

A CASE FOR CALCULATING $f_{\text{DM}}(p)$ (AND HOW TO... MAKE DM COOL AGAIN)

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Based on:

work (in progress) with **E. Cervantes & S. Lederer**

work with **S. Chatterjee** 2604.14688

+ some earlier works with **T. Binder, T. Bringmann, M. Gustafsson, M. Laletin**

BE CAREFUL WHAT YOU AVERAGE OVER

Dynamics of particle production is often governed by various **rates**:

$$n \langle \sigma v \rangle \quad \text{or} \quad \langle \Gamma \rangle$$

where the average is done **over the (momentum) distribution $f(p)$** :

$$\langle \dots \rangle \equiv \langle \dots \rangle_f$$

The question is: what is $f(p)$? Is it **thermal**? Does it matter if not?...

(often it is... but in many cases it is not)

it depends...

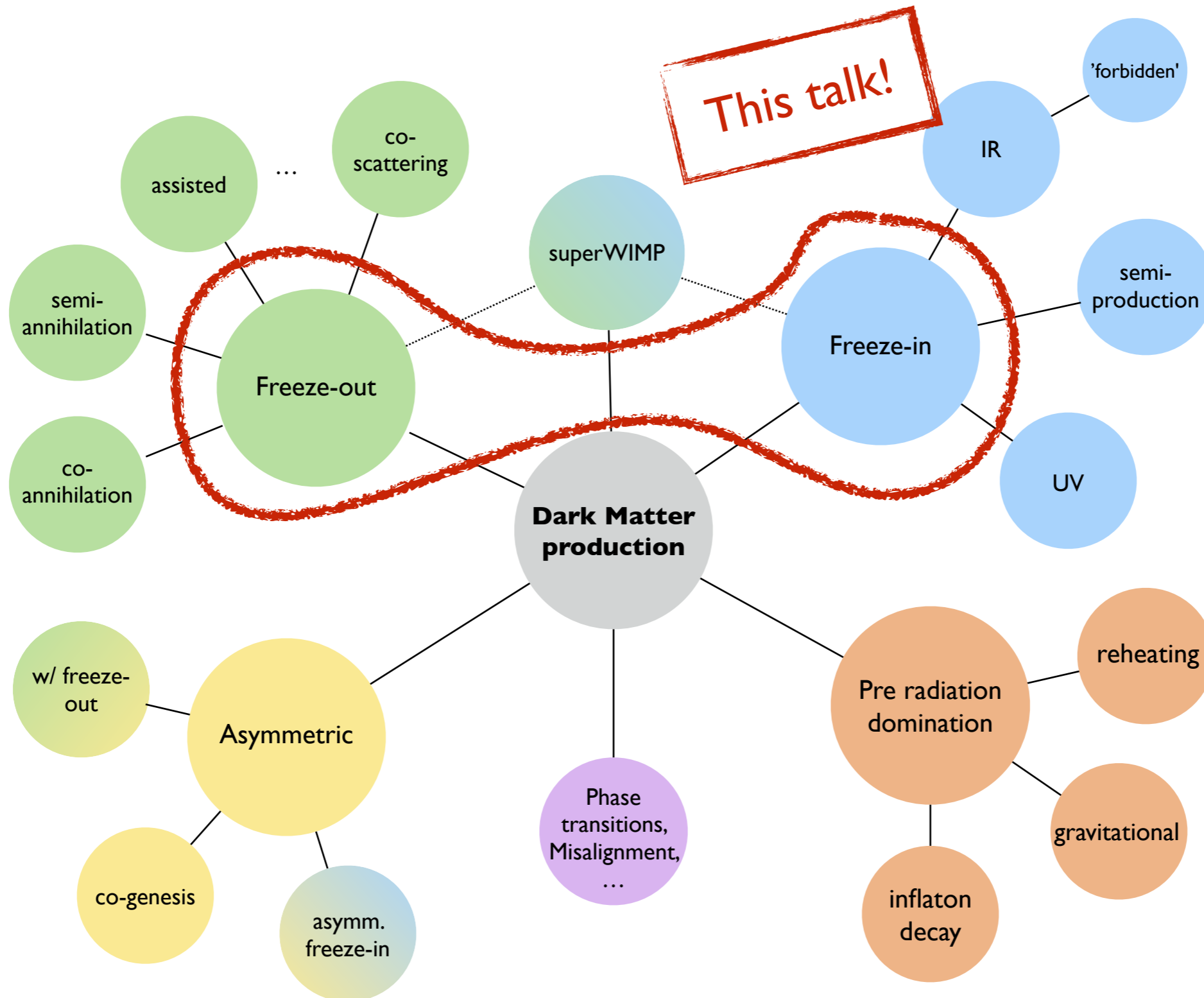
MOTIVATION & OBJECTIVES

A (step in a) program of describing
Dark Matter production
in systems
departing from local thermal equilibrium

Implementation of
freeze-in production
& two-component
systems in **DRAKE2** 

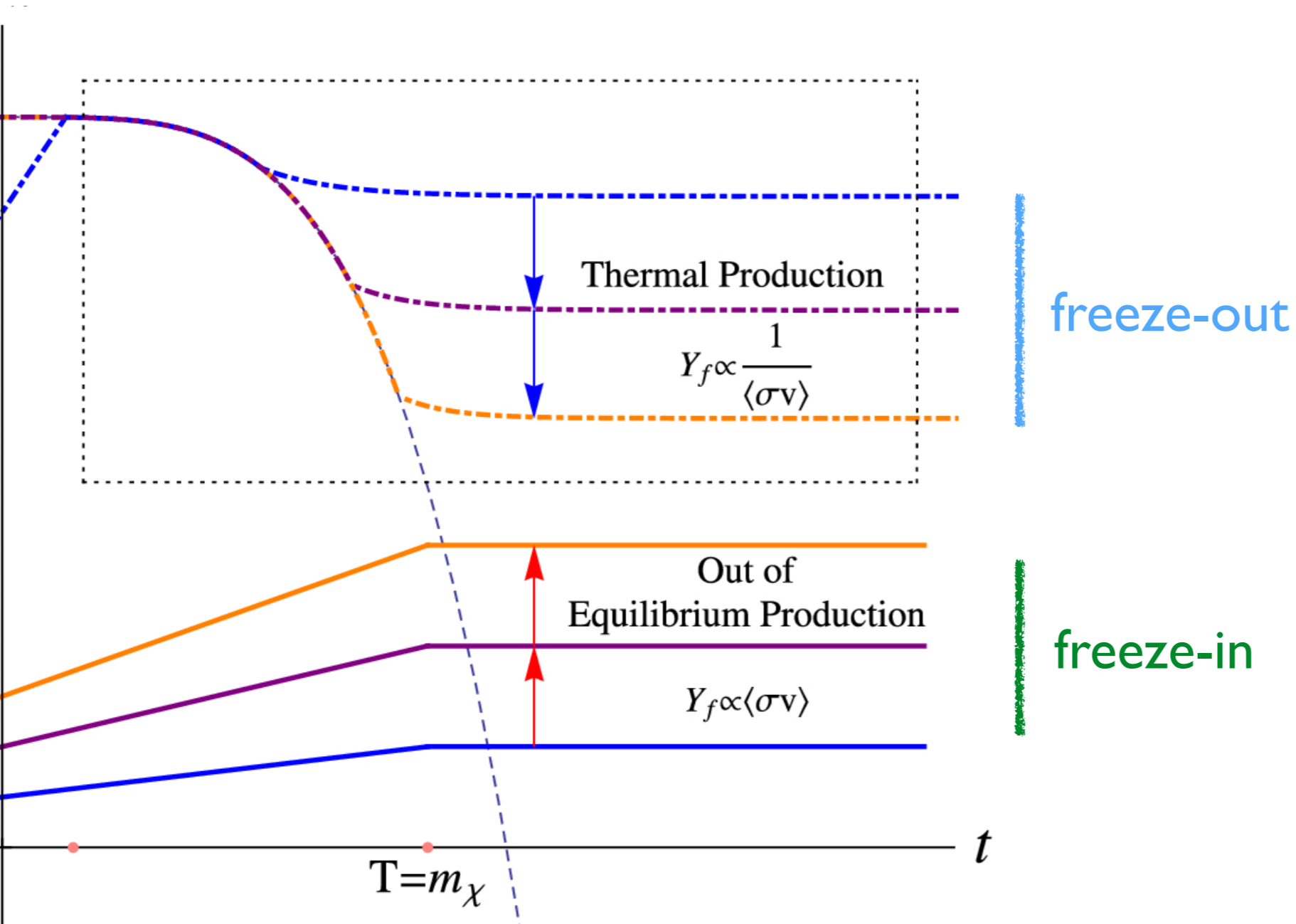
New mechanism for
cooling DM & loosening
Lyman- α bound

DARK MATTER ORIGIN



FREEZE-IN vs. FREEZE-OUT

Freeze-in is in a sense the 'opposite' of freeze-out



Does DM interact (somewhat) with any particle of the SM?

Yes

No (or nearly no)

At high temperature DM thermalises, has very high abundance

Is initial population (e.g. from reheating) small?

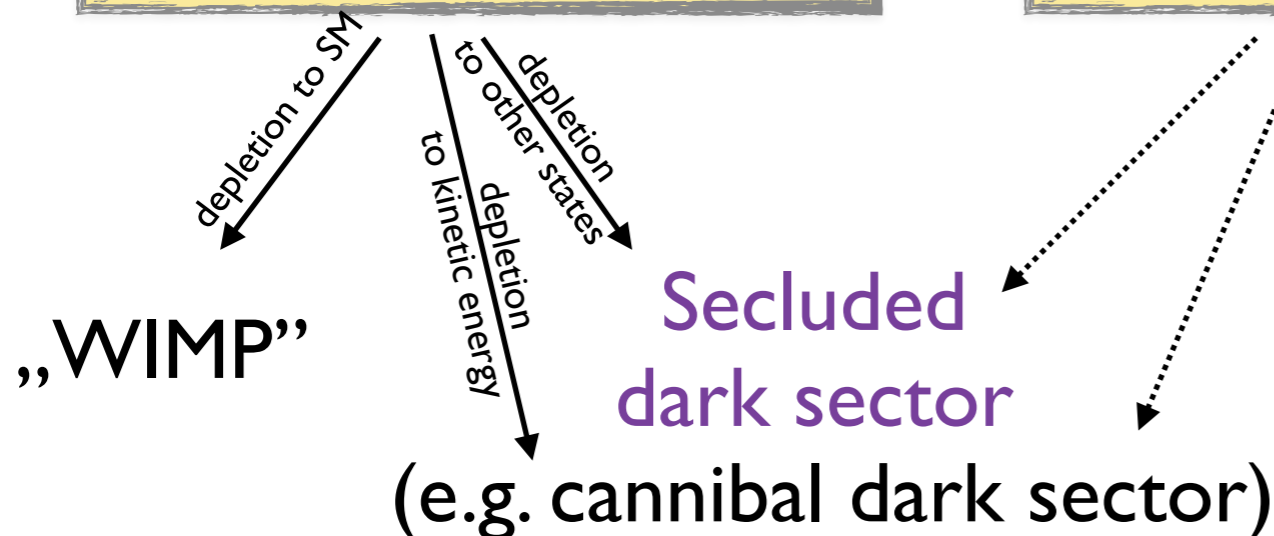
Yes

No

An **efficient depletion** is needed (freeze-out)

A **slow production** is needed (freeze-in)

„Initial condition” dependence



CANNIBAL DM

Explains depletion of DM solely
through **self number changing**
reactions!

