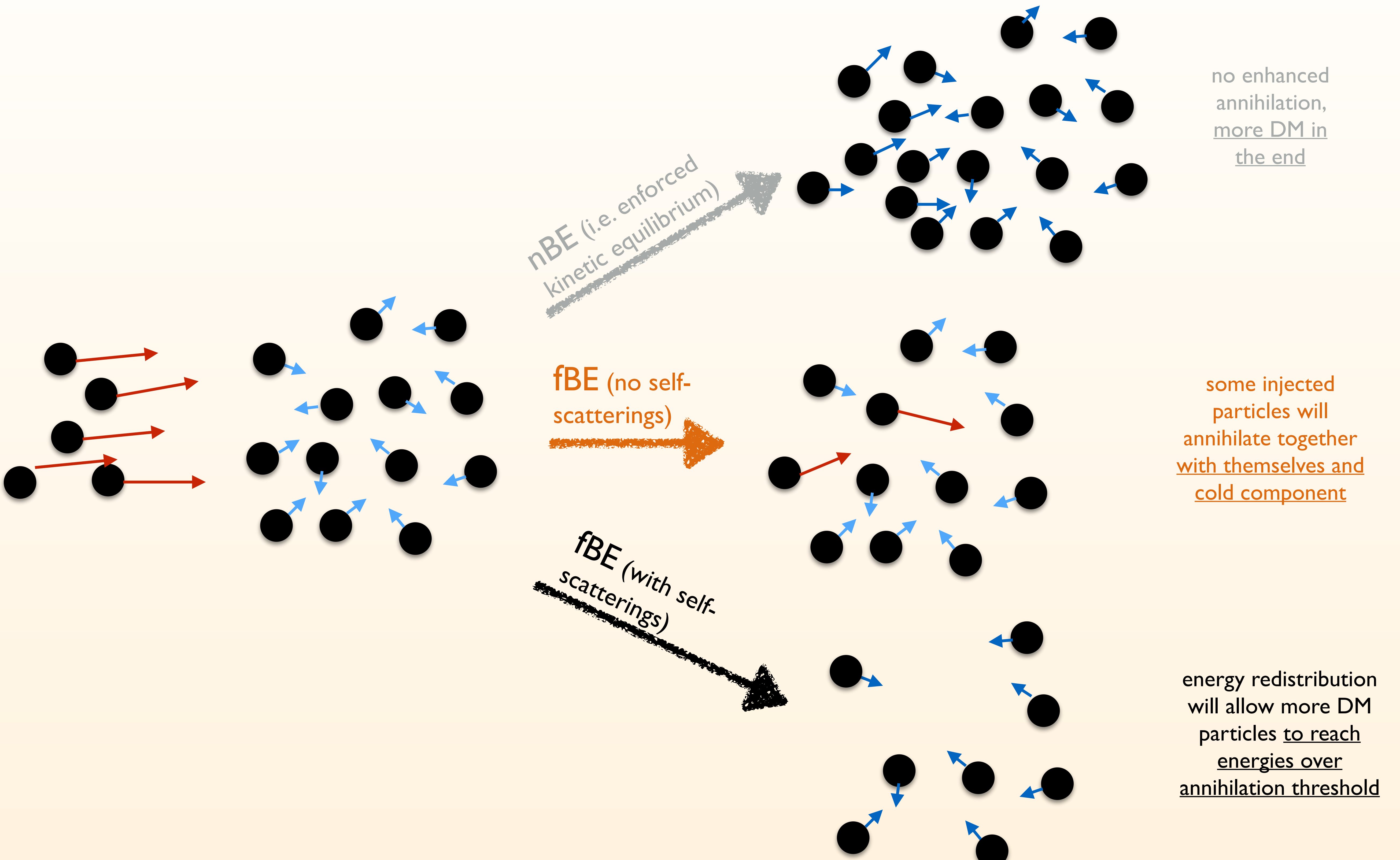
D) Multi-component dark sectors

Sudden injection of more DM particles distorts $f_\chi(p)$
(e.g. from a decay or annihilation of other states)

- this can modify the annihilation rate (if still active)
- how does the thermalization due to elastic scatterings happen?

