RELIC ABUNDANCE THEORY: NEW DEVELOPMENTS

Andrzej Hryczuk



A personal selection of recent ideas in the field

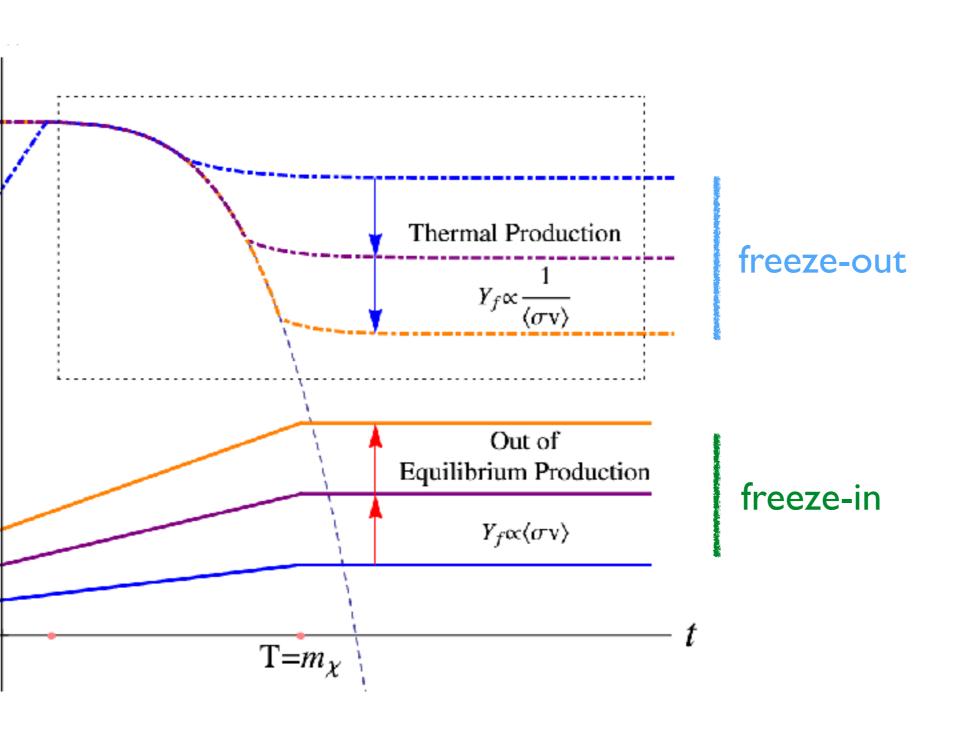
+ some results based on:

T. Binder, T. Bringmann, M. Gustafsson & A.H. <u>1706.07433</u>, <u>2103.01944</u>
A.H. & M. Laletin 2204.07078

DARK MATTER ORIGIN

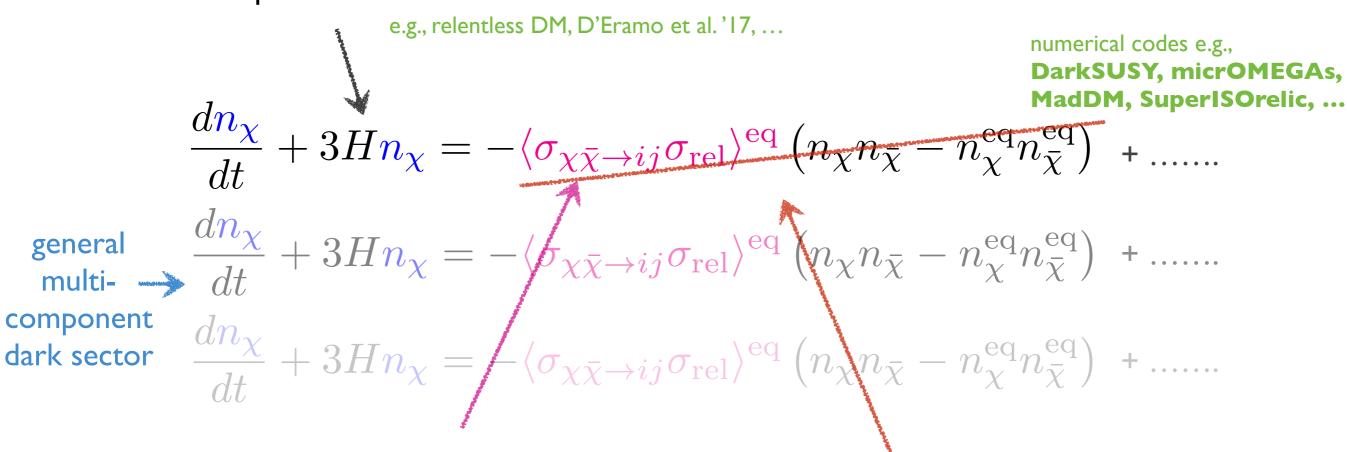


FREEZE-IN vs. FREEZE-OUT



THERMAL RELIC DENSITY STANDARD SCENARIO & BEYOND

modified expansion rate



modified cross section

Sommerfeld enhancement Bound State formation

NLO

finite T effects

where the thermally averaged cross section:

$$\langle \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel} \rangle^{\rm eq} = -\frac{h_{\chi}^2}{n_{\chi}^{\rm eq} n_{\bar{\chi}}^{\rm eq}} \int \frac{d^3 \vec{p}_{\chi}}{(2\pi)^3} \frac{d^3 \vec{p}_{\bar{\chi}}}{(2\pi)^3} \, \sigma_{\chi\bar{\chi}\to ij} v_{\rm rel} \, f_{\chi}^{\rm eq} f_{\bar{\chi}}^{\rm eq}$$

assumptions leading to different form of the equation, e.g. violation of kinetic equilibrium

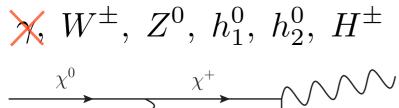
CHAPTER I: PARTICLE PHYSICS EFFECTS

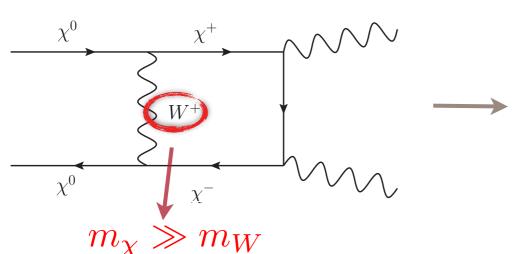
THE SOMMERFELD EFFECT

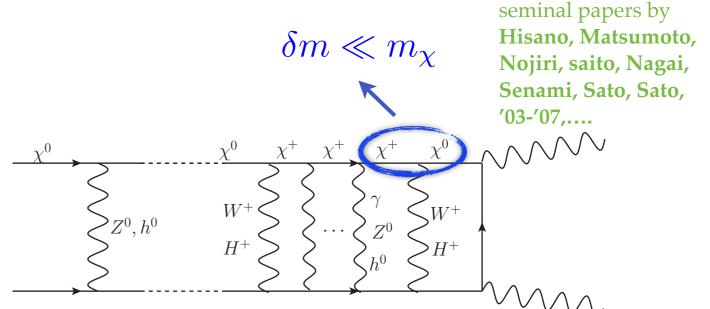
FROM EW INTERACTIONS

Thermal relic abundance, $\Omega_{\rm DM}h^2$ 0.2 2 m (TeV)

force carriers in the MSSM:







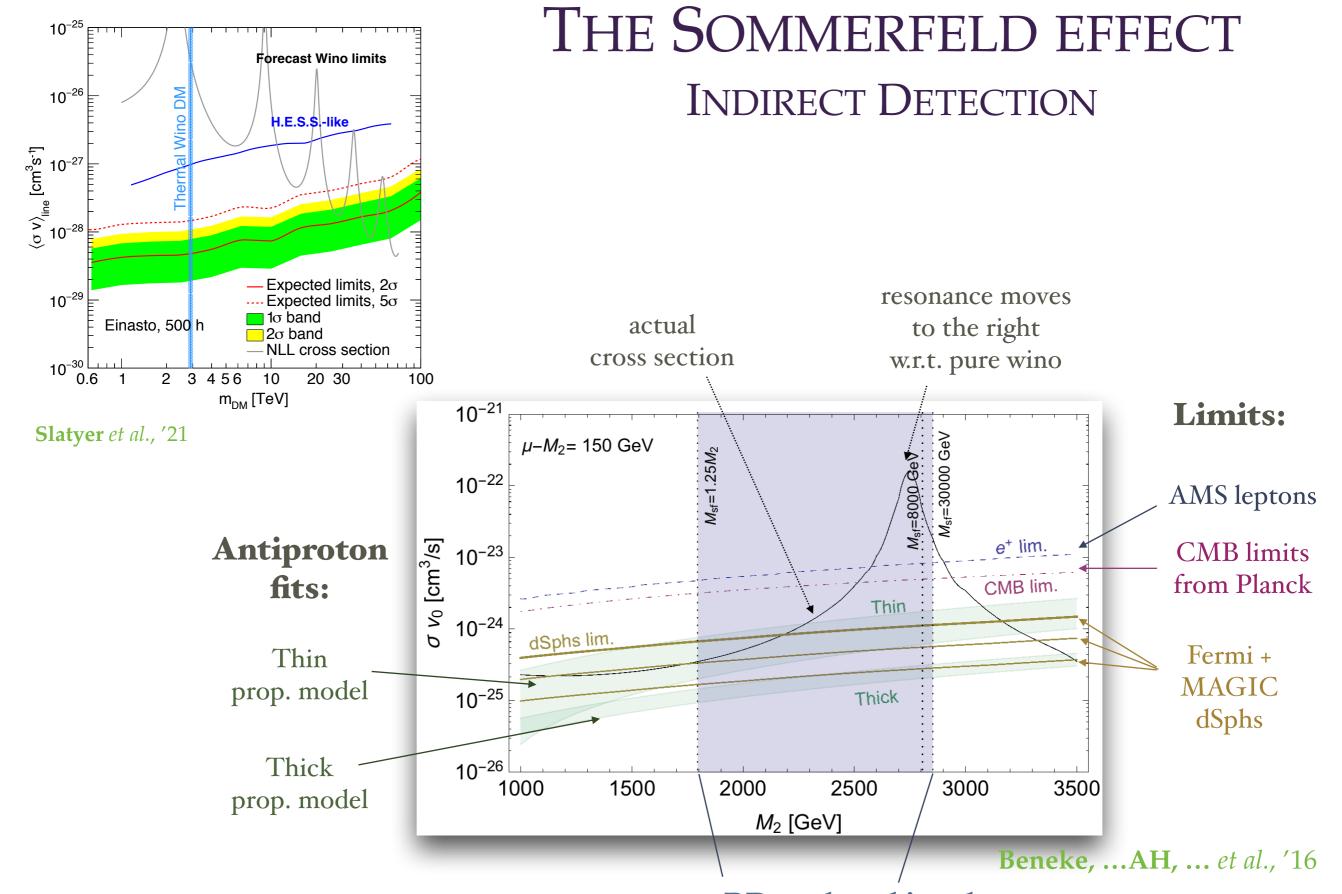
at TeV scale \implies generically effect of $\mathcal{O}(1-100\%)$