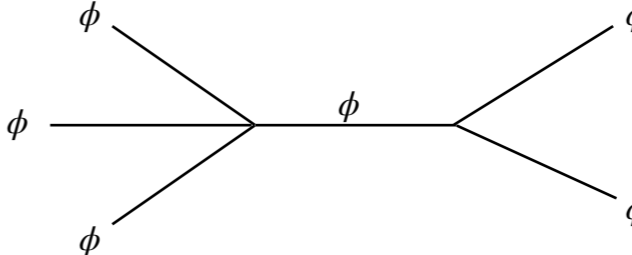
SELF-INTERACTING DARK MATTER
ERIC D. CARLSON
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AND

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Received 1992 March 17; accepted 1992 April 20

Simplest example, scalar ϕ with interactions (**no coupling to SM!**):

$$\frac{g}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4 \Rightarrow$$


To obtain correct relic abundance: $m_\phi \sim \mathcal{O}(10 - 100 \text{ MeV})$

..., Hochberg et al. '14; ...

cannibalisation: mass \rightarrow kinetic energy

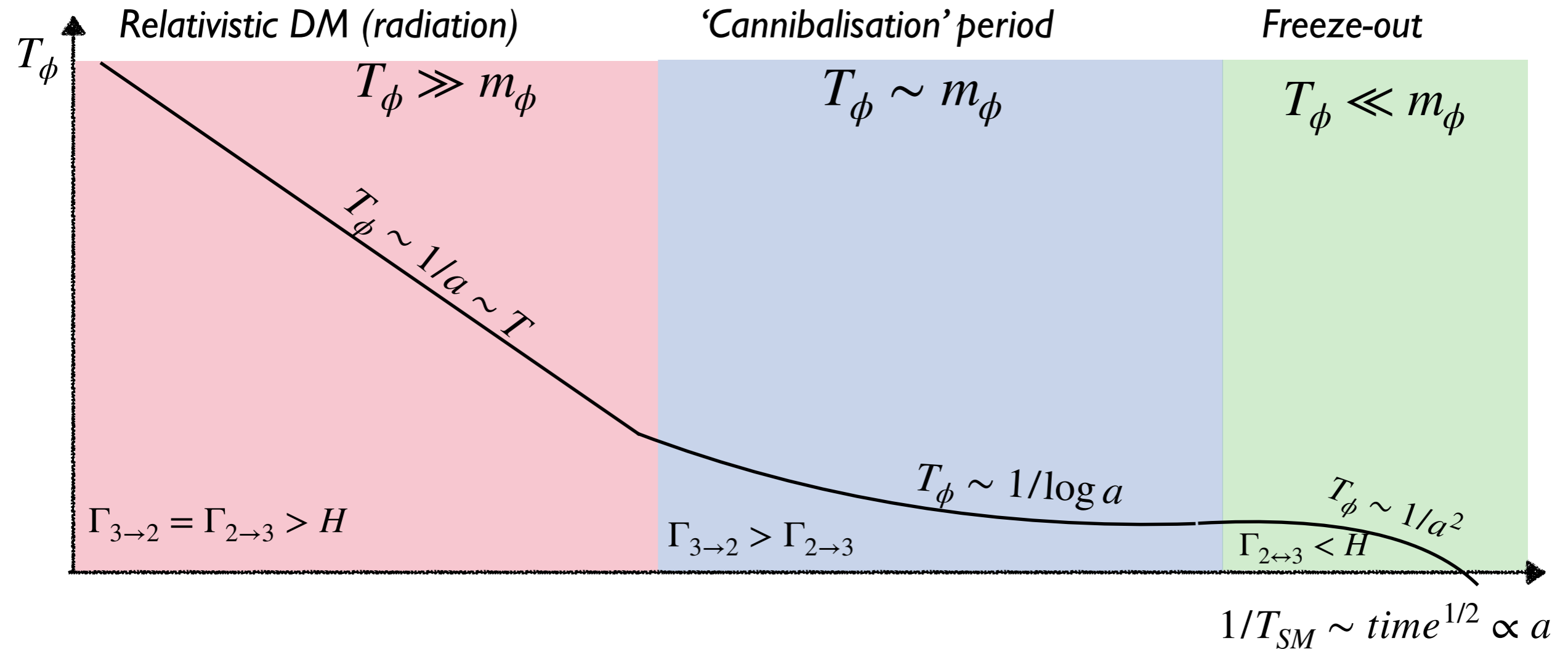
expansion: kinetic energy redshifts \rightarrow depletion of E of the dark sector

Problem: structure formation...

de Laix et al. '95

CANNIBAL FREEZE-OUT

Self-heating of DM makes it warmer, **erasing the formation of structures!**
Tracking its temperature evolution is essential!



- Initially DM is *relativistic* ($T_{DM} \gg m_{DM}$);
- During freeze-out the dark sector uses its rest mass as *fuel* to keep itself warm;
- The system decouples and behaves as a non-relativistic gas.

EXAMPLE: SCALAR SINGLET DM

The simplest DM model: $\mathcal{L}_{SM} \supset -\frac{m^2}{2}\varphi^2 - \frac{\lambda}{4!}\varphi^4 - \frac{1}{2}\lambda_{h\varphi}\varphi^2 |H|^2$

$\langle\varphi\rangle = 0$ & $\lambda_{h\varphi} \gtrsim 10^{-4}$ „WIMP” (thermalizes, undergoes freeze-out)

$\langle\varphi\rangle = 0$ & $\lambda_{h\varphi} \lesssim 10^{-9}$ FIMP (feebly-interacting, undergoes freeze-in)

$\langle\varphi\rangle \neq 0$ Cannibal DM: spontaneous \mathbb{Z}_2 breaking \rightarrow Higgs mixing terms

Hufnagel, Tytgat '22

Evolution of number density $Y_\varphi := n_\varphi/s$ and 'temperature' $x_\varphi := m_\varphi/T_\varphi$ governed by set of Boltzmann equations:

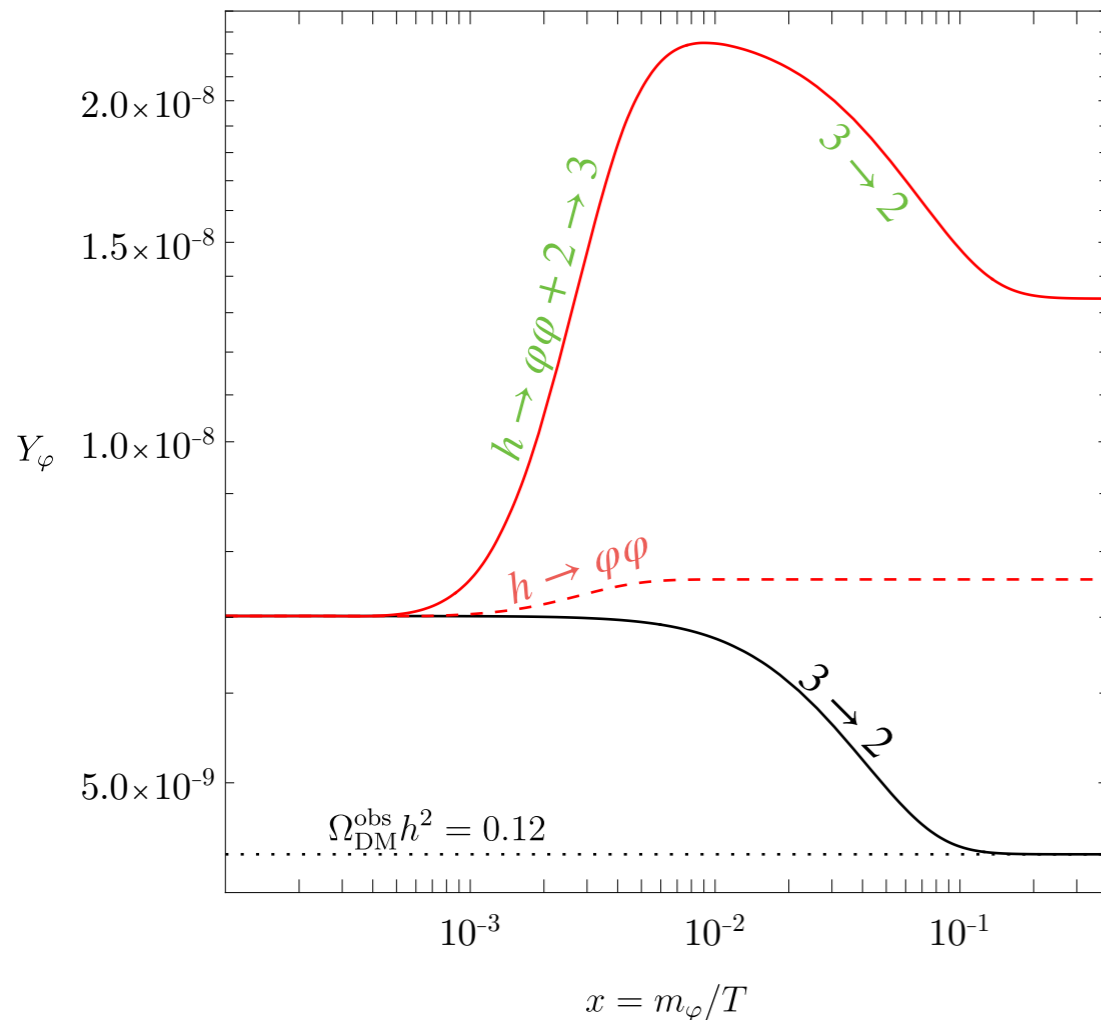
$$\frac{Y'_\varphi}{Y_\varphi} = \frac{1}{x\tilde{H}} \left(\langle C_{h\rightarrow\varphi\varphi} \rangle + \langle C_{hh\rightarrow\varphi\varphi} \rangle + \langle C_{3\leftrightarrow 2} \rangle \right)$$

$$-\frac{x'_\varphi}{x_\varphi} = \frac{1}{x\tilde{H}} \left(\langle C_{h\rightarrow\varphi\varphi} \rangle_2 + \langle C_{hh\rightarrow\varphi\varphi} \rangle_2 + \langle C_{\phi h \leftrightarrow \phi h} \rangle_2 + \langle C_{3\leftrightarrow 2} \rangle_2 \right) - \frac{Y'_\varphi}{Y_\varphi} + \frac{H}{x\tilde{H}} \frac{\langle p^4/E^3 \rangle}{3T_\varphi} + \frac{2s'}{3s}$$

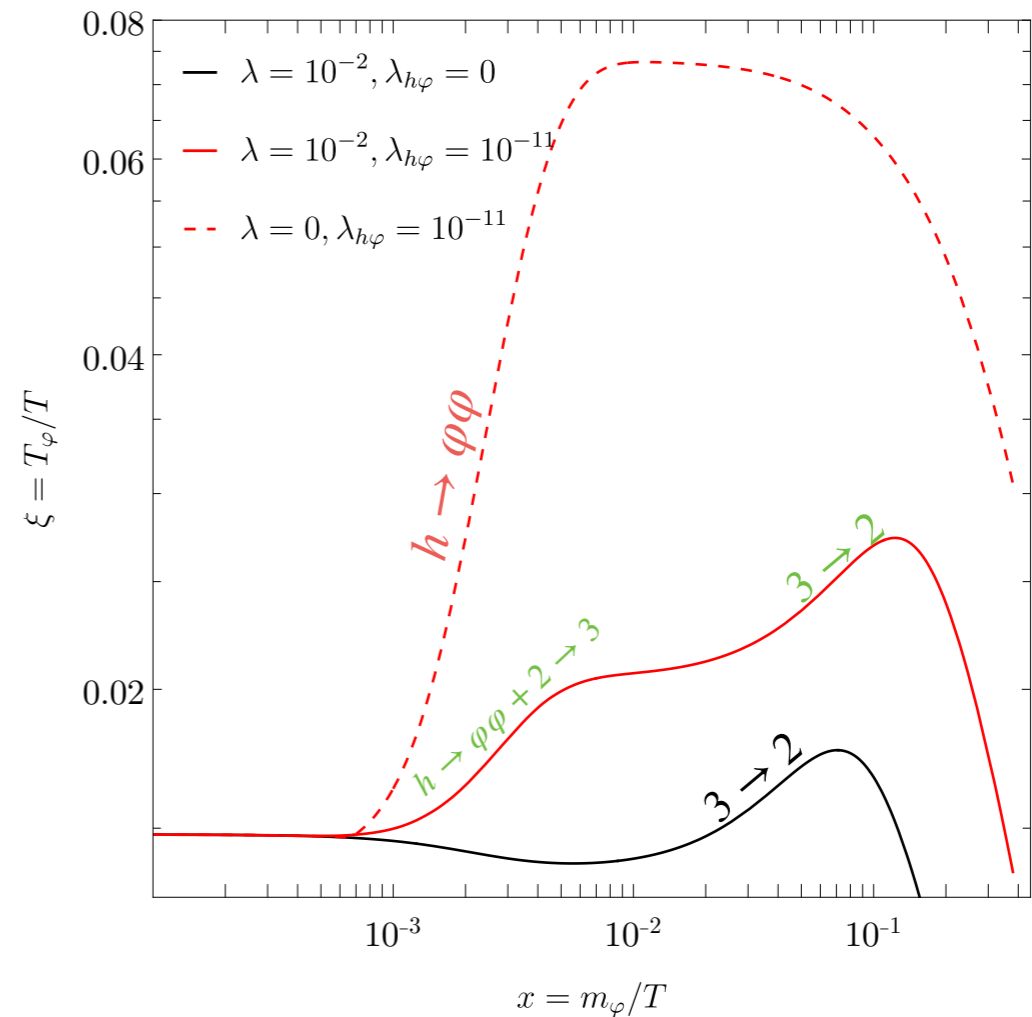
Freeze-in Freeze-in/out El. scattering Cannibal

BOOSTING FREEZE-IN

Consider an initial cold, low populated dark sector: $T_{DM}^i = 10^{-2} T_{SM}^i$



only cannibalization



only freeze-in

cannibal+freeze-in

IDEA:
MAKE DM COOL AGAIN

WHAT IF ϕ IS THE MEDIATOR?