1) DM produced via:
- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

2) DM annihilation has a threshold
e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$



AH, Laletin 2204.07078

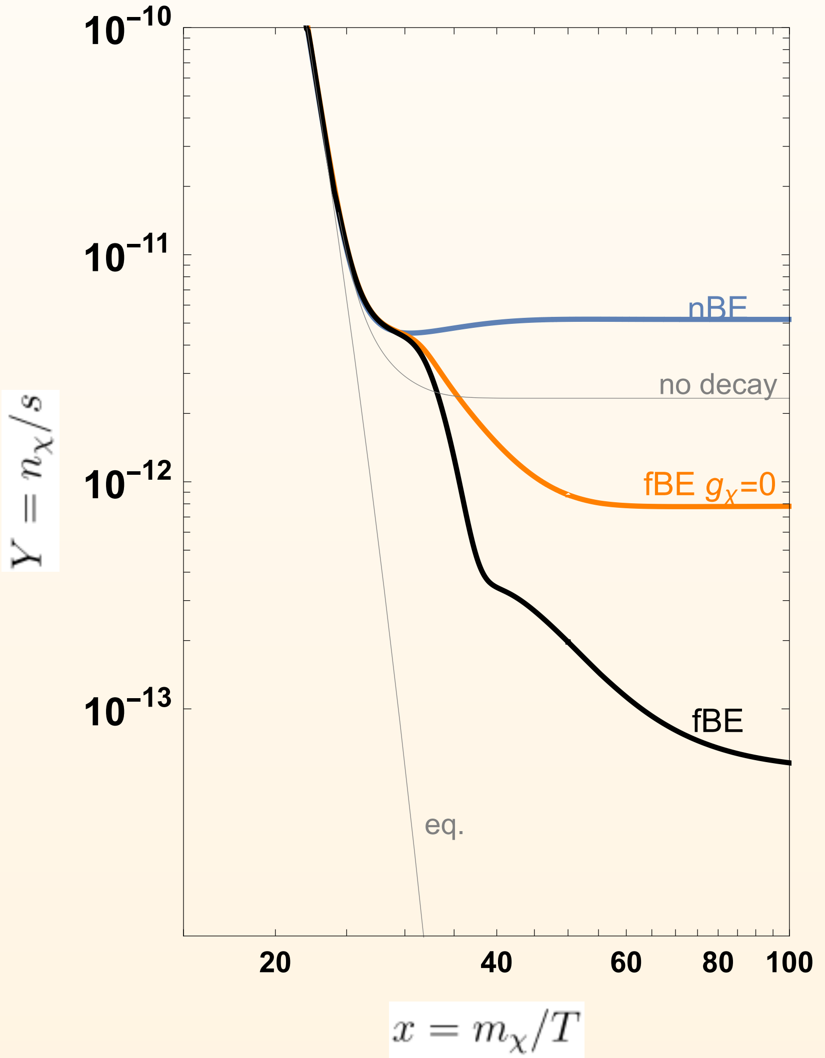
EXAMPLE EVOLUTION

1) DM produced via:
- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

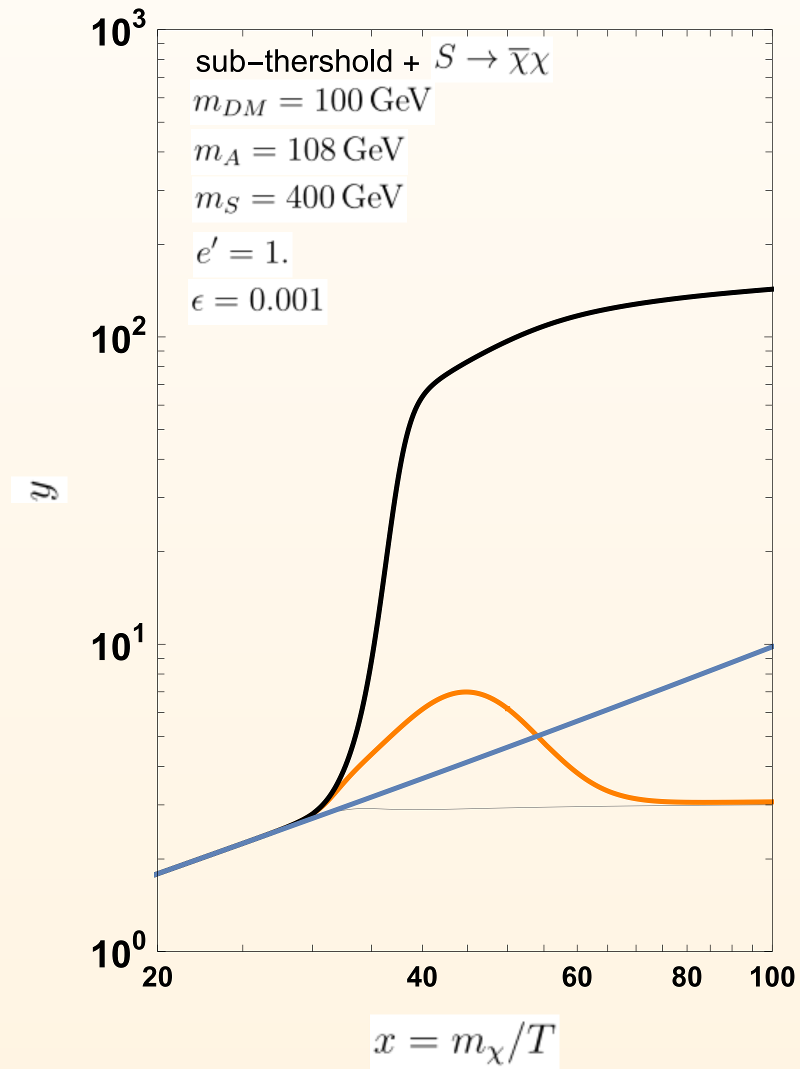
2) DM annihilation has a threshold

e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$

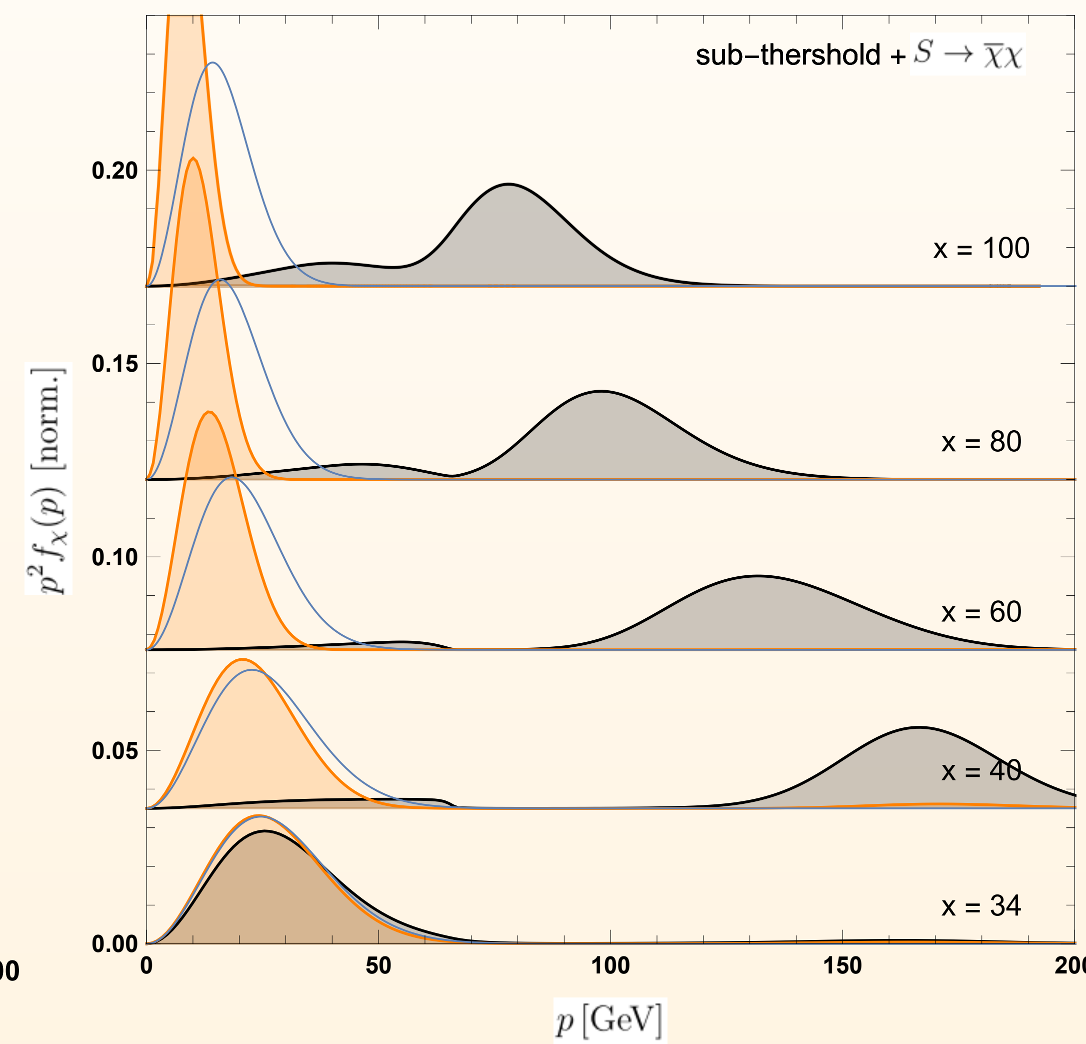
$Y \sim$ number density



$y \sim$ temperature



$p^2 f(p) \sim$ momentum distribution



FREEZE-IN:

C) with semi-annihilation process

HOW ABOUT SEMI-PRODUCTION?

AH, Laletin 2104.05684
(see also Bringmann et al. 2103.16572)

Consider process of production that is the inverse of semi-annihilation:

$$\chi\phi \to \chi\chi$$

DM mediator or a SM state

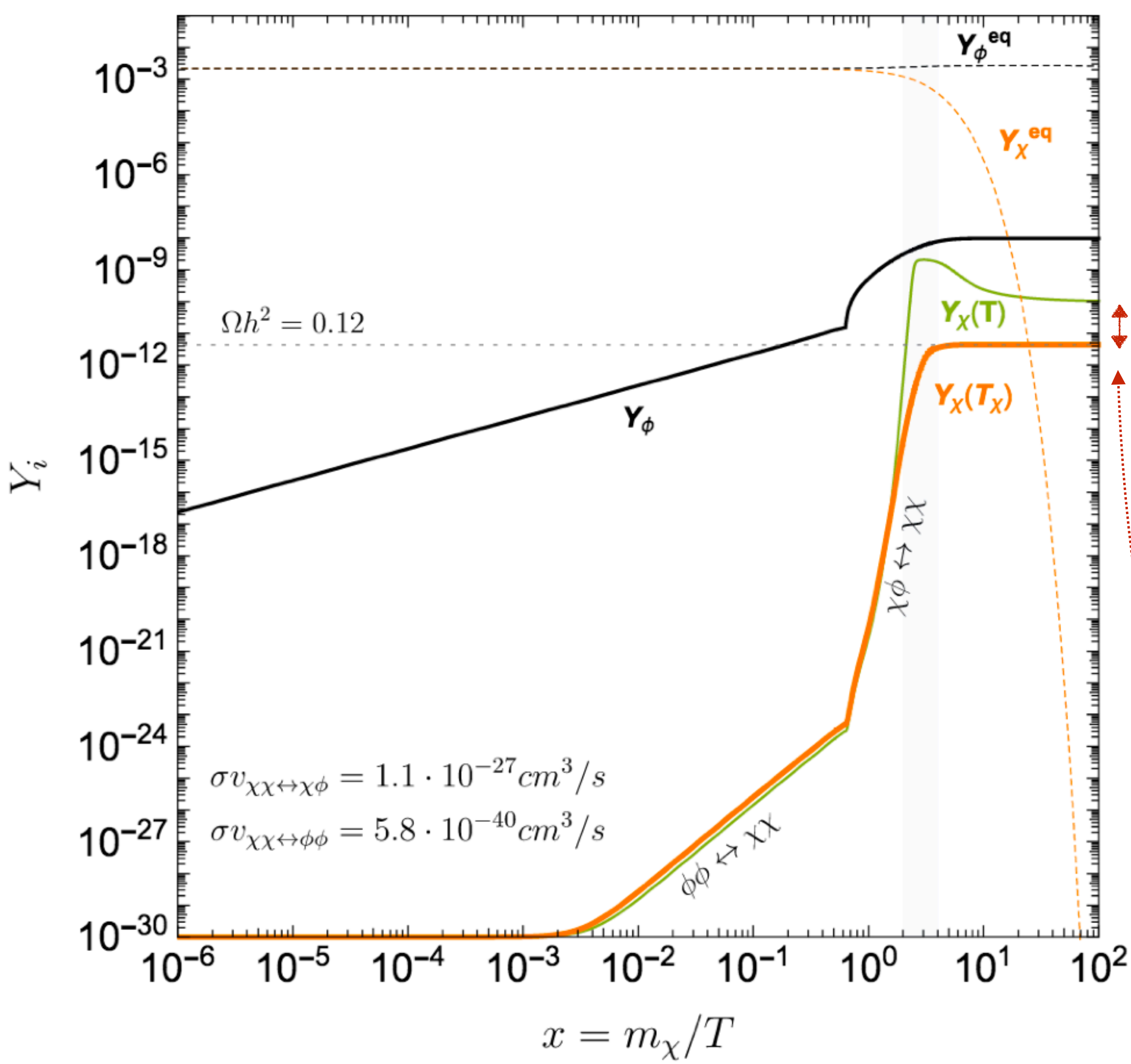
What is different (from the decay/pair-annihilation freeze-in)?

- The production rate is proportional to the DM density. (Smaller initial abundance → larger cross section…)