on top of that resonance structure

can be understood as being close to a threshold of lowest bound state

 \rightarrow effect of $\mathcal{O}(\text{few})$ for the relic density

AH, R. Iengo, P. Ullio. '10; AH '11 AH et al. '17, M. Beneke et al.; '16



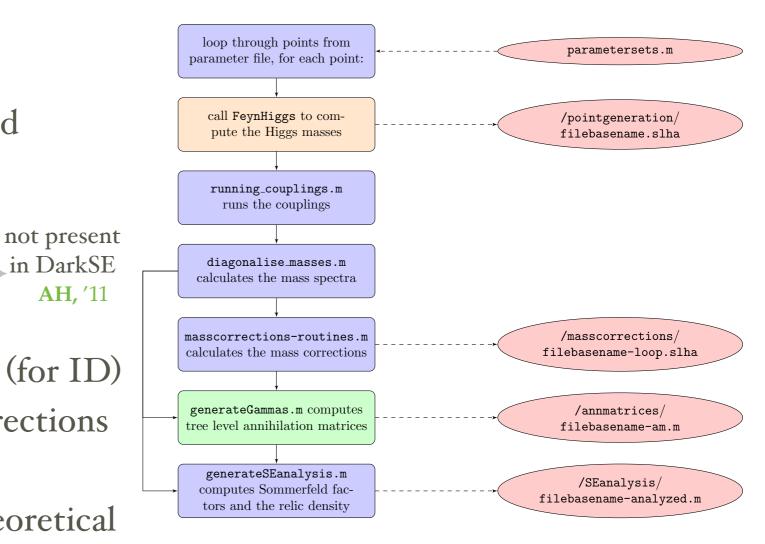
correct RD can be achieved: when varying sfermion masses

NEW NUMERICAL TOOL

based on EFT, improving accuracy in numerous ways

AH. '11

- suitable for (large scale) scans
- implemented full MSSM
- one-loop on-shell mass splittings and running couplings
- the Sommerfeld effect for P- and O(v²) S-wave
- off-diagonal annihilation matrices
- present day annihilation in the halo (for ID)
- possibility of including thermal corrections
- accuracy at O(%), dominated by theoretical uncertainties of EFT



Status: all works as intended, making the code ready for public release

Beneke,..., AH,... et al. in preparation

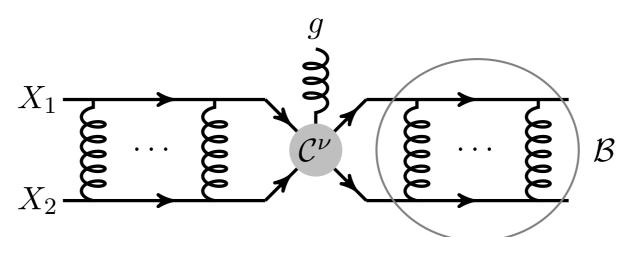
BOUND STATE FORMATION

As noticed before Sommerfeld effect has resonances when Bohr radius ~ potential range, i.e. when close to a bound state threshold

Can DM form actual bound states from such long range interactions?

¥ Yes, it can!

Q: How to describe such bound states and their formation?



free DM states

DM bound state

^{*}the effect was first studied in simplified models with light mediators, then gradually extended to non-Abelian interactions, double emissions, co-annihilations, etc.

^{**}vide also "WIMPonium"

March-Russel, West '10

Example: BSF for TeV scale WIMP

Electroweak interactions are stronger and longer ranged than Higgs mediated... but also more complicated (non-Abelian + massive mediators)

here as far as I know work is still in progress...

Higgs mediated \Rightarrow

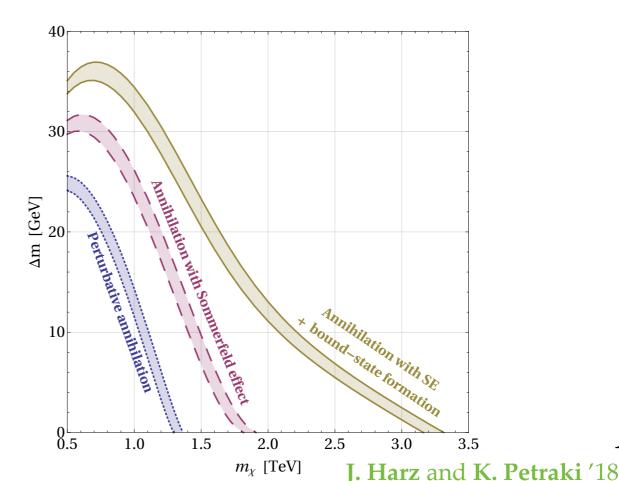
Could lead to DM bound states, but for usual TeV DM models, biggest effect observed is more indirect

> e.g. produces tighter bound states of squarks - less inefficient dissociation - more efficient DM depopulation

> > J. Harz and K. Petraki '19

but e.g.: co-annihilation with squarks and QCD squark bound states

significant modification of the annihilation rate - large effects on the DM models, especially in the TeV scale



BOUND STATES IN DM WORKSHOP LAST JUNE 2021 @ IMPU https://indico.ipmu.jp/event/389/



Przeglad

Rejestracja

Lista uczestników

Group photos

Harmonogram

Prorgam

Lista wkładów

Link

Kavli IPMU Code of Conduct

Overview:

This online theory workshop will bring together Quarkonia and Dark Matter physicists. The main goal of this workshop is to exchange theoretical knowledge on the intersection of both fields. In particular, this includes state-of-the-art effective field theoretical descriptions of heavy pair annihilation and bound-state formation/dissociation inside a plasma, as well as non-equilibrium quantum field theories for describing the systems dynamics, such as open quantum system treatments and the Keldysh-Schwinger formalism. For this initial meeting, speakers are experts in the fields and selected by invitation only. Participants are encouraged to apply for a poster contribution.

Dates:

- 15th June 2021 18th June 2021
- Daily from 8:00 PM to 11:59 PM JST (1 PM 5 PM CET, 7 AM 11 AM EDT)

DARK MATTER AT NLO

```
Bergstrom '89; Drees et al., 9306325;
                                        helicity suppression lifting
    Ullio & Bergstrom, 9707333
      Bergstrom et al., 0507229;
                                        spectral features in indirect searches
     Bringmann et al., 0710.3169
     Ciafaloni et al., 1009.0224
      Cirelli et al., 1012.4515
                                         large EW corrections
    Ciafaloni et al., 1202.0692
     AH & lengo, 1111.2916
   Chatterjee et al., 1209.2328
      Harz et al., 1212.5241
    Ciafaloni et al., 1305.6391
                                         thermal relic density
    Hermann et al., 1404.2931
    Boudjema et al., 1403.7459
    Bringmann et al., 1510.02473
      Klasen et al., 1607.06396
```

$$\Omega_{DM}h^2 = 0.1187 \pm 0.0017. \qquad \mbox{< I.5\% uncertainty!} \\ \mbox{Planck+WMAP pol.+highL+BAO; I303.5062}$$