A mediator field does not need a stabilising symmetry \Rightarrow cubic interactions exist generically and generate $2 \leftrightarrow 3$ interactions

...example: a scalar field with ϕ^3 or a non-abelian vector e.g. SU(N)

In the following we take the case:

$$\mathcal{L}_s \ni \frac{g_h}{2} |H|^2 \phi^2 + \frac{\sqrt{3}\lambda}{3!} m_\phi \phi^3 + \frac{\lambda}{4!} \phi^4 + \kappa \phi \bar{\chi} \chi$$

Higgs portal

cubic + quartic

decay to χ

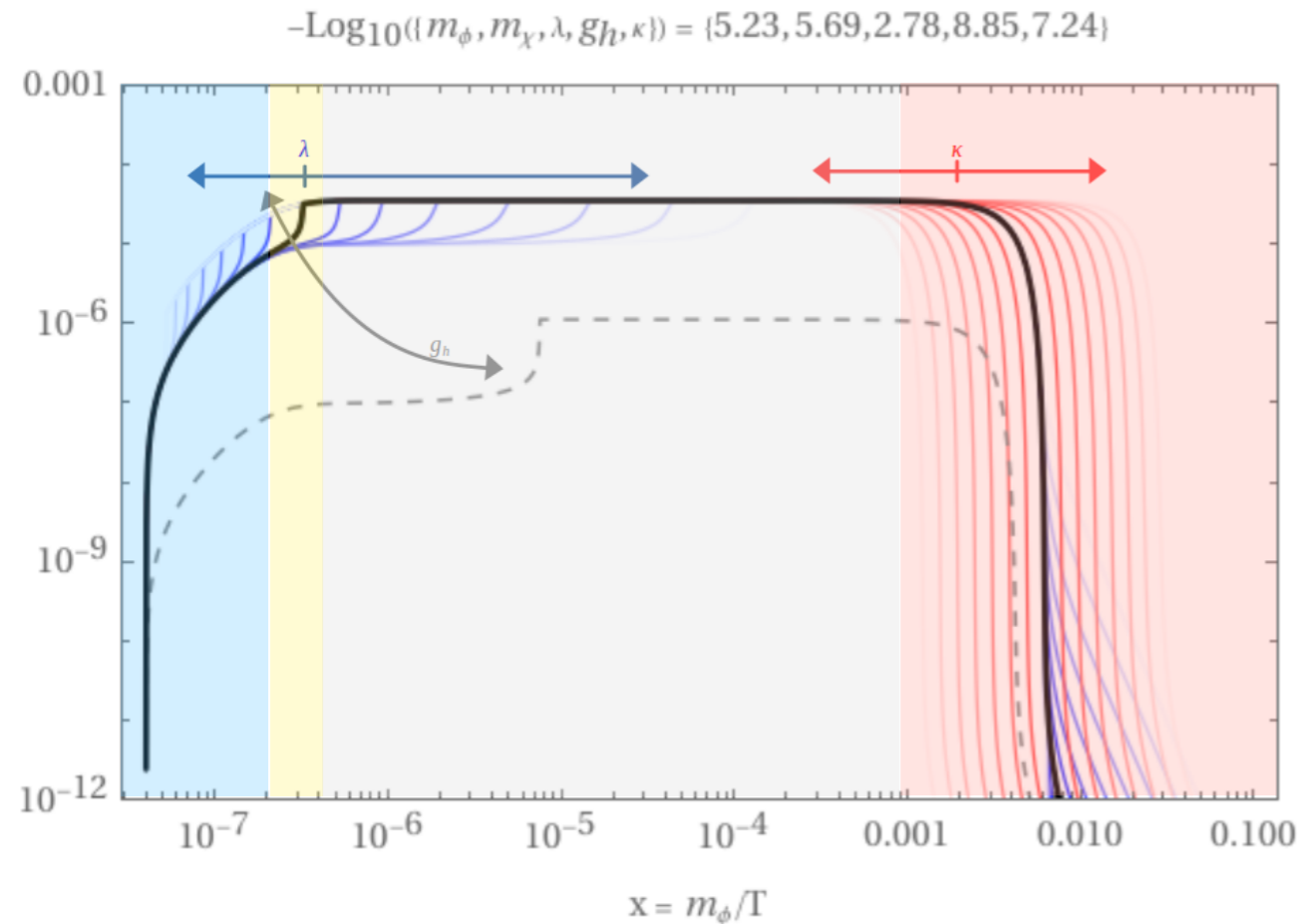
ϕ freeze-in

FI boost & self-cooling

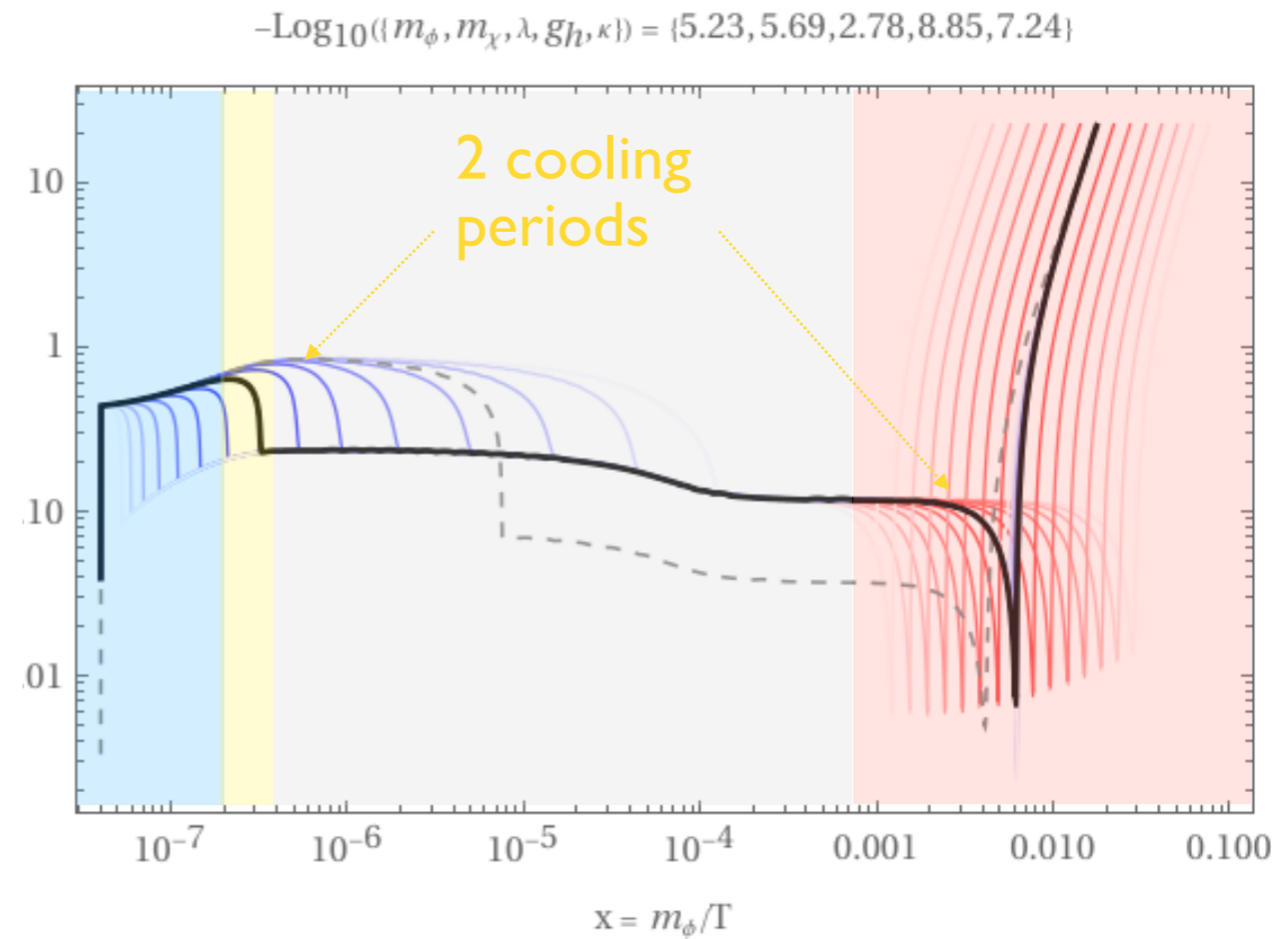
DM freeze-in

SELF-COOLING: HOW DOES IT WORK?

ϕ yield evolution:



T_ϕ evolution:



1. Higgs decay provides ϕ freeze-in

2. Expansion rate drops, yield grows
 $\Rightarrow 2 \rightarrow 3$ efficiency grows

3. Self-thermalization \Rightarrow plateau

4. Decay to DM

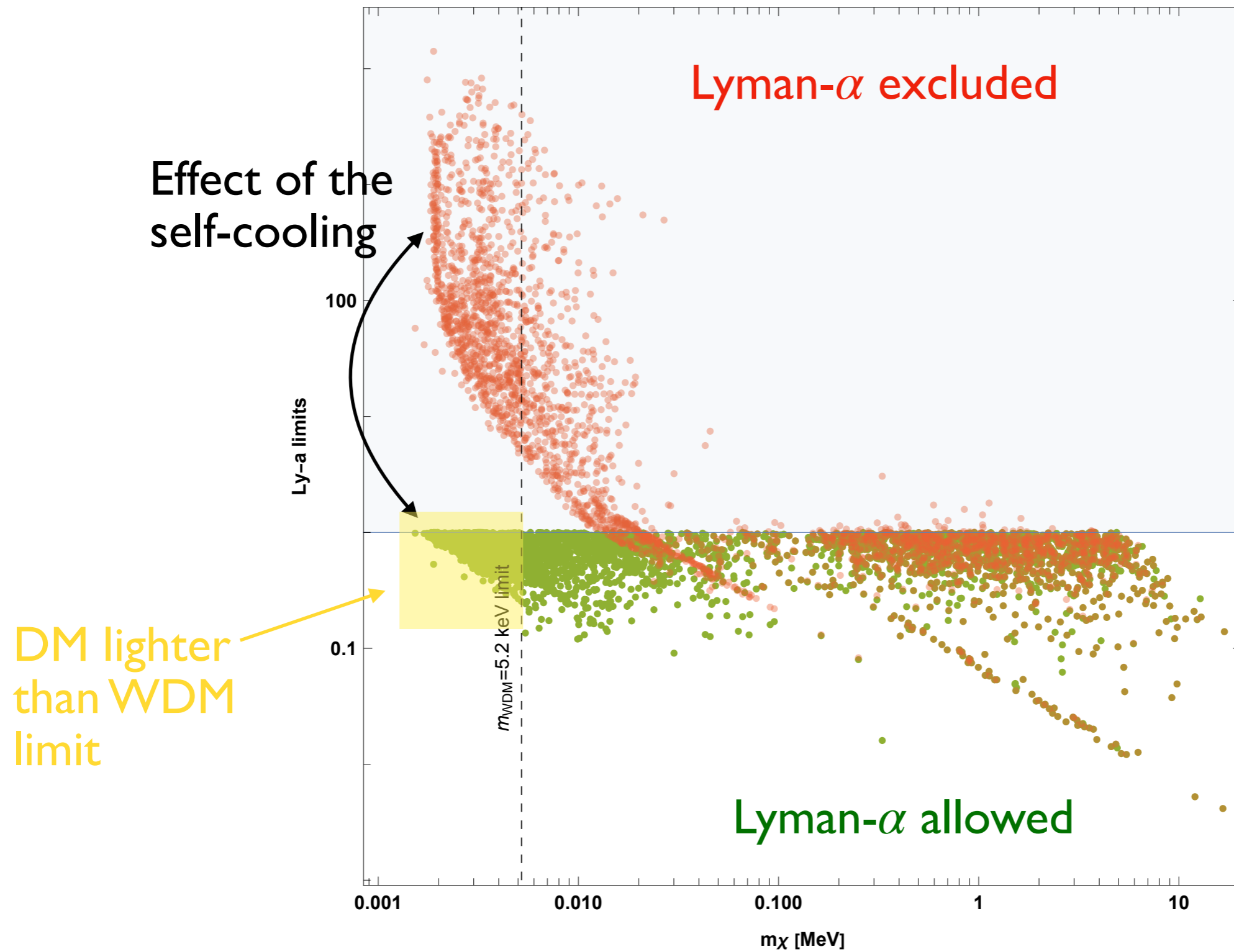
1. Temperature $\sim T/2$

2. Self-cooling drop

3. Equilibrium expansion

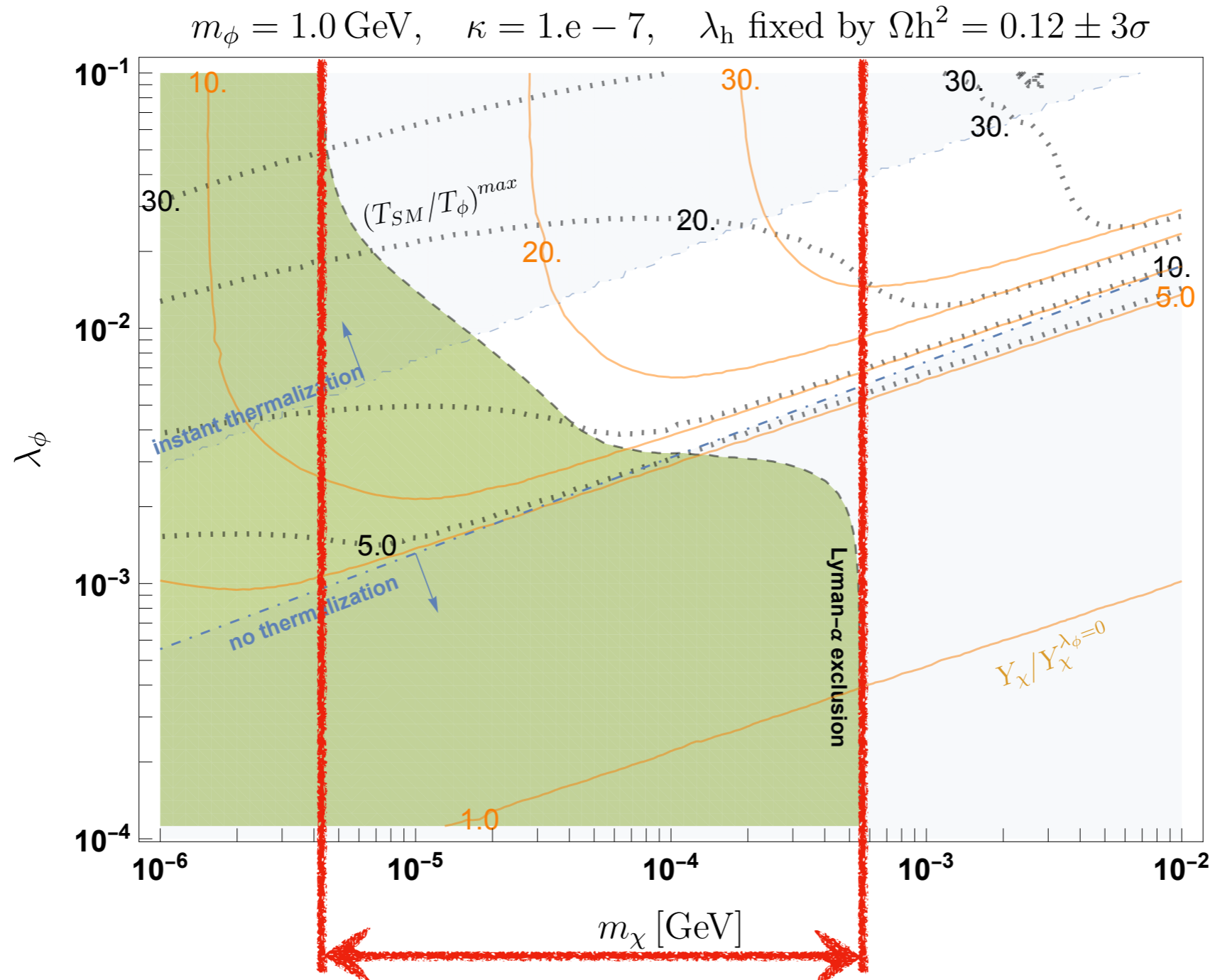
4. Apparent heating: skewing due to time dilatation