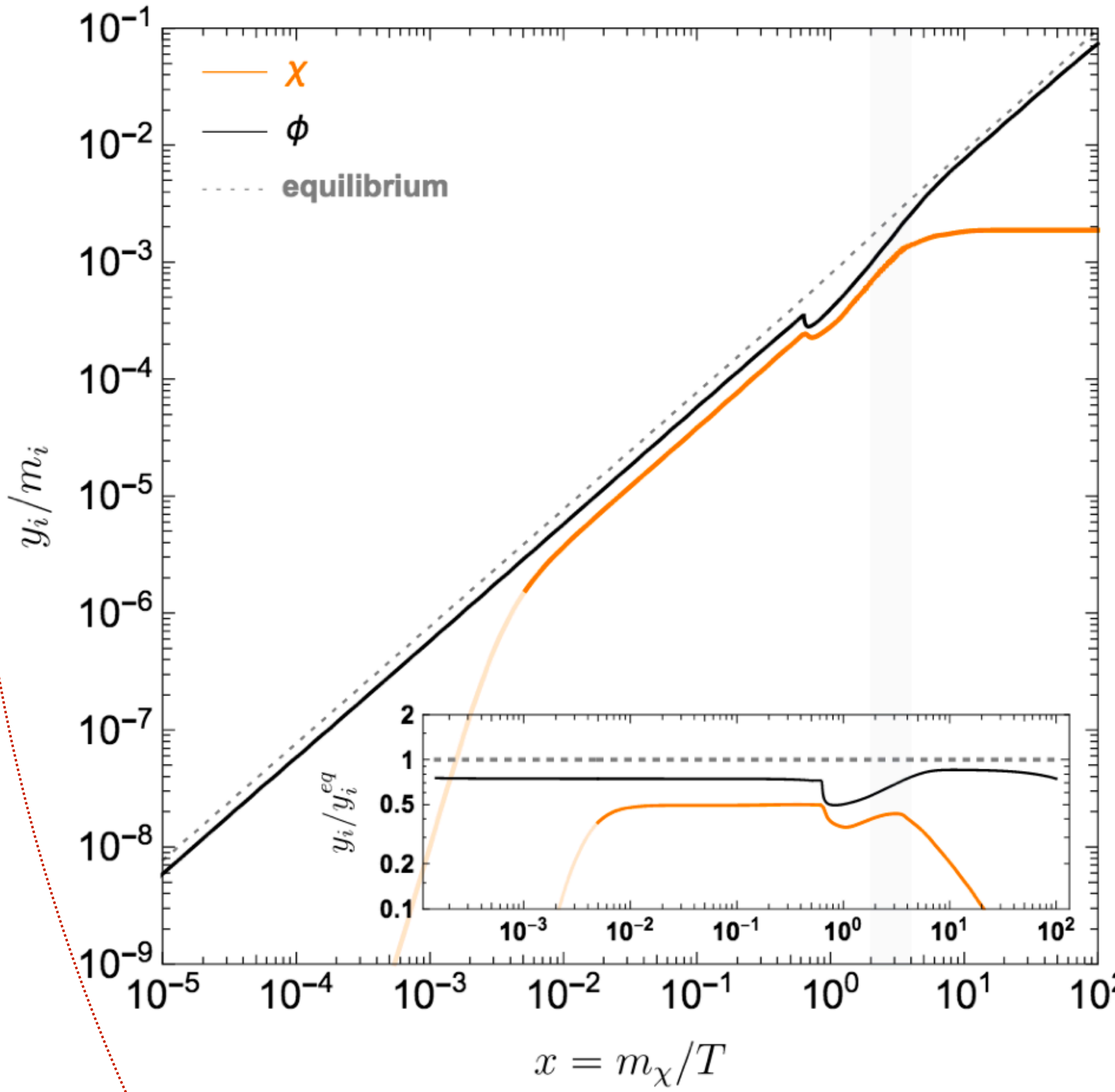
- Semi-production modifies the energy of DM particles in a non-trivial way, so the temperature evolution can affect the relic density