SloopS, DM@NLO, PPC4DMID

NLO codes

RELIC DENSITY AT NLO

Recall at LO:

$$C_{\text{LO}} = -h_{\chi}^{2} \int \frac{d^{3}\vec{p}_{\chi}}{(2\pi)^{3}} \frac{d^{3}\vec{p}_{\bar{\chi}}}{(2\pi)^{3}} \, \sigma_{\chi\bar{\chi}\to ij} v_{\text{rel}} \, \left[f_{\chi} f_{\bar{\chi}} (1 \pm f_{i}) (1 \pm f_{j}) - f_{i} f_{j} (1 \pm f_{\chi}) (1 \pm f_{\bar{\chi}}) \right]$$

crucial point:
$$p_\chi + p_{ar\chi} = p_i + p_j \Rightarrow f_\chi^{
m eq} f_{ar\chi}^{
m eq} pprox f_i^{
m eq} f_j^{
m eq}$$
 in Maxwell approx

at NLO both virtual one-loop and 3-body processes contribute:

$$\begin{split} C_{1-\mathrm{loop}} &= -h_{\chi}^2 \int \frac{d^3\vec{p}_{\chi}}{(2\pi)^3} \, \frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3} \, \sigma_{\chi\bar{\chi}\to ij}^{1-\mathrm{loop}} v_{\mathrm{rel}} \ [f_{\chi}f_{\bar{\chi}}(1\pm f_i)(1\pm f_j) - f_if_j(1\pm f_{\chi})(1\pm f_{\bar{\chi}})] \\ C_{\mathrm{real}} &= -h_{\chi}^2 \int \frac{d^3\vec{p}_{\chi}}{(2\pi)^3} \, \frac{d^3\vec{p}_{\bar{\chi}}}{(2\pi)^3} \, \sigma_{\chi\bar{\chi}\to ij\gamma} v_{\mathrm{rel}} \ [f_{\chi}f_{\bar{\chi}}(1\pm f_i)(1\pm f_j)(1+f_{\gamma}) - f_if_jf_{\gamma}(1\pm f_{\chi})(1\pm f_{\bar{\chi}})] \\ p_{\chi} + p_{\bar{\chi}} &= p_i + p_j \pm p_{\gamma} \Rightarrow \begin{array}{c} \mathrm{photon\ can\ be} \\ \mathrm{arbitrarily\ soft} \end{array}$$

Maxwell approx. not valid anymore...

...problem: *T*-dependent IR divergence!

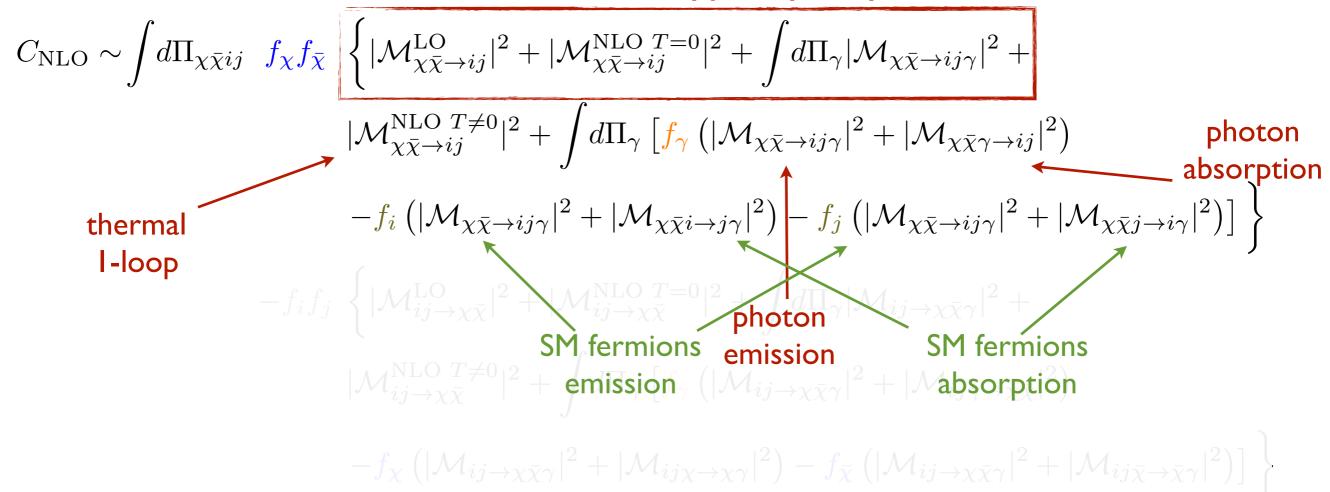
 $f_{\gamma} \sim \omega^{-1}$

RELIC DENSITY

WHAT REALLY HAPPENS AT NLO?

Beneke, Dighera, AH, 1409.3049

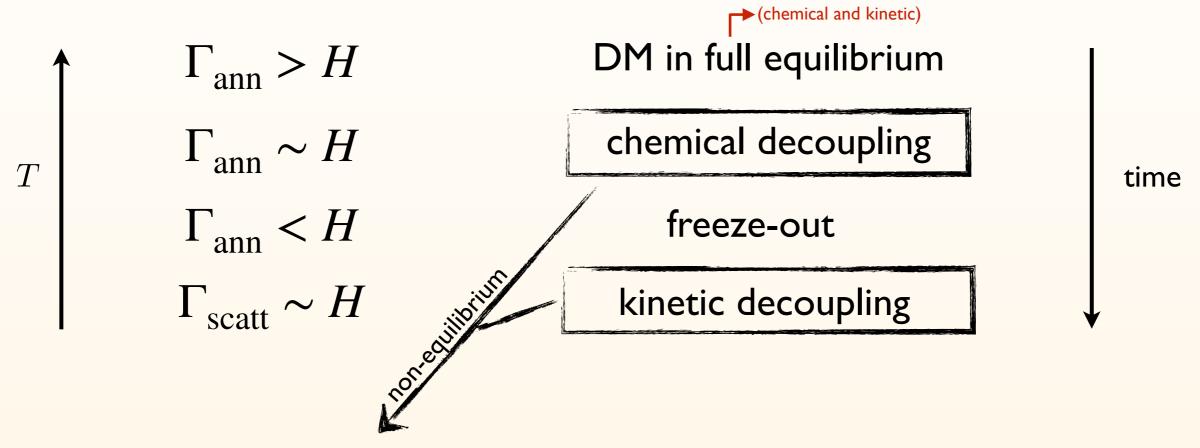
typically only this used in NLO literature



SOLUTION: non-equilibrium thermal field theory

CHAPTER II: NON-EQUILIBRIUM EFFECTS

THERMAL RELIC DENSITY STANDARD SCENARIO



time evolution of $f_{\chi}(p)$ in kinetic theory:

$$E\left(\partial_t - H\vec{p} \cdot \nabla_{\vec{p}}\right) f_{\chi} = \mathcal{C}[f_{\chi}]$$

Liouville operator in FRW background

the collision term

THERMAL RELIC DENSITY STANDARD APPROACH

Boltzmann equation for $f_{\chi}(p)$:

$$E\left(\partial_t - H\vec{p} \cdot \nabla_{\vec{p}}\right) f_{\chi} = \mathcal{C}[f_{\chi}]$$

integrate over *p* (i.e. take 0th moment)

*assumptions for using Boltzmann eq: classical limit, molecular chaos,...

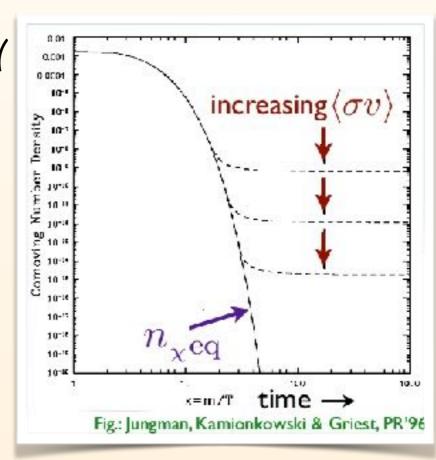
...for derivation from thermal QFT see e.g., 1409.3049

$$\frac{dn_{\chi}}{dt} + 3Hn_{\chi} = -\langle \sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq} \left(n_{\chi}n_{\bar{\chi}} - n_{\chi}^{\rm eq}n_{\bar{\chi}}^{\rm eq}\right)$$

Critical assumption:

kinetic equilibrium at chemical decoupling

$$f_{\chi} \sim a(T) f_{\chi}^{\text{eq}}$$



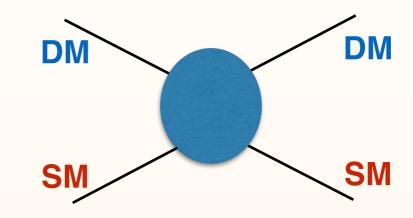
FREEZE-OUT VS. DECOUPLING

annihilation



$$\sum \left| \mathcal{M}^{\text{pair}} \right|^2 = F(p_1, p_2, p_1', p_2')$$

(elastic) scattering



$$\sum_{\text{spins}} \left| \mathcal{M}^{\text{scatt}} \right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM



scatterings typically more frequent

dark matter frozen-out but typically still kinetically coupled to the plasma

$$au_{
m r}(T_{
m kd}) \equiv N_{
m coll}/\Gamma_{
m el} \sim H^{-1}(T_{
m kd})$$

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

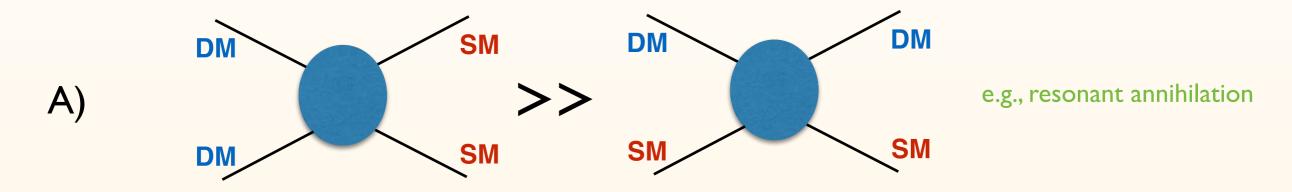
Two consequences:

- During freeze-out (chemical decoupling) typically: $f_{\chi} \sim a(\mu) f_{\chi}^{\text{eq}}$
- If kinetic decoupling much, much later: possible impact on the matter power spectrum 2. i.e. kinetic decoupling can have observable consequences and affect e.g. missing satellites problem

EARLY KINETIC DECOUPLING?