MCMC SCAN FOR LYMAN- α



MODEL PARAMETER SPACE

Model has 5 free parameters. We fix two with smallest impact (m_ϕ, κ), λ_h fix by relic abundance and show remaining two (m_χ - DM mass & λ_ϕ - self-interaction strength):



Loosening limits due to self-cooling!

ONE TO BIND IT ALL:

DRAKE2 

THERMAL RELIC DENSITY

STANDARD APPROACH

Boltzmann equation for $f_\chi(p)$:

$$E (\partial_t - H \vec{p} \cdot \nabla_{\vec{p}}) f_\chi = \mathcal{C}[f_\chi]$$

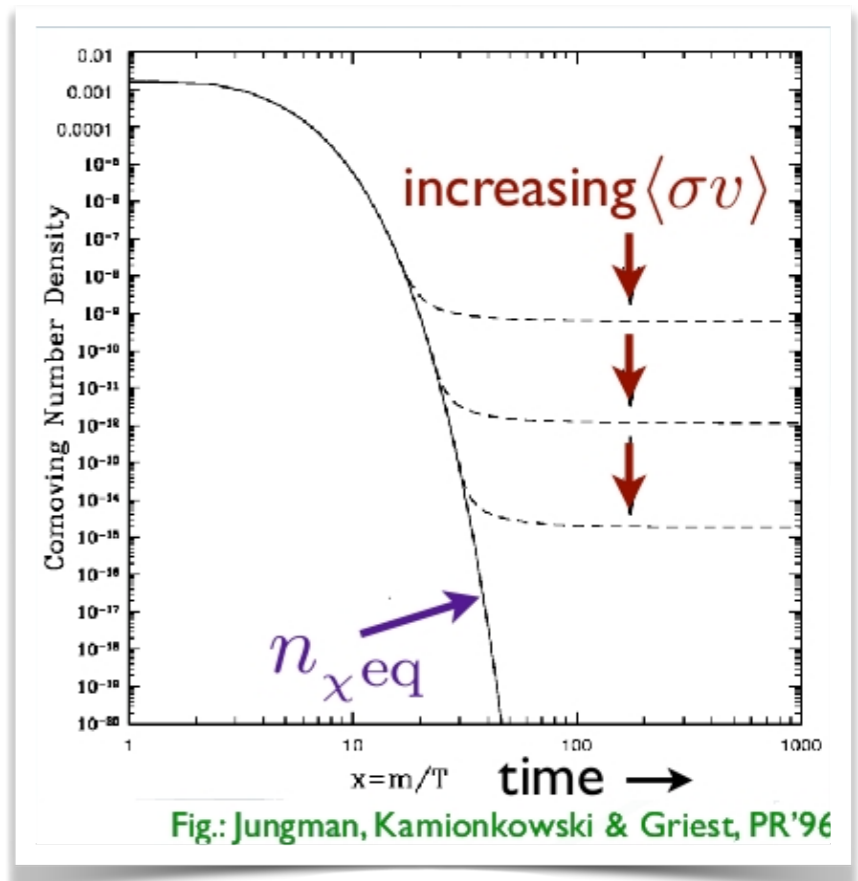


$$\frac{dn_\chi}{dt} + 3Hn_\chi = - \langle \sigma_{\chi\bar{\chi} \rightarrow ij} \sigma_{\text{rel}} \rangle^{\text{eq}} (n_\chi n_{\bar{\chi}} - n_\chi^{\text{eq}} n_{\bar{\chi}}^{\text{eq}})$$



Critical assumption:
kinetic equilibrium at chemical decoupling

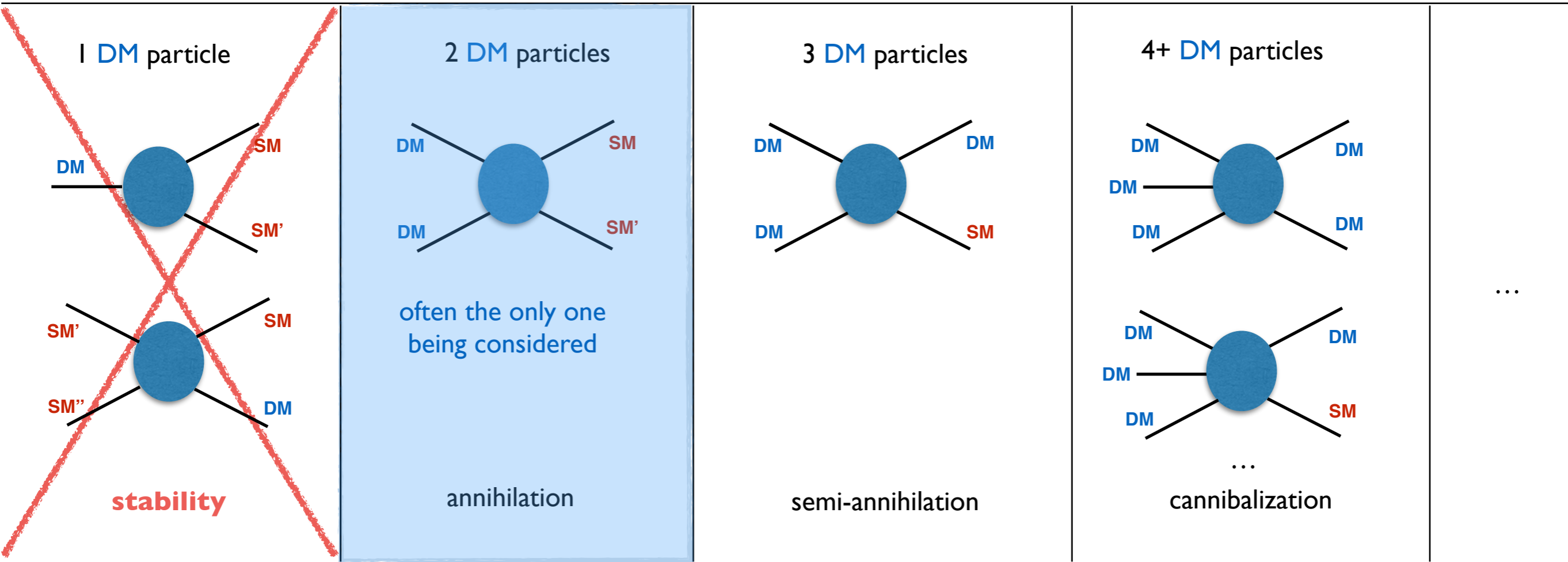
$$f_\chi \sim a(T) f_\chi^{\text{eq}}$$



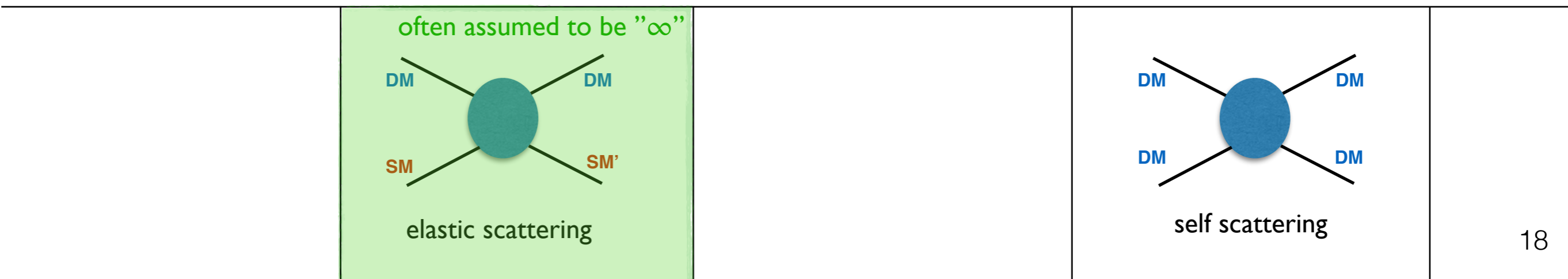
WHAT GOES INTO C IN GENERAL?

For now assume a minimal theory of **SM** + one **DM** field

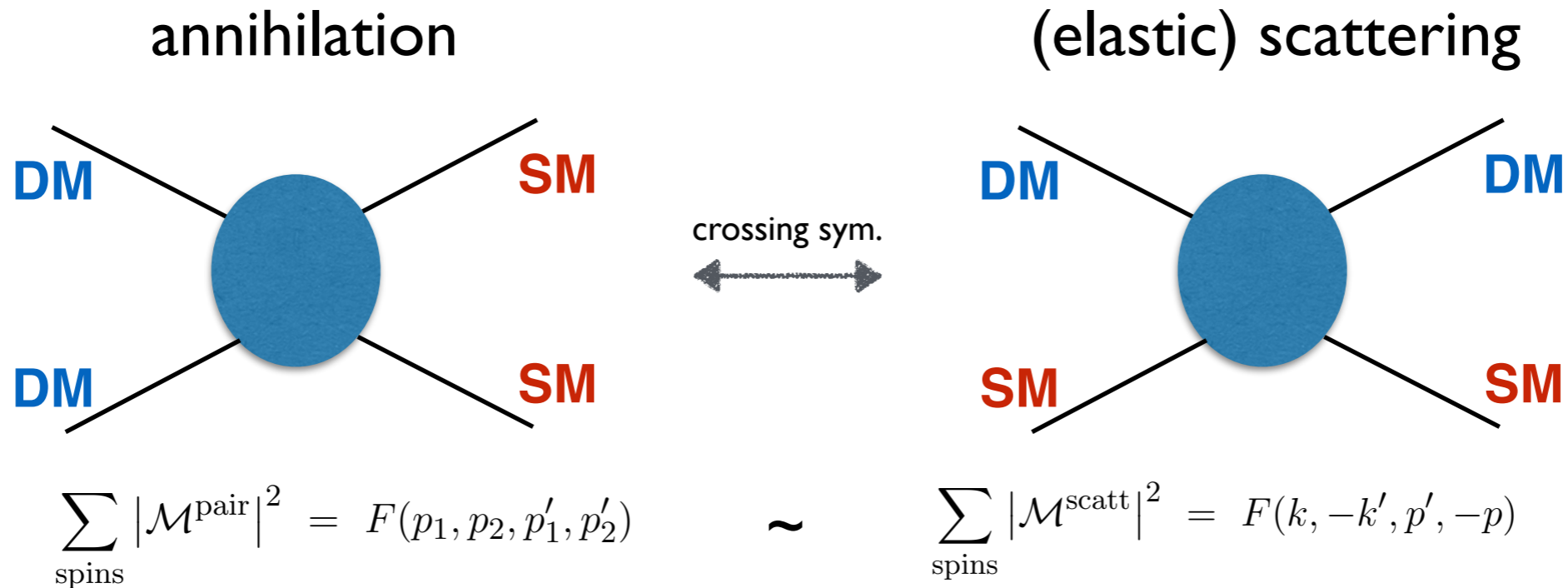
changing processes \Rightarrow number density



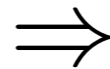
conserving processes \Rightarrow energy density



FREEZE-OUT vs. DECOUPLING



Boltzmann suppression of **DM** vs. **SM**



scatterings typically more frequent

dark matter frozen-out but typically still kinetically coupled to the plasma

$$\tau_r(T_{\text{kd}}) \equiv N_{\text{coll}}/\Gamma_{\text{el}} \sim H^{-1}(T_{\text{kd}})$$

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Two consequences:

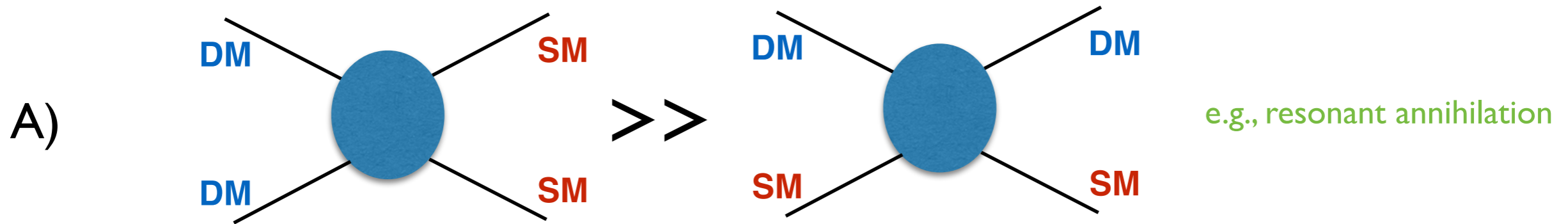
1. During freeze-out (chemical decoupling) typically: $f_\chi \sim a(\mu) f_\chi^{\text{eq}}$
2. If kinetic decoupling much, much later: possible impact on the matter power spectrum
i.e. kinetic decoupling can have observable consequences and affect e.g. missing satellites problem

see e.g., Bringmann, Ihle, Karsten, Valia '16

DEPARTURE FROM KINETIC EQUILIBRIUM?