EVOLUTION

co-moving number density



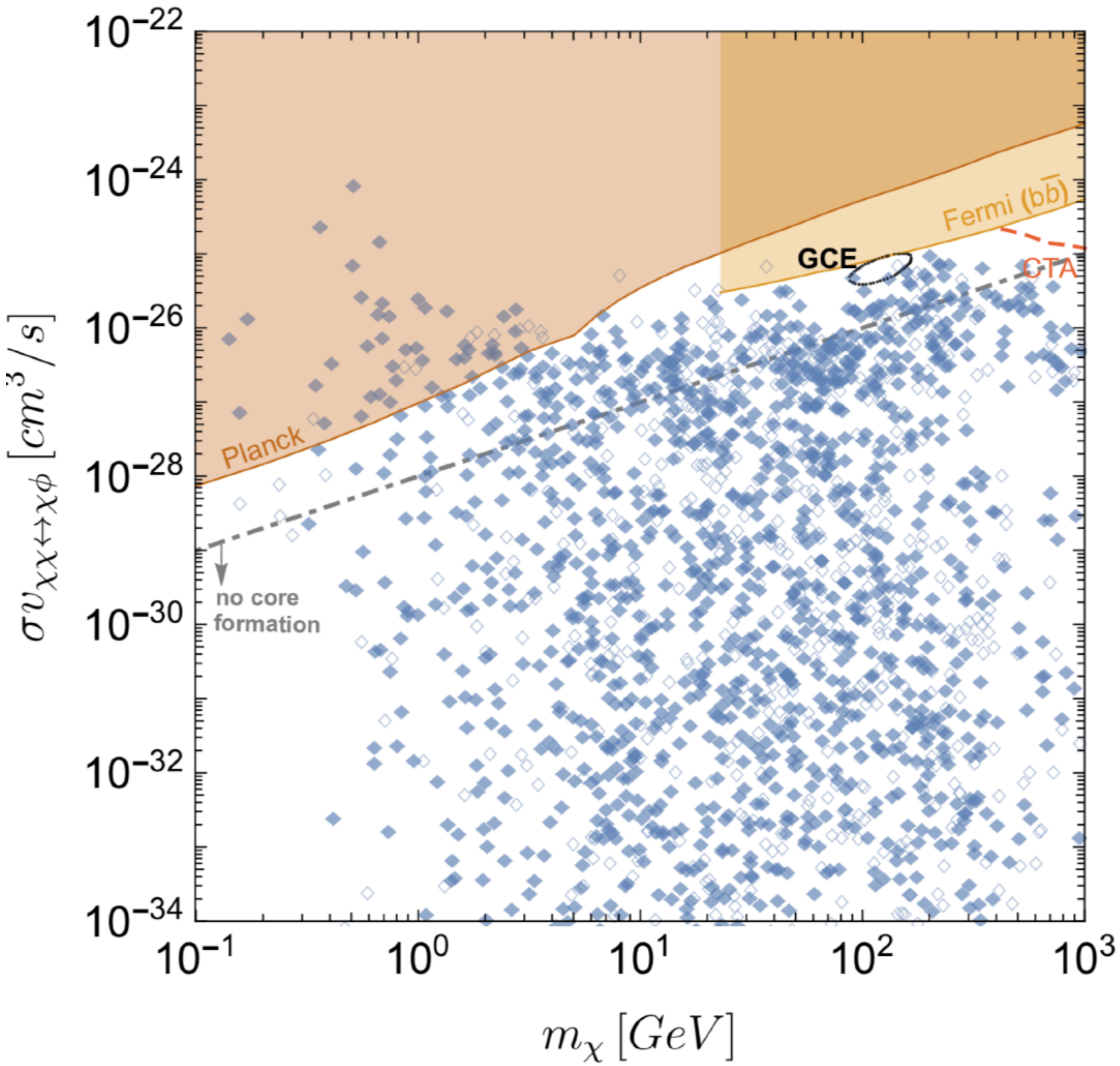
'temperature'



$m_\chi = 100$ GeV, $\mu_\phi = 1$ GeV, $\lambda_1 = 1.1 \times 10^{-2}$, $\lambda_2 = 10^{-8}$, $\lambda_{h\phi} = 6 \times 10^{-11}$

The full calculation compared to one assuming $T_\chi = T$
can differ by more than order of magnitude!

Indirect Detection



- The results of the scan in the parameter space for the DM production dominated by the semi-annihilation processes.
- The coloured squares indicate the points, which are within the reach of the future searches for the mediator ϕ and the empty ones are beyond these prospects.
- The points above the grey dot-dashed line can potentially explain the core formation in dSph [1803.09762]

SUMMARY

1. Kinetic equilibrium is a <u>necessary</u> (often implicit) assumption for <u>standard</u> relic density calculations in all the numerical tools…
…while it is not always warranted!

2. Much more <u>accurate</u> treatment comes from solving the full phase space Boltzmann equation (fBE) to obtain result for $f_{\rm DM}(p)$ where one can study also self-thermalization from self-scatterings

3. Introduced DRAKE : a <u>new tool</u> to extend the current capabilities to the regimes beyond kinetic equilibrium

4. Multi-component sectors, when studied at the fBE level, can reveal quite unexpected behavior