A necessary and sufficient condition: scatterings weaker than annihilation i.e. rates around freeze-out: $H \sim \Gamma_{\rm ann} \gtrsim \Gamma_{\rm el}$

Possibilities:



B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,...

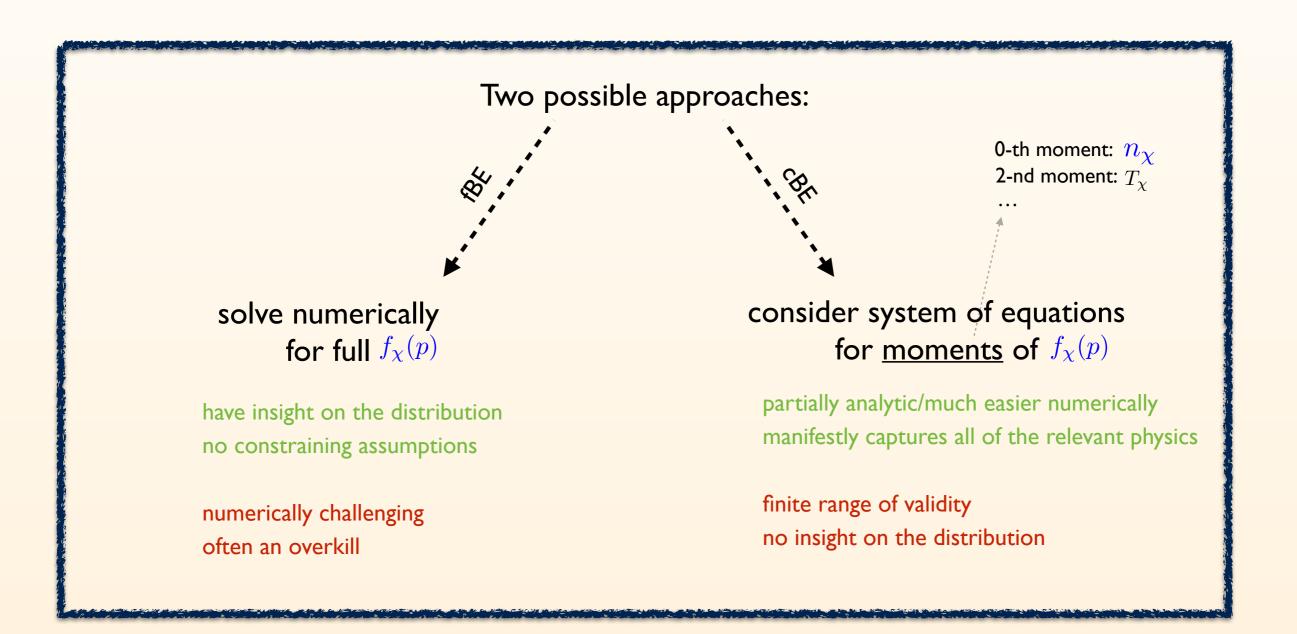
D) Multi-component dark sectors

HOW TO GO BEYOND KINETIC EQUILIBRIUM?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p} \cdot \nabla_{\vec{p}}\right) f_{\chi} = \mathcal{C}[f_{\chi}]$$
 contains both scatterings and annihilations



NEW TOOL!

T. Binder, T. Bringmann,
M. Gustafsson & A.H. 2103.01944

GOING BEYOND THE STANDARD APPROACH

- Home
- Downloads
- Contact



Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.

DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,
 Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]

Currently, an user guide can be found in the Appendix A of this reference. Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for any (user defined) model*

Interplay between chemical and kinetic decoupling

Prediction for the DM phase space distribution

Late kinetic decoupling and impact on cosmology

see e.g., 1202.5456

. . .

(only) prerequisite: Wolfram Language (or Mathematica)



SNAPSHOTS FROM AN EXAMPLE NOTEBOOK

I. Load DRAKE

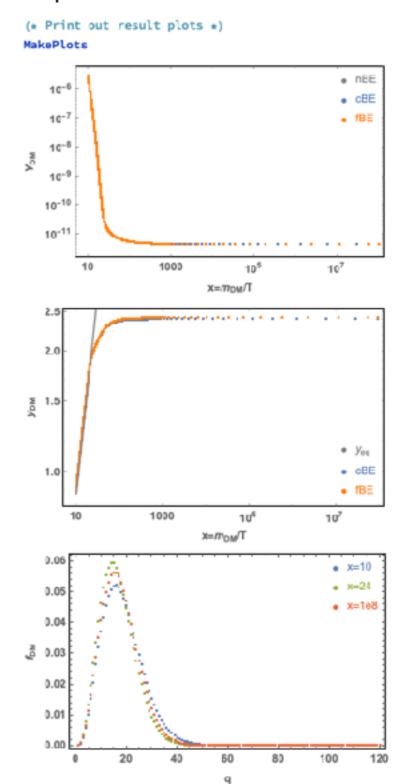
Needs["DRAKE`"]

Oh2fBE = 0.120037

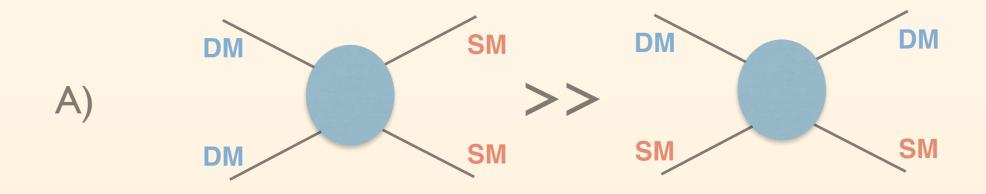
2. Initialize model

3. Run

4. Print plots

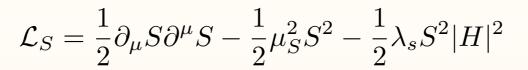


EXAMPLE A:SCALAR SINGLET DM



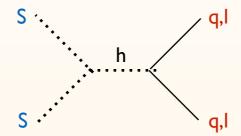
EXAMPLE A SCALAR SINGLET DM

To the SM Lagrangian add one singlet scalar field S with interactions with the Higgs:



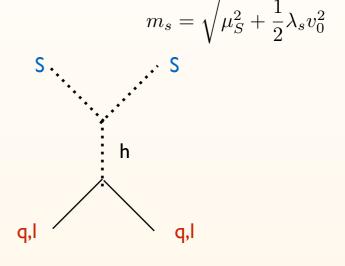
Annihilation processes:

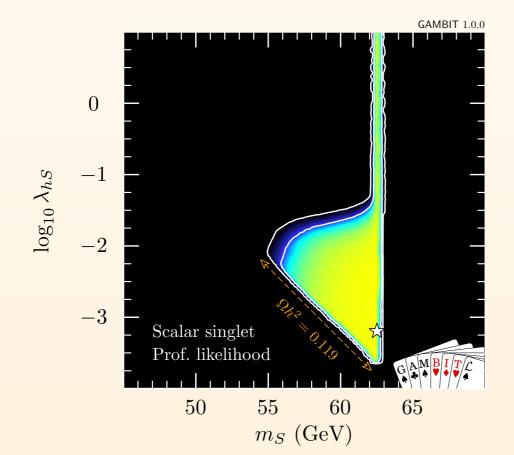
resonant

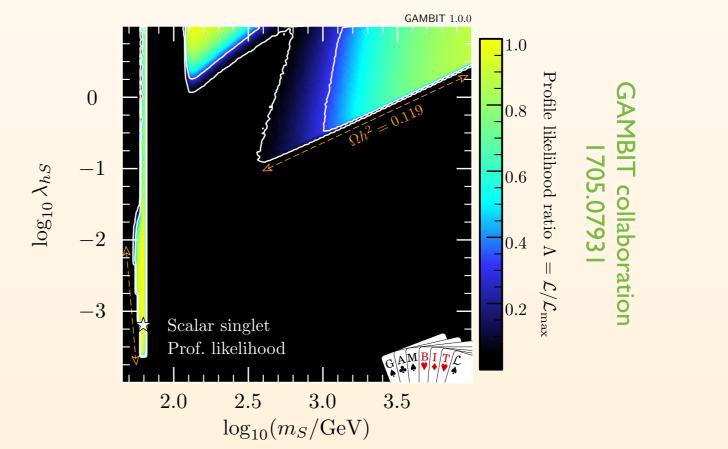


El. scattering processes:

non-resonant

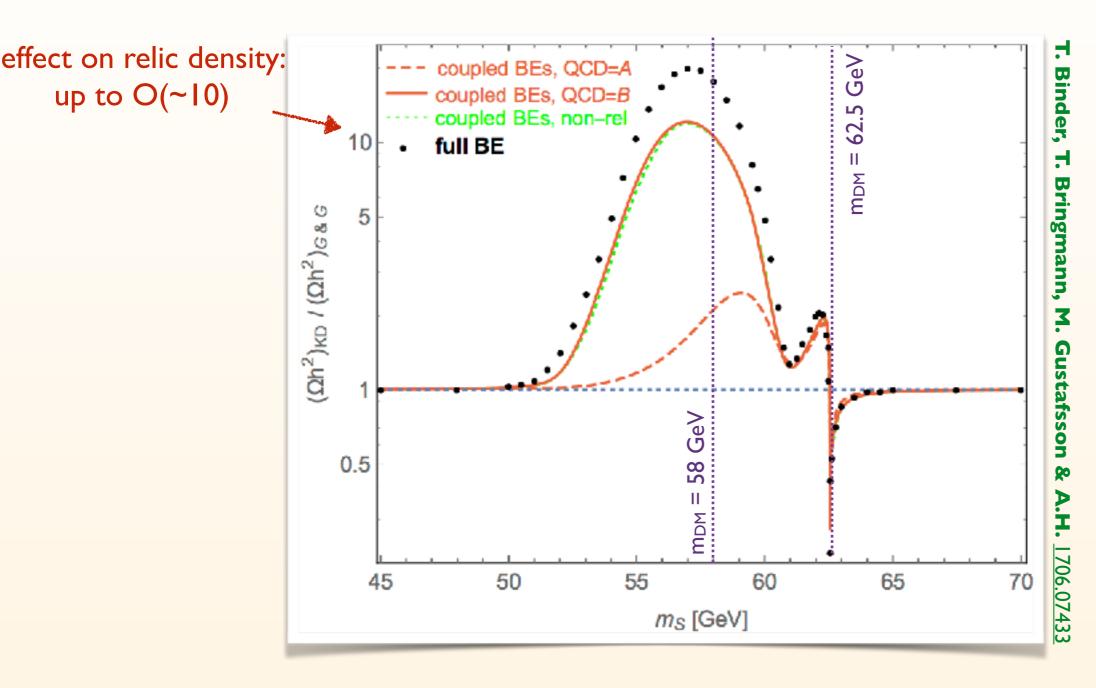






RESULTS

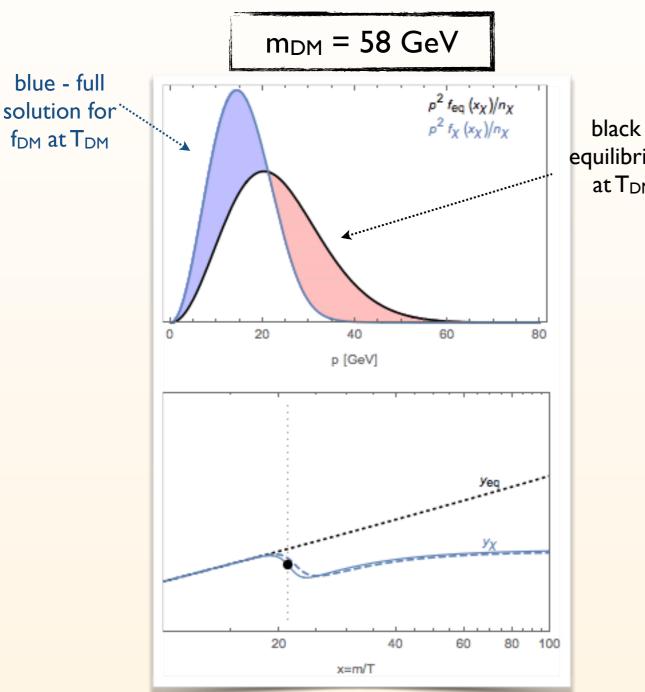
EFFECT ON THE Ωh^2



[... Freeze-out at few GeV — what is the <u>abundance of heavy quarks</u> in QCD plasma?

two scenarios: QCD = A - all quarks are free and present in the plasma down to T_c = 154 MeV QCD = B - only light quarks contribute to scattering and only down to $4T_c$

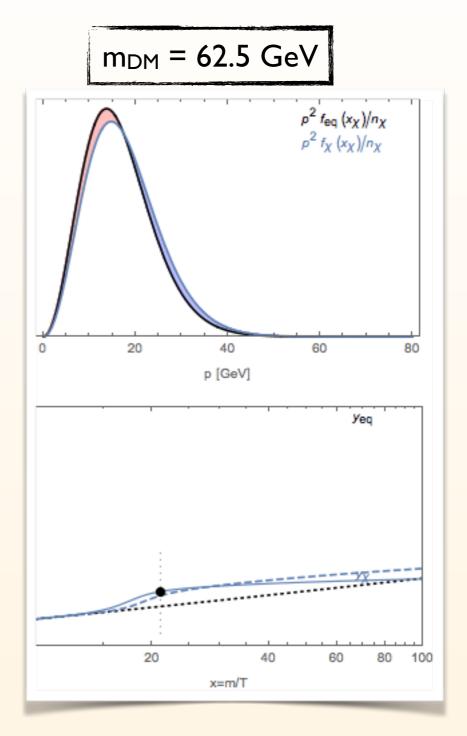
FULL PHASE-SPACE EVOLUTION



black equilibrium at T_{DM}

significant deviation from equilibrium shape already around freeze-out

→ effect on relic density largest, both from different T and f_{DM}



large deviations at later times, around freeze-out not far from eq. shape

effect on relic density ~only from different T

CBE vs. FBE

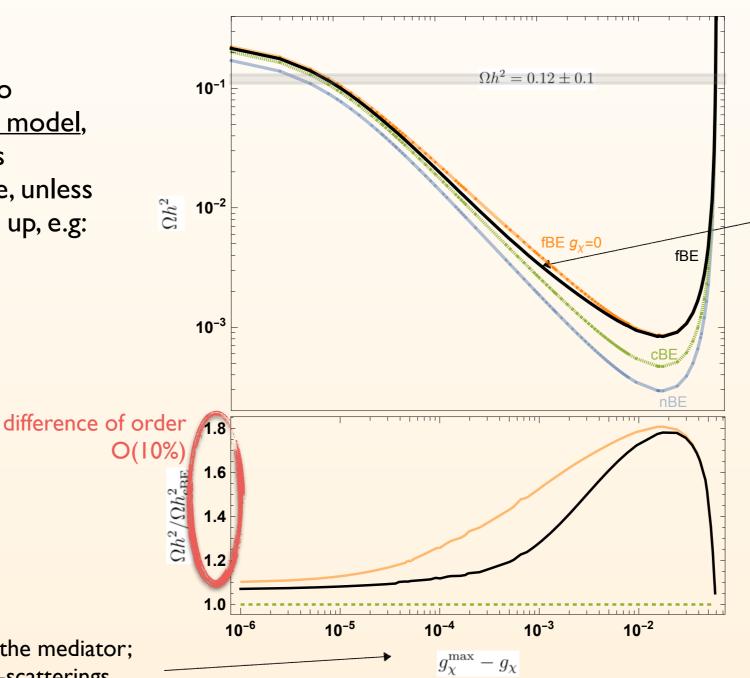
WHICH IS MORE ACCURATE?! A.H. & M. Laletin 2204.07078

They correspond to the opposite limits of self-interaction strengths:

very efficient - cBE

inefficient - fBE

Which limit is closer to reality depends on the model, but it seems that fBE is typically more accurate, unless self-scattering is tuned up, e.g:

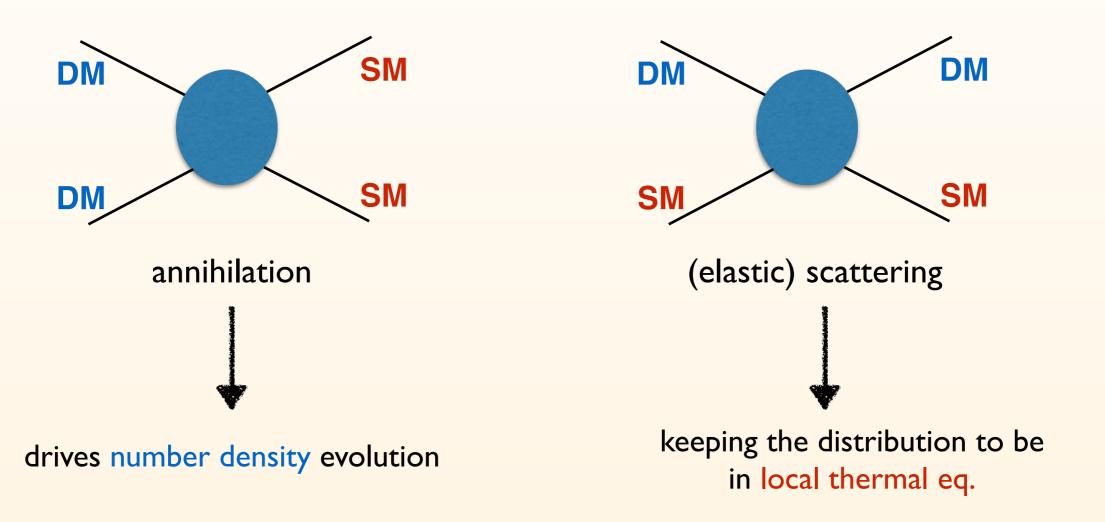


black line gives the result including self-scattering processes! (being between pure fBE and cBE)

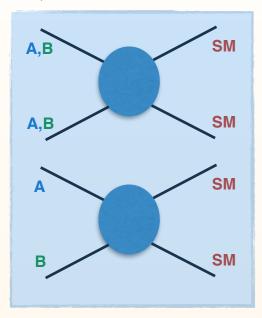
coupling to the mediator; governs self-scatterings