A **necessary** and **sufficient** condition: scatterings weaker than annihilation
i.e. rates around freeze-out: $H \sim \Gamma_{\text{ann}} \gtrsim \Gamma_{\text{el}}$

Possibilities:



B) Boltzmann suppression of **SM** as strong as for **DM**
e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure
e.g., semi-annihilation, 3 to 2 models, ...

D) Multi-component dark sectors
e.g., additional sources of DM from late decays, ...

HOW TO GO BEYOND KINETIC EQUILIBRIUM?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E (\partial_t - H\vec{p} \cdot \nabla_{\vec{p}}) f_{\chi} = \mathcal{C}[f_{\chi}]$$

contains both scatterings and annihilations

Two possible approaches:

fBE

solve numerically
for full $f_{\chi}(p)$

have insight on the distribution
no constraining assumptions

numerically challenging
often an overkill

CBE

consider system of equations
for moments of $f_{\chi}(p)$

partially analytic/much easier numerically
manifestly captures all of the relevant physics

finite range of validity
no insight on the distribution

0-th moment: n_{χ}
2-nd moment: T_{χ}

...

PUBLIC TOOL!

Binder, Bringmann, Gustafsson, AH 2103.01944

GOING BEYOND THE STANDARD APPROACH

- Home
- Downloads
- Contact



Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models. DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,** Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]

Currently, a user guide can be found in the Appendix A of this reference. Please cite also quoted other works applying for specific cases.

v1.0 « [Click here to download DRAKE](#)

(March 3, 2021)

<https://drake.hepforge.org>

Applications:

DM relic density for
any (user defined) model*

Interplay between chemical and
kinetic decoupling

Prediction for the DM
phase space distribution

Late kinetic decoupling
and impact on cosmology

see e.g., [1202.5456](#)

...

(only) prerequisite:
Wolfram Language (or Mathematica)

*at the moment for a single DM species and w/o
co-annihilations... but stay tuned for extensions!

New features:

Two-component dark sectors
(also with potentially unstable states)

Freeze-out & **Freeze-in**

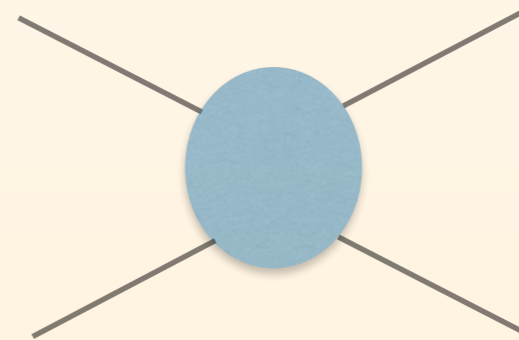
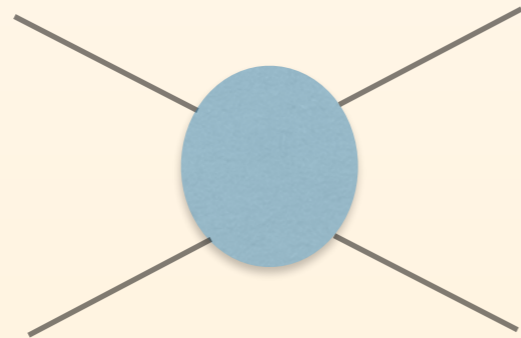
Automatic **model generation**
[linking to **FeynRules etc.**]

Improvements:

Increased efficiency
[e.g. more extended use of
compiled functions, parallelisation,
matrix formulation]

Updated *user interface*

EXAMPLE A:
SCALAR SINGLET DM



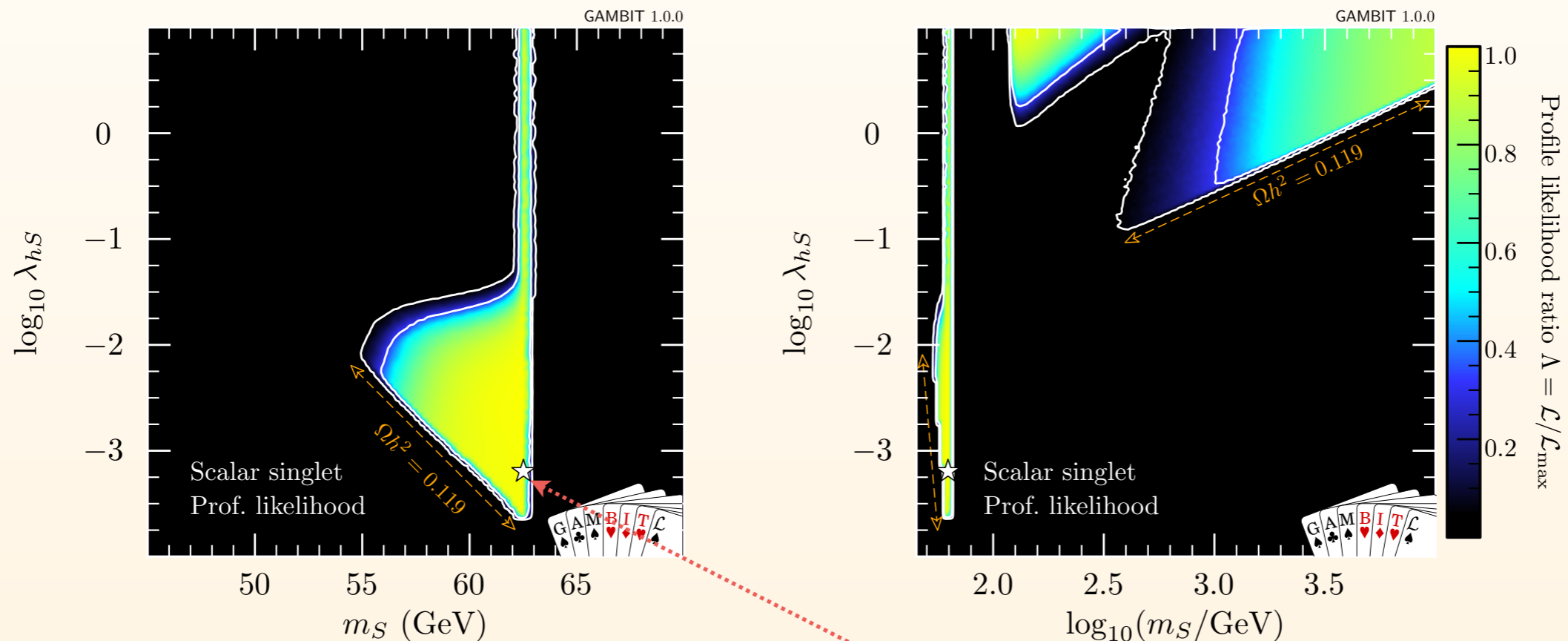
EXAMPLE A

SCALAR SINGLET DM

To the SM Lagrangian add one singlet scalar field S with interactions with the Higgs:

$$\mathcal{L}_S = \frac{1}{2} \partial_\mu S \partial^\mu S - \frac{1}{2} \mu_S^2 S^2 - \frac{1}{2} \lambda_s S^2 |H|^2$$

$$m_s = \sqrt{\mu_S^2 + \frac{1}{2} \lambda_s v_0^2}$$



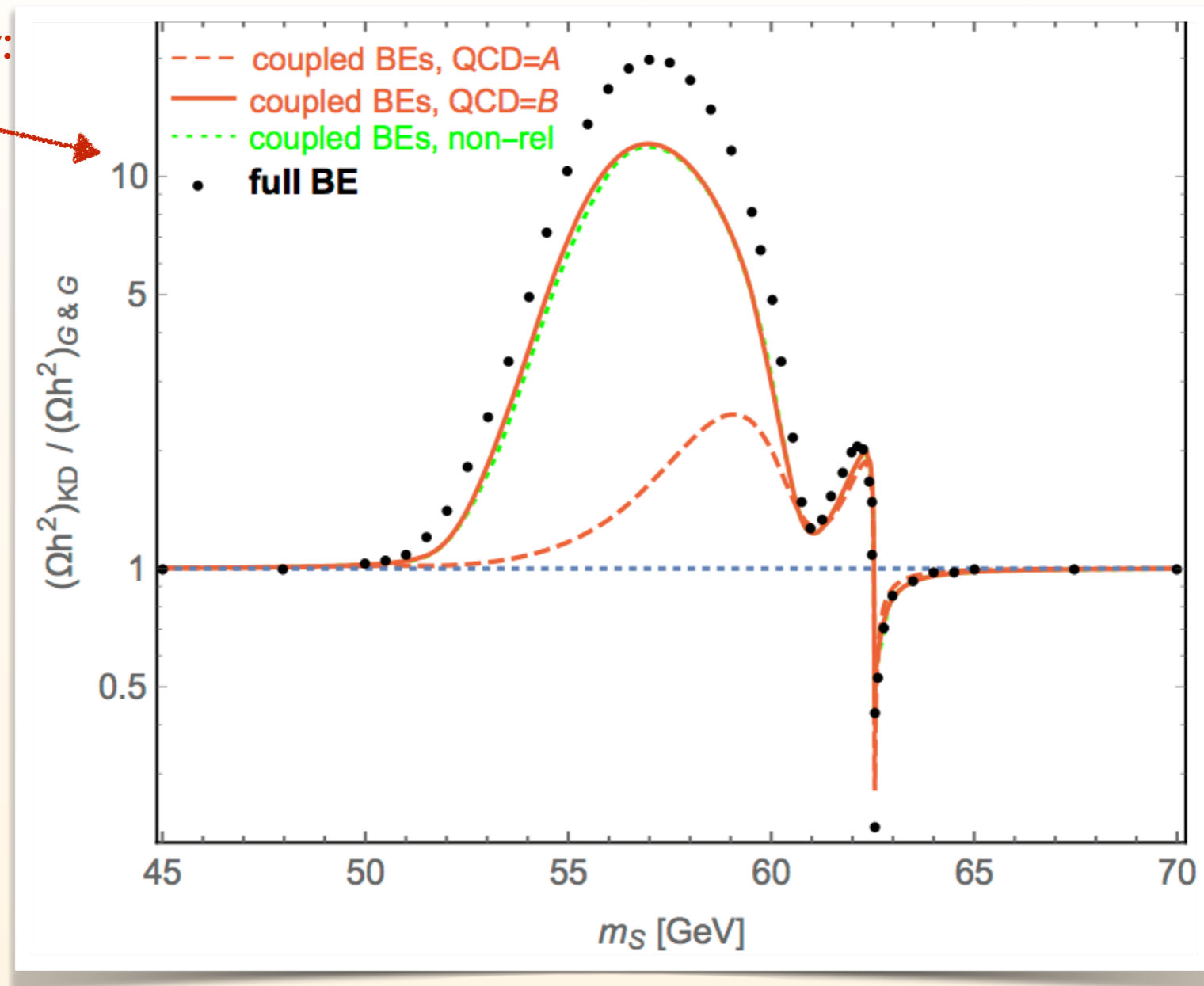
Most of the parameter space excluded, but... even such a simple model is hard to kill

best fit point hides in the **resonance region!**

RESULTS

EFFECT ON THE Ωh^2

effect on relic density:
up to $O(\sim 10)$

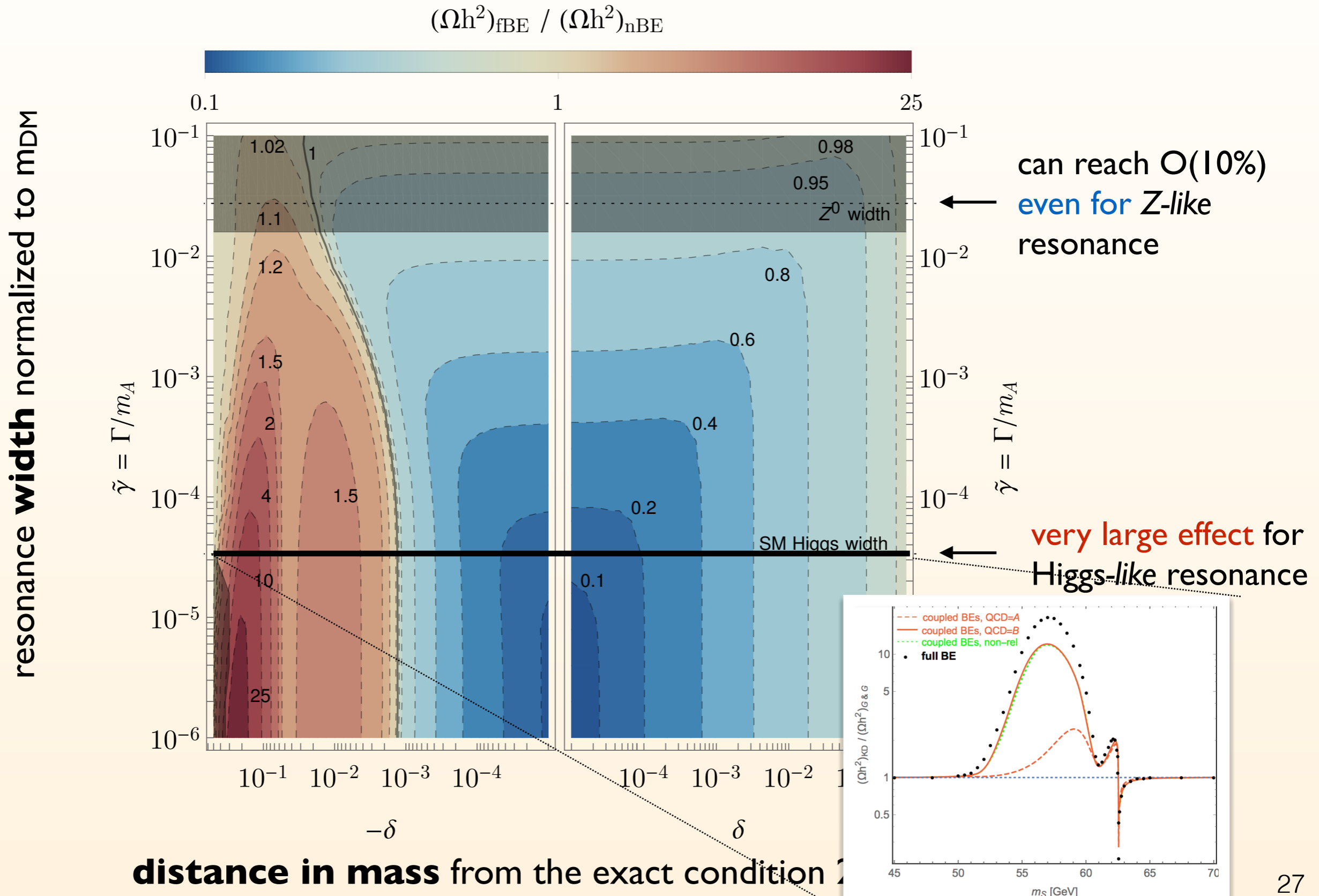


[... Freeze-out at few GeV \rightarrow what is the abundance of heavy quarks in QCD plasma?

two scenarios: QCD = A - all quarks are free and present in the plasma down to $T_c = 154$ MeV
 QCD = B - only light quarks contribute to scattering and only down to $4T_c$...] 26

GENERIC RESONANT ANNIHILATION

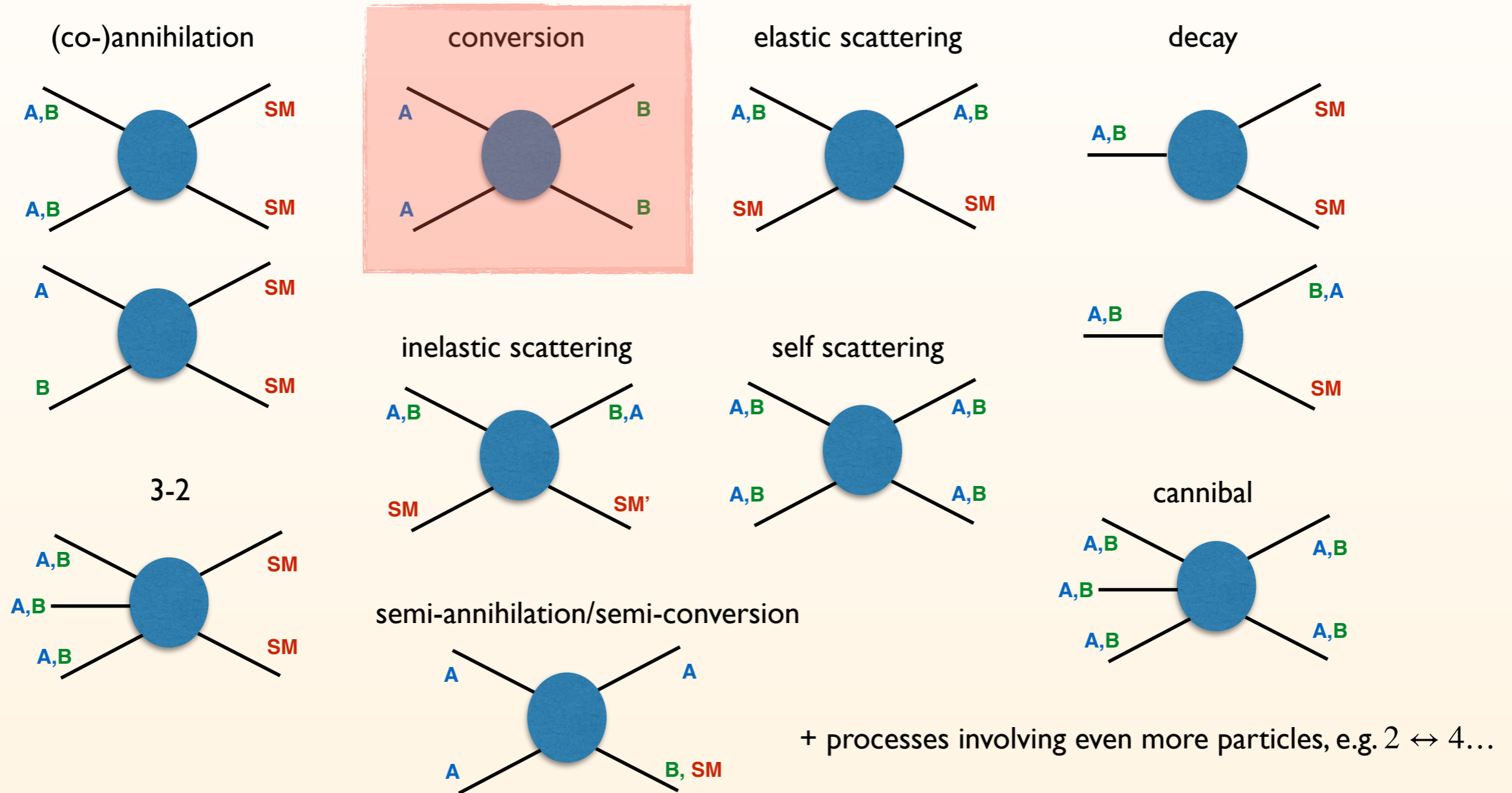
EXAMPLE EFFECT OF EARLY KD ON RELIC DENSITY



EXAMPLE B:
EFFECT OF CONVERSION PROCESSES

WHAT IF A NON-MINIMAL SCENARIO?

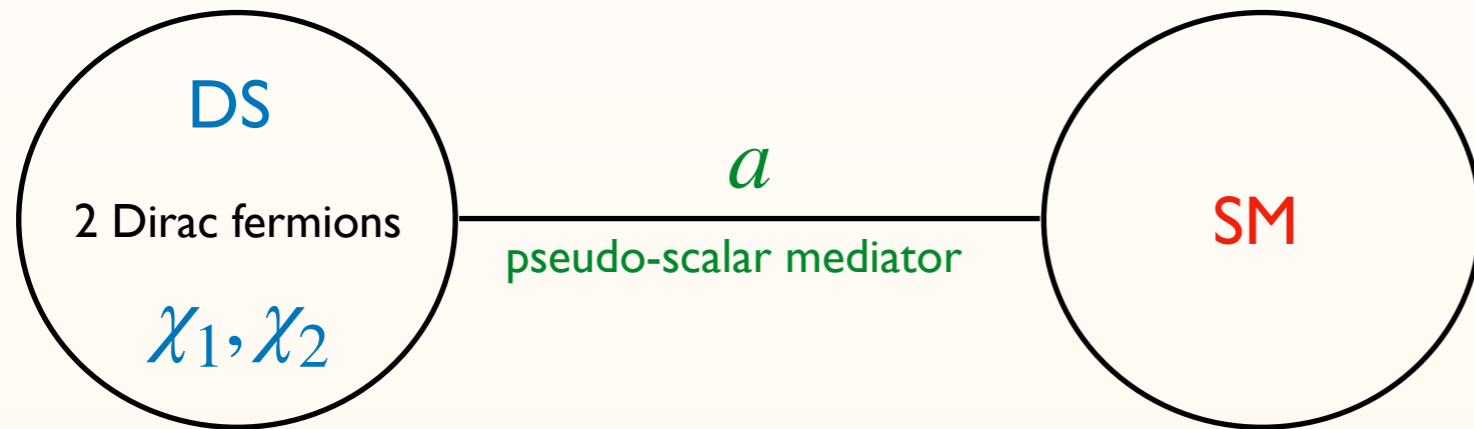
A,B — two different dark sector states (at least one needs to be stable)



Note: some of these processes affect **not only # density**, but also strongly modify the **energy distribution of DM particles!**

RESULTS: THE MODEL

Let's take one of the simplest two-component DM models:



$$\mathcal{L}_{int} = - \sum_{i=1,2} i\lambda_i a \bar{\chi}_i \gamma^5 \chi_i - i\lambda_y \frac{m_f}{v} a \bar{f} \gamma^5 f$$

coupled directly to SM fermions in a MFV way

New fields: χ_1, χ_2, a New params: m_1, m_2, m_a
 $\lambda_1, \lambda_2, \lambda_y$

Parametrically:

$$\sigma_{11 \rightarrow SM} \sim \sigma_{1SM \rightarrow 1SM} \sim \lambda_1^2 \lambda_y^2$$

$$\sigma_{22 \rightarrow SM} \sim \sigma_{2SM \rightarrow 2SM} \sim \lambda_2^2 \lambda_y^2$$

$$\sigma_{11 \rightarrow 22} \sim \lambda_1^2 \lambda_2^2$$



Varying:

$$\lambda_1 \rightarrow c \lambda_1$$

$$\lambda_2 \rightarrow c \lambda_2$$

$$\lambda_y \rightarrow \lambda_y / c$$

Keeps everything fixed, except conversions

Main motivation (for models in the literature with pseudo-scalar mediator):

Evasion of the direct detection bounds... while giving strong signal in indirect detection, in particular **for explaining the Galactic Centre excess**

(see e.g. „Coy DM”)