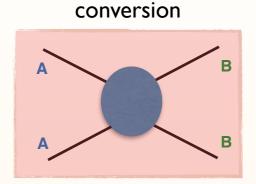
CHAPTER III: MULTI-COMPONENT DARK MATTER

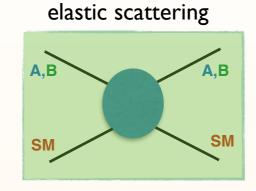
In a minimal WIMP case only two types of processes are relevant:

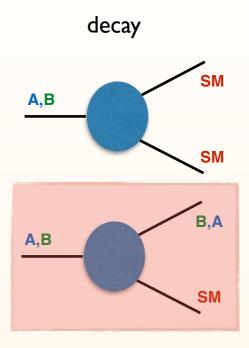


A,B annihilation to SM



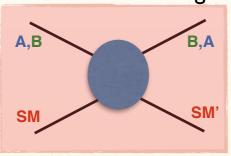


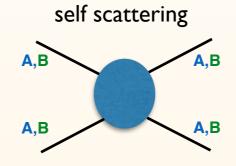




inelastic scattering

or





Co-annihilation ----

Griest, Seckel '91

due to efficient conversion processes one can trace only number density of sum of the states with shared conserved quantum number using weighted annihilation cross section



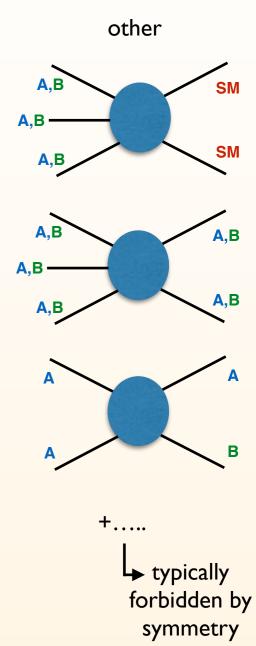
what one calculates

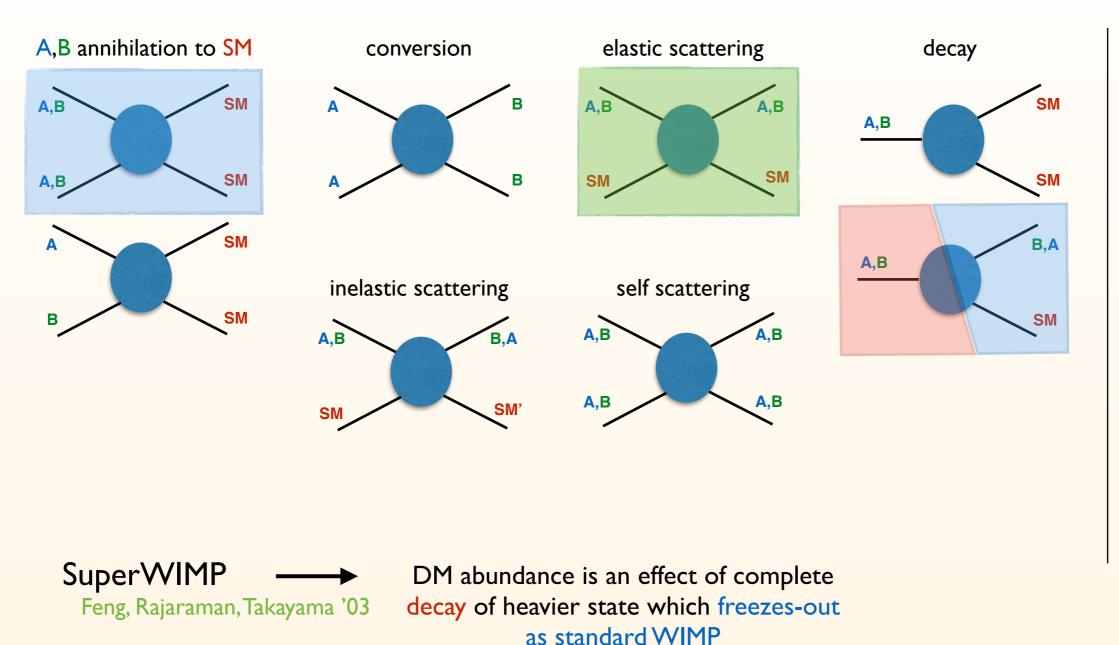


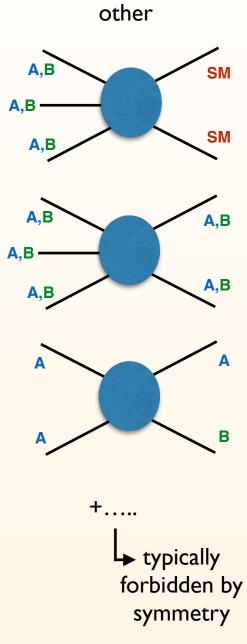
"defines" the mechanism (necessary for it to work)



assumed in calculation (but not necessary)







what one calculates



"defines" the mechanism (necessary for it to work)



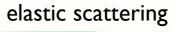
assumed in calculation (but not necessary)

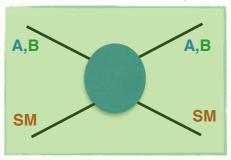
A,B annihilation to SM

A,B SM

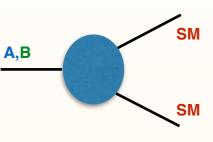
A,B SM

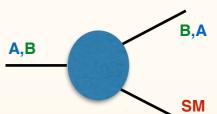


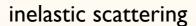


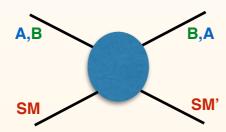


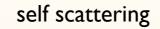


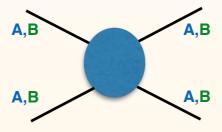










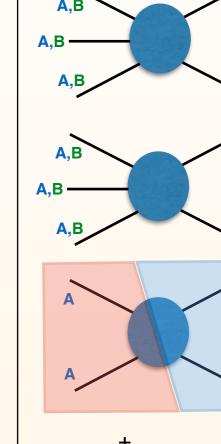


Semi-annihilation

D'Eramo, Thaler '10

SM

new type of annihilation precess that can dominate the freeze-out dynamics; occurs when new "flavour" or "baryon" structure in dark sector

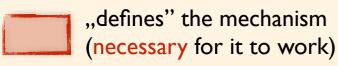


other

SM

A,B

what one calculates

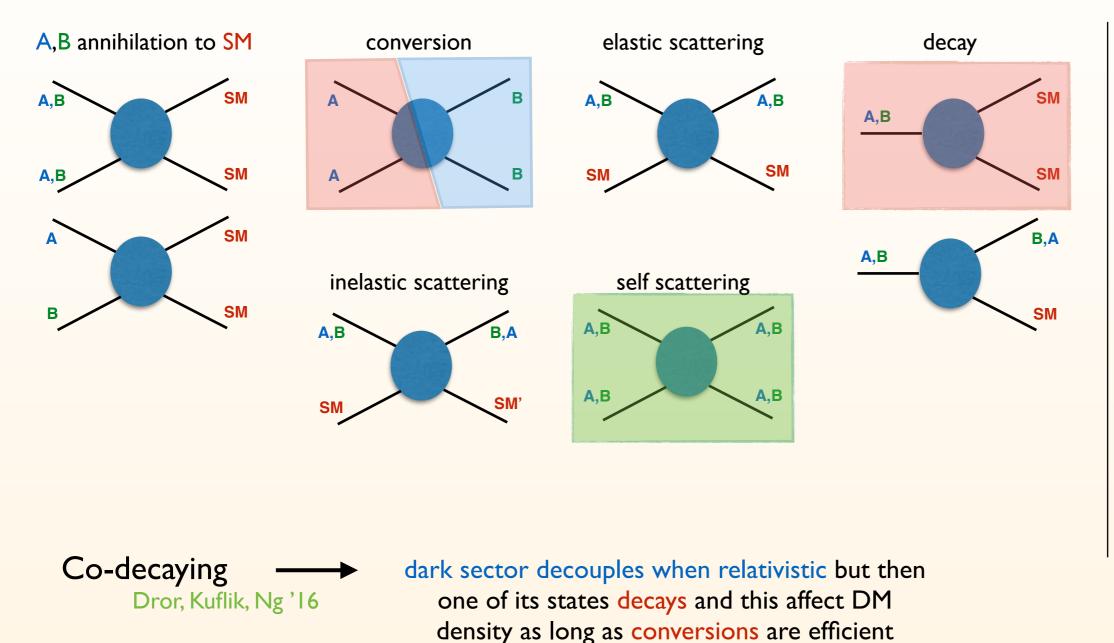


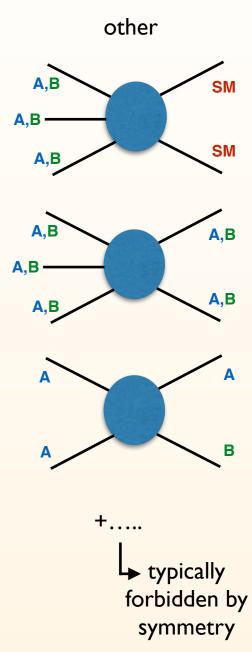
assumed in calculation (but not necessary)