MOTIVATIONAL PLOT

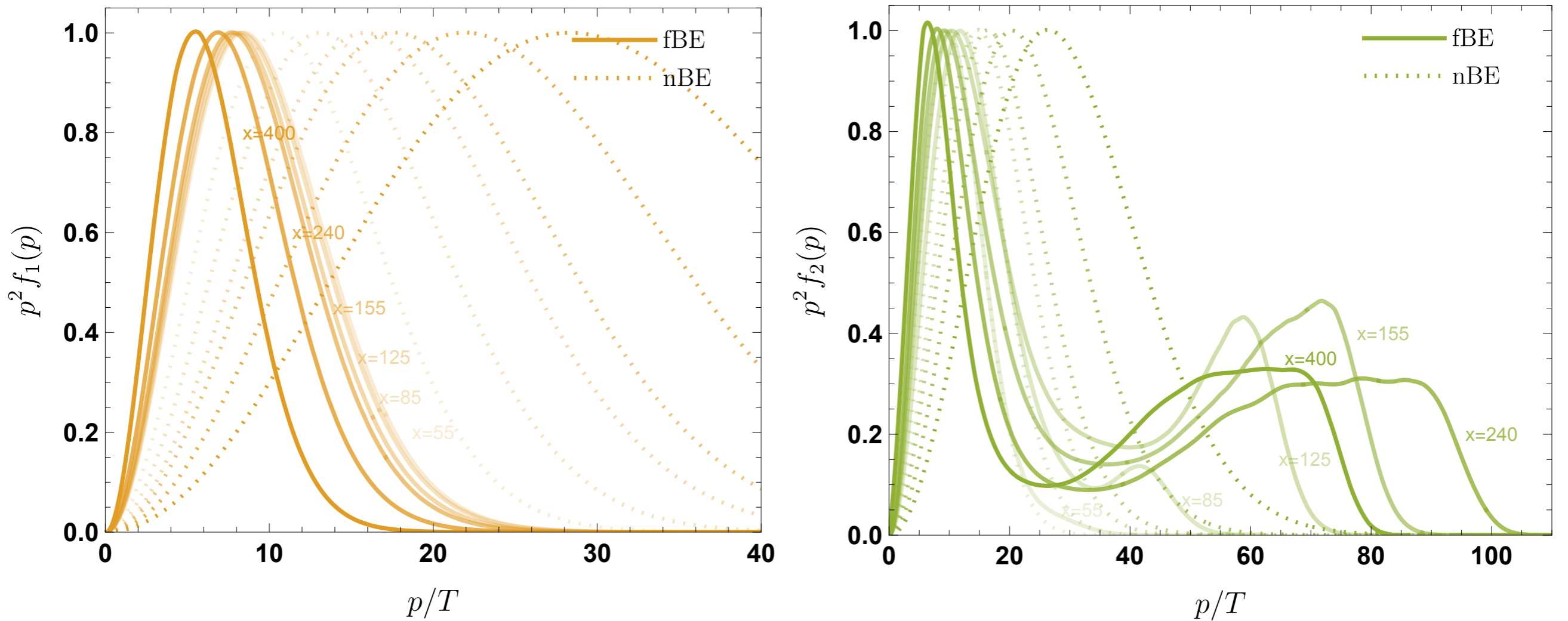
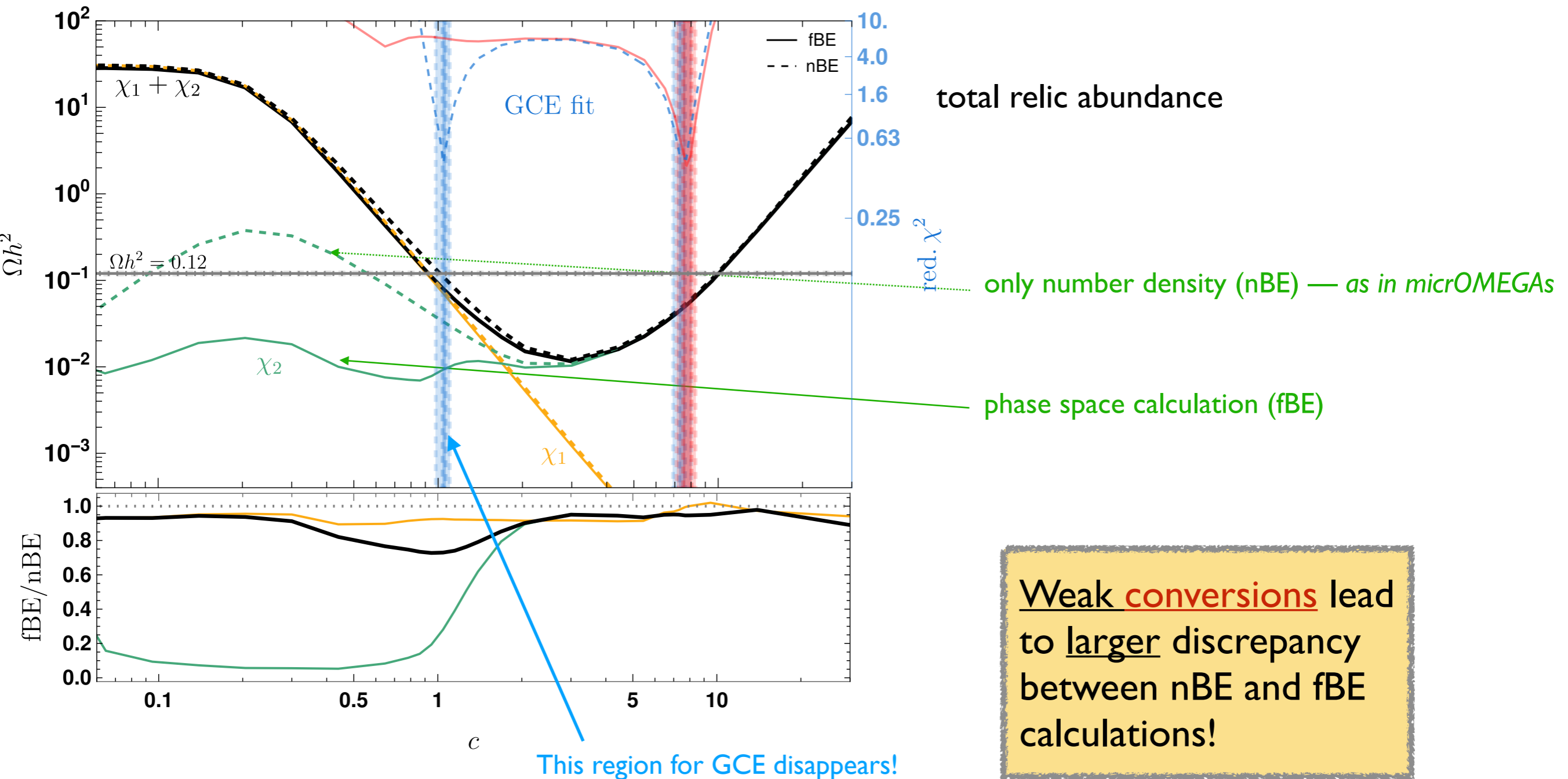


Figure 1: Time snapshots of the evolution of the normalised momentum distributions for χ_1 (left) and χ_2 (right) in p/T , for the benchmark point showcasing the interplay of conversions and resonant annihilations with early kinetic decoupling. The solid lines show the particle distribution functions f_{χ_i} while the dotted lines show the corresponding equilibrium distributions.

RESULTS: CONVERSION IMPACT

Varying: $\lambda_1 \rightarrow c \lambda_1$ $\lambda_2 \rightarrow c \lambda_2$ $\lambda_y \rightarrow \lambda_y/c$ Only **conversions** change!

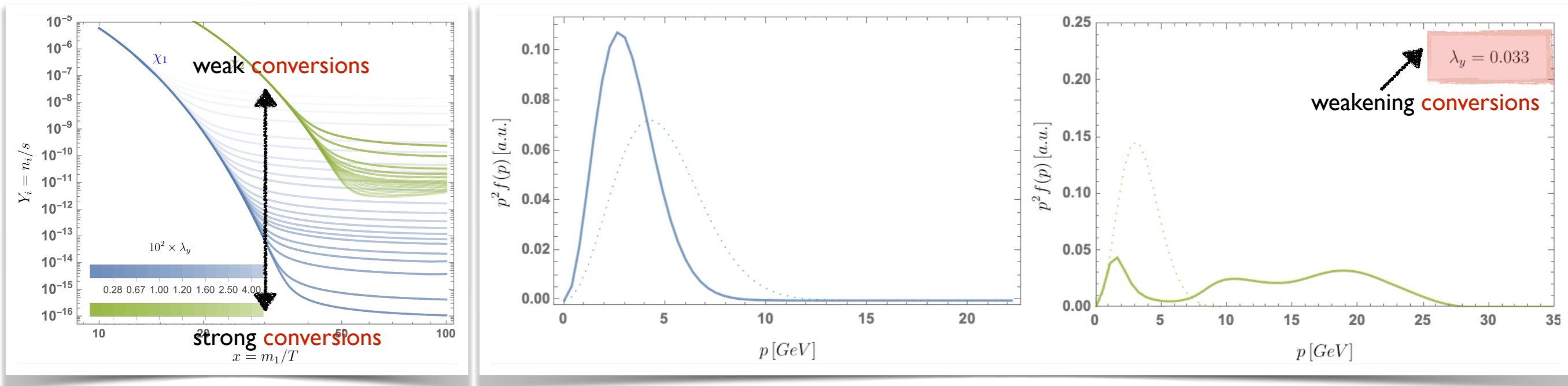
← weak **conversions** strong **conversions** →



Weak conversions lead to larger discrepancy between nBE and fBE calculations!

RESULTS: CONVERSION IMPACT

weaker conversions \Rightarrow less depletion of $\chi_1 \Rightarrow$ around χ_2 freeze-out more χ_1 in the plasma
 \Rightarrow larger distortion of thermal shape



Conversions are ubiquitous in multicomponent models... but not the only processes affecting the distributions:

- decays of heavier to lighter dark sector states
- inelastic scatterings
- semi-annihilations
- cannibal ($3 \leftrightarrow 2$)

A.H. & Laletin 2204.07078 (see also Beuchadne & Chiang 2401.03657)

A.H. & Laletin 2104.05684

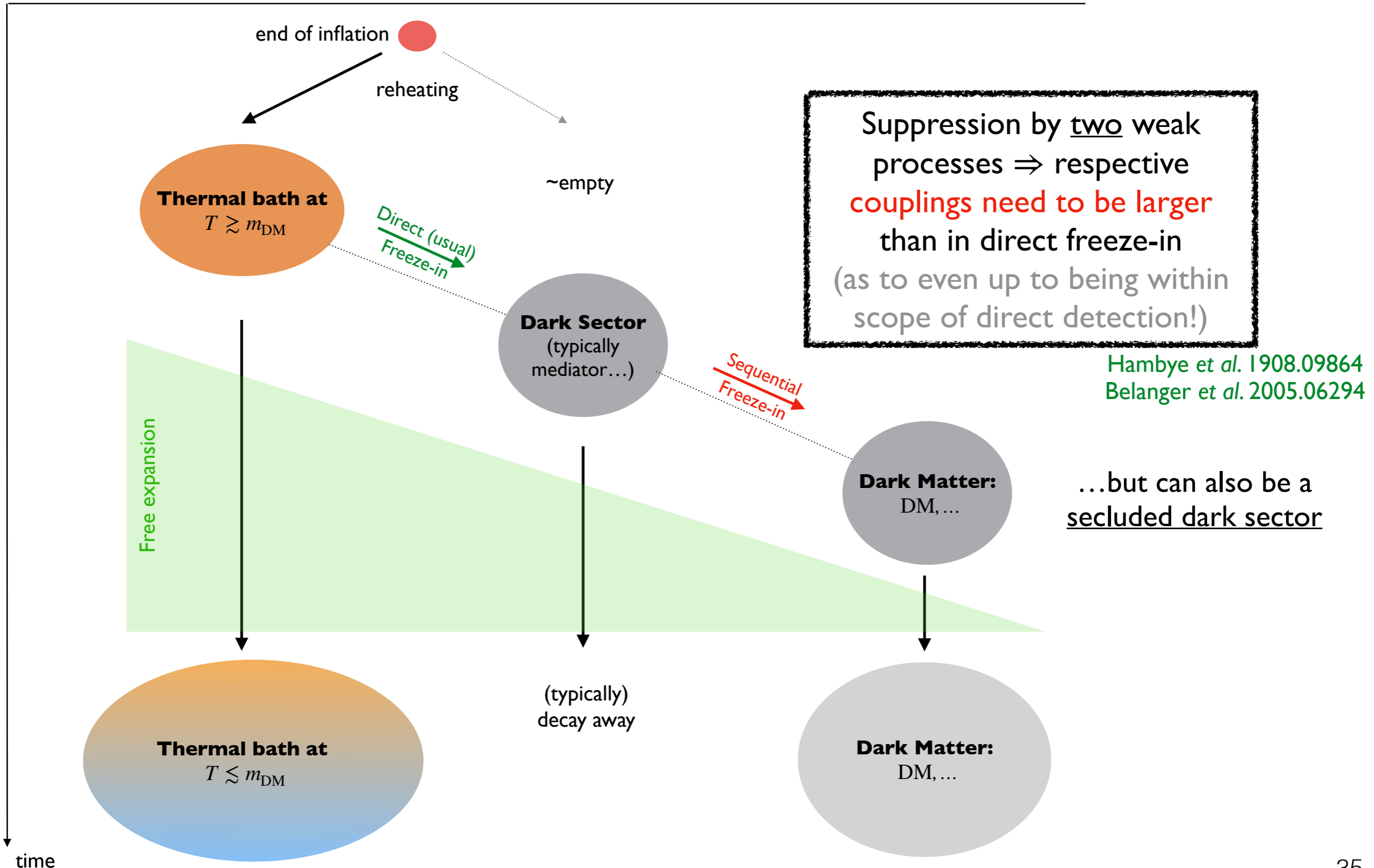
Cervantes & A.H. 2407.12104

EXAMPLE C:
(SEQUENTIAL) FREEZE-IN

WHAT IS SEQUENTIAL FREEZE-IN?



Visible Sector

Dark Sector

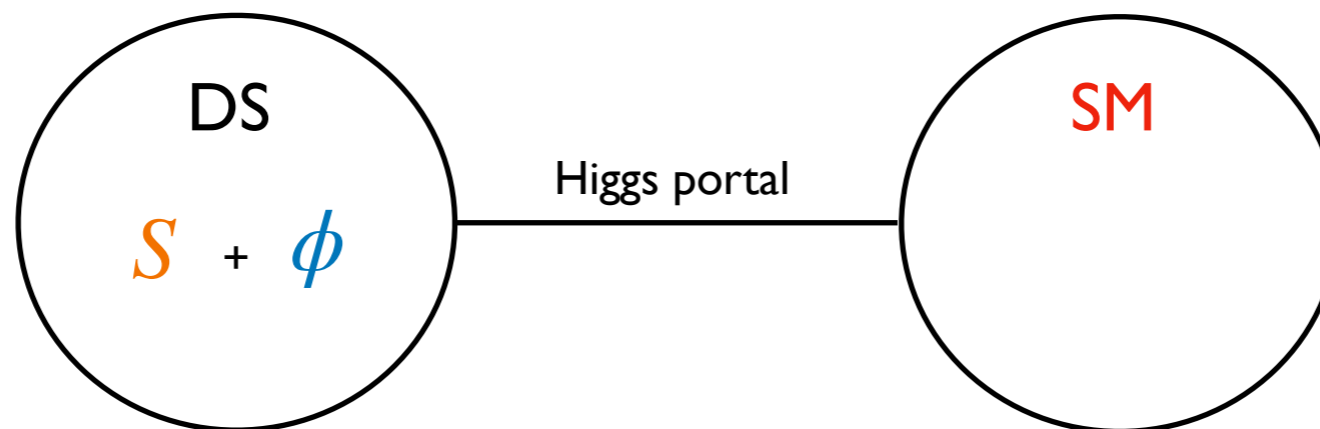


A TALE OF TWO SCALARS

Postulate two new scalars (singlets w.r.t SM gauge group):

	S	\mathbb{Z}_2 -symmetric	stable	dark matter	feeble int. with SM
	ϕ	\mathbb{Z}_2 explicitly broken	unstable	"mediator"	feeble int. with SM

^



$$V \supset -\lambda_{h\phi} \phi^2 H^\dagger H - \frac{\lambda_{Sh}}{2} S^2 H^\dagger H - \frac{1}{4} \lambda_{S\phi} S^2 \phi^2$$

mediator-Higgs
DM-Higgs
DM-mediator

mediator-Higgs mixing

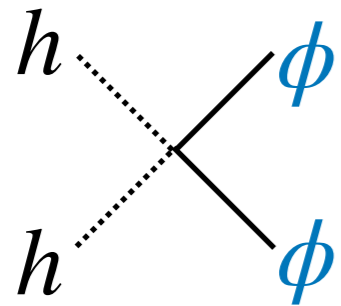
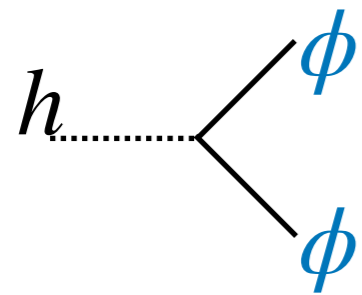
$$\sin \theta = \frac{A v}{m_h^2 - m_\phi^2} \left(1 - \frac{\lambda_{h\phi} v^2}{2m_\phi^2} \right) \quad 36$$

Such models are not unheard of. Most similar in the literature:

...; Wang, Han '14; Claude, Godfrey '21; ...

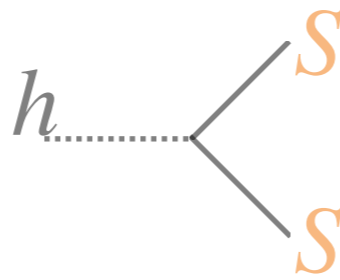
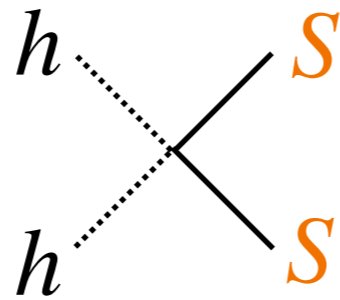
A TALE OF TWO SCALARS

mediator freeze-in:



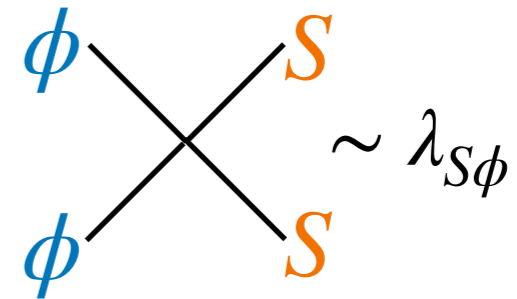
$$\sim \lambda_{h\phi}$$

DM freeze-in:



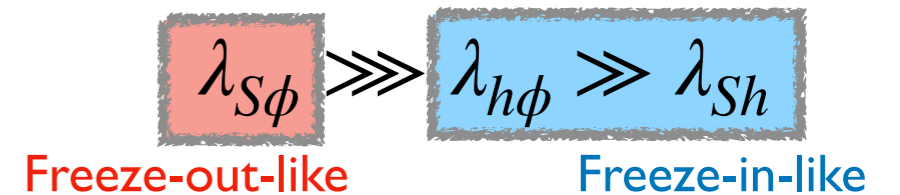
$$\sim \lambda_{Sh}$$

sequential freeze-in:

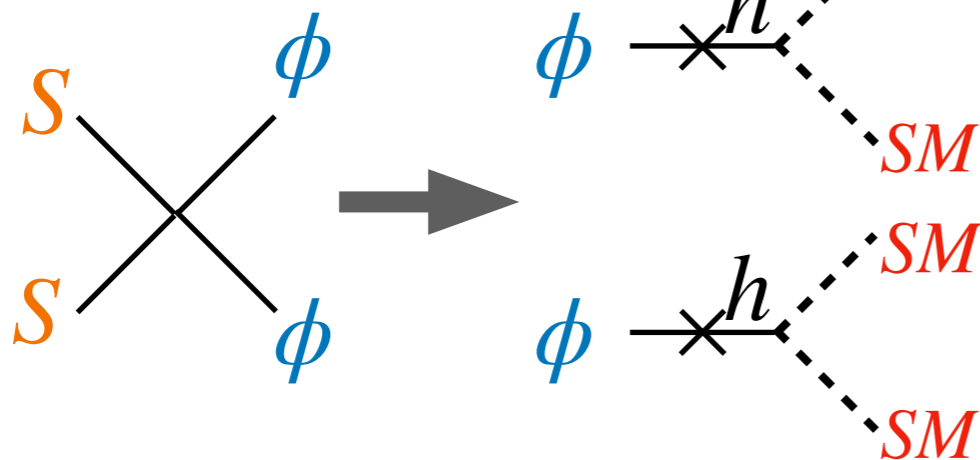


$$\sim \lambda_{S\phi}$$

Typical hierarchy:

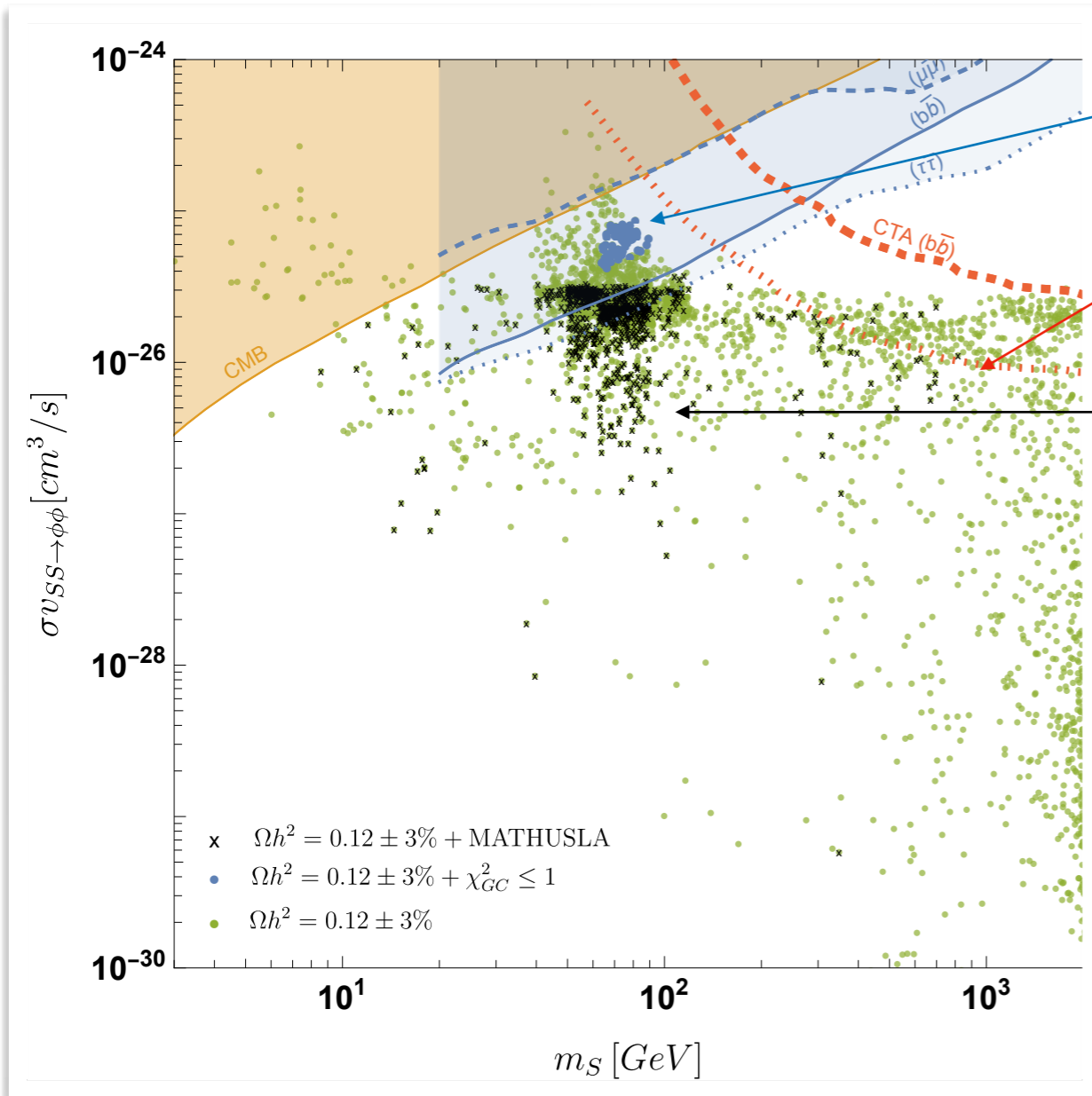


Indirect detection through a cascade decay (iff $m_S > m_\phi$):



ID signal = requirement of sub-threshold sequential freeze-in

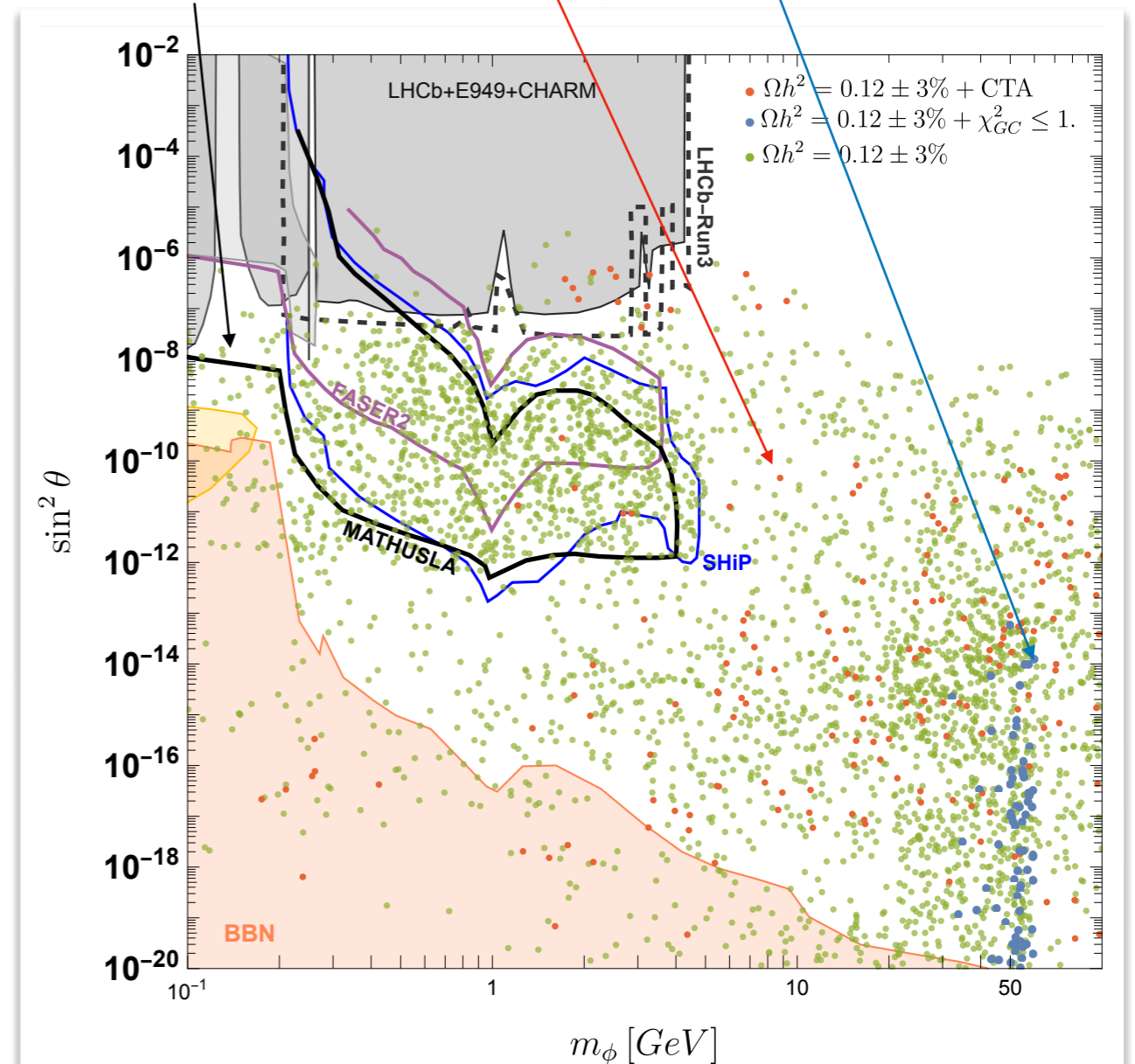
SCAN RESULTS: ID AND FORWARD PHYSICS



Points giving good fit to GCE

Points within (optimistic) reach of CTA

within reach of MATHUSLA



All points satisfy relic density constraint