L typically

forbidden by

symmetry

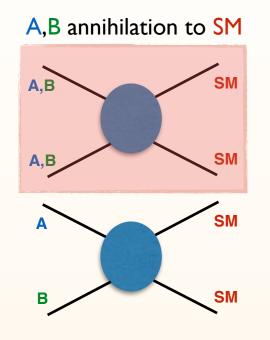


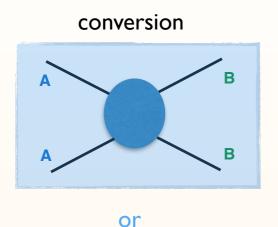


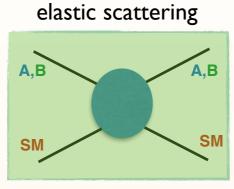
what one calculates

"defines" the mechanism (necessary for it to work)

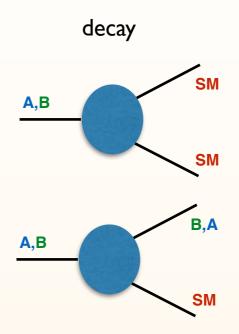
assumed in calculation (but not necessary)

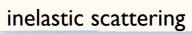


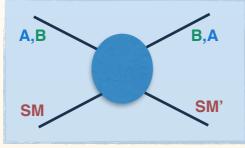


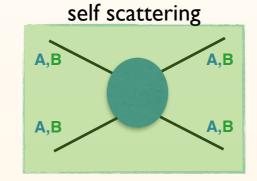


or











Garny, Heisig, Lulf, Vogl '17



Co-scattering

D'Agnolo, Pappadopulo, Ruderman '17

one of the dark sector states annihilates very efficiently, but conversions stop being efficient which blocks co-annihilation



what one calculates



"defines" the mechanism (necessary for it to work)



assumed in calculation (but not necessary)

L typically

forbidden by

symmetry

other

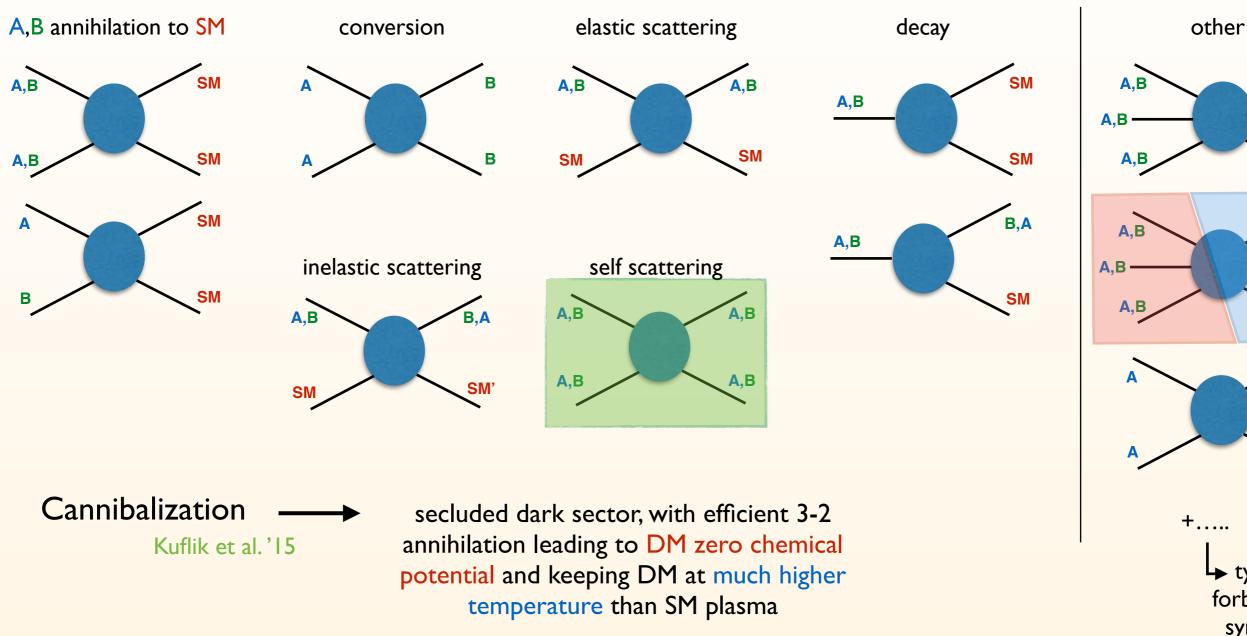
A,B

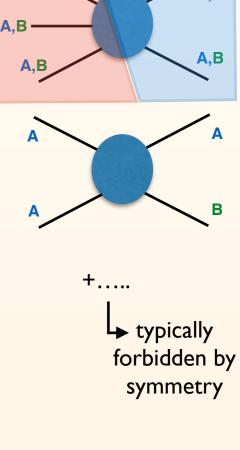
A,B-

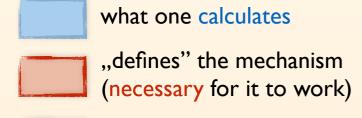
A,B

SM

A.B







SM

SM

A,B

Example: assume two particles in the dark sector: A and B

Scenario Process	Corannihilation	supervitAP	Cordecaying	Conversion driven	Cannibal Sernir	Forbidden-like	.·•
annihilation A A <-> SM SM A B <-> SM SM B B <-> SM SM							
conversion AA<->BB inelastic scattering ASM<->BSM							
elastic scattering A SM <-> A SM B SM <-> B SM							in all scenarios kinetic equilibrium
el. self-scattering AA<->AA BB<->BB							assumption crucial, but not always "automatic"!
decays A <-> B SM A <-> SM SM B <-> SM SM							
semi-ann/3->2 A A A <-> A A A A <-> A B A A A <-> SM A							36

EXAMPLE D:

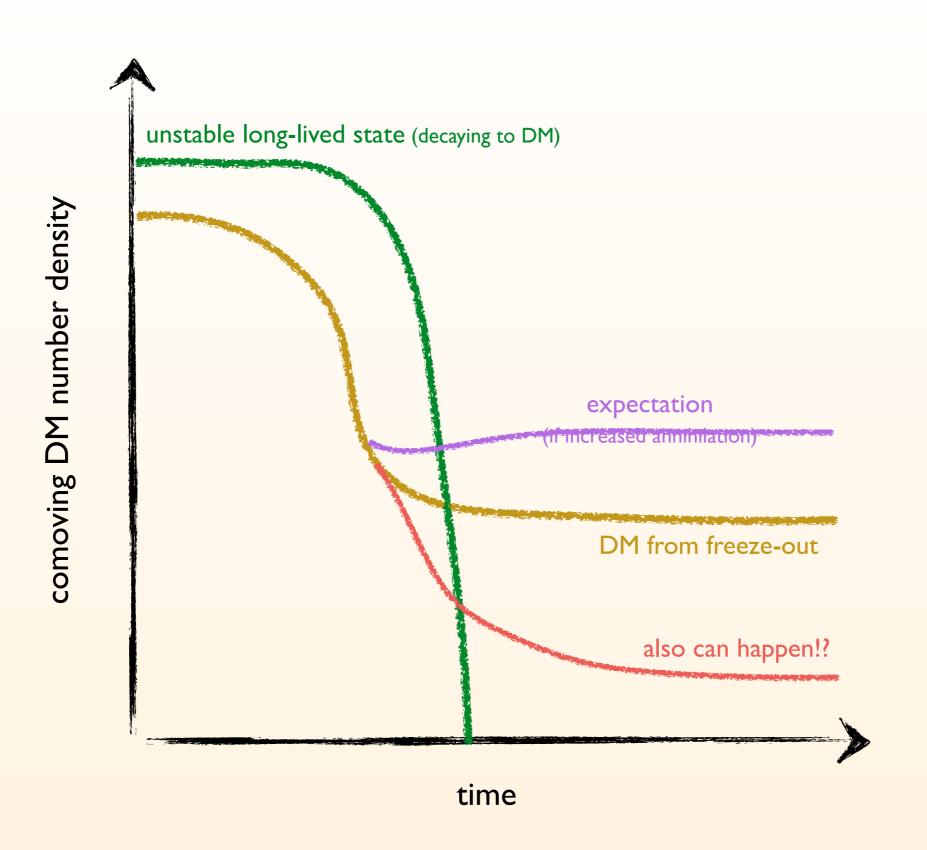
When additional influx of DM arrives

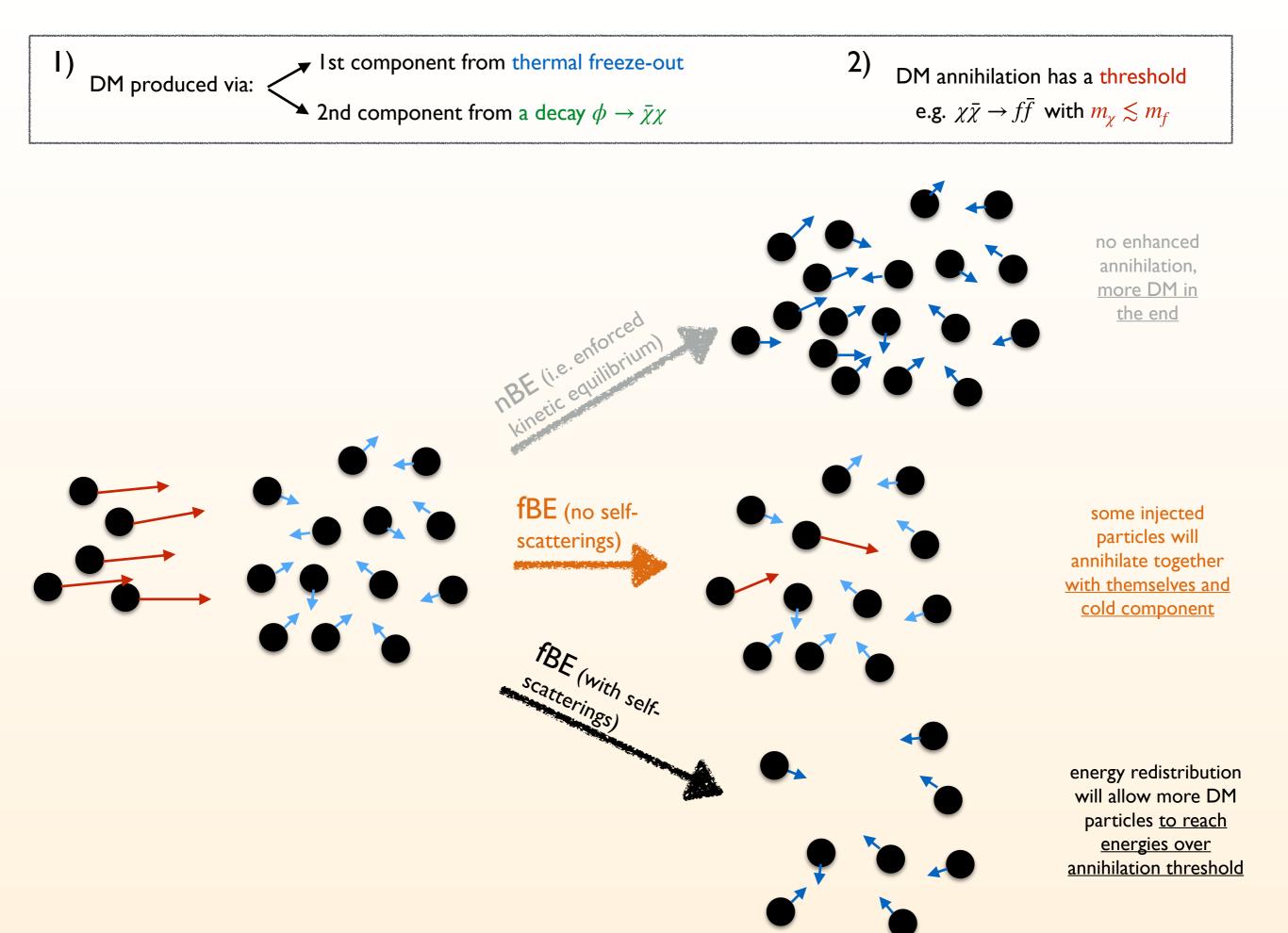
D) Multi-component dark sectors

Sudden injection of more DM particles distorts $f_{\chi}(p)$ (e.g. from a decay or annihilation of other states)

- this can modify the annihilation rate (if still active)
- how does the thermalization due to elastic scatterings happen?

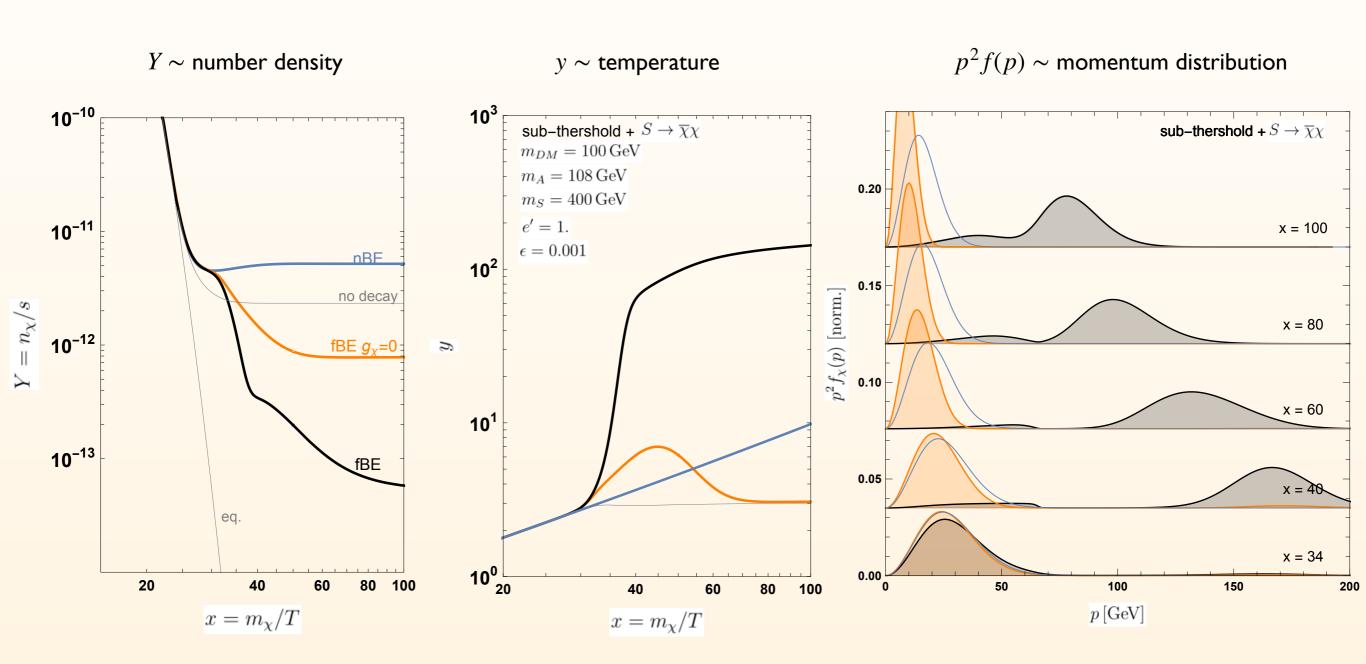
WEIRD THINGS CAN HAPPEN...





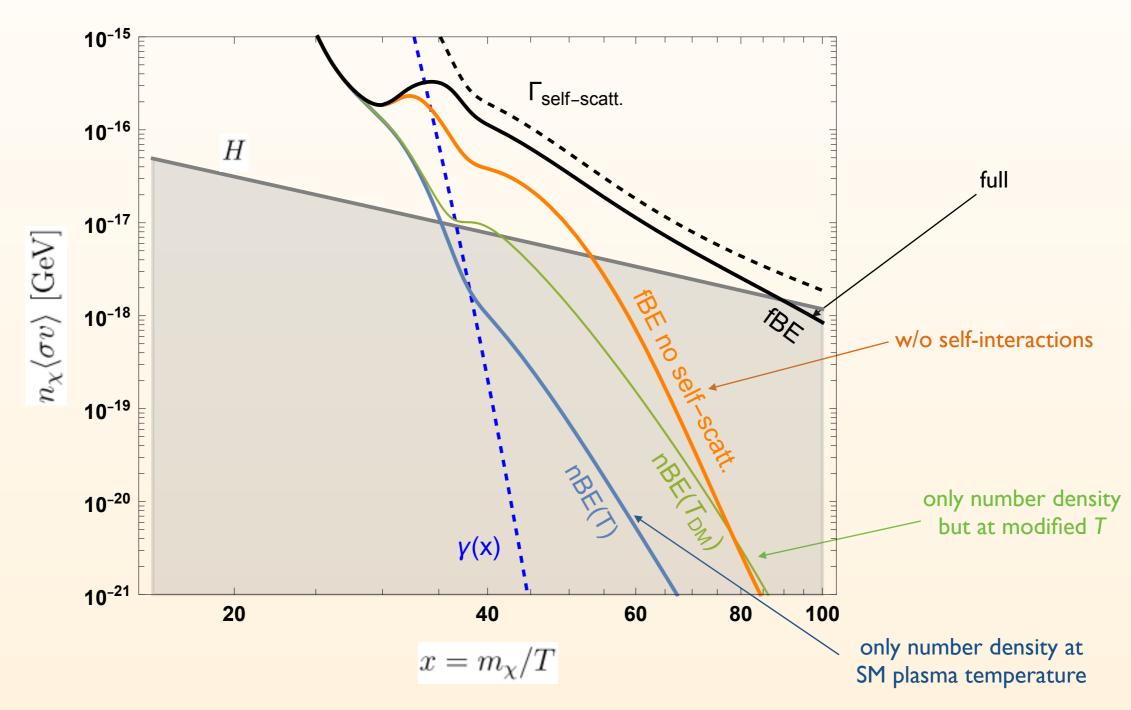
DM produced via: Ist component from thermal freeze-out 2nd component from a decay $\phi \to \bar{\chi} \chi$

2) DM annihilation has a threshold e.g. $\chi \bar{\chi} \to f \bar{f}$ with $m_{\chi} \lesssim m_f$



WHY DOES <u>INJECTING MORE</u> DM PARTICLES CAN LEAD TO <u>DECREASE</u> OF THE RELIC ABUNDANCE?

Let's look on the interaction rates for different cases:



CHAPTER IV: CONCLUSION

TAKEAWAY MESSAGE

When computing relic density of dark matter one needs carefully to check if the standard treatment is sufficient for the case at hand

"Everything should be made as simple as possible, but no simpler."

attributed to* Albert Einstein

^{*}The published quote reads:

[&]quot;It can scarcely be denied that the supreme goal of all theory is to make the irreducible basic elements as simple and as few as possible without having to surrender the adequate representation of a single datum of experience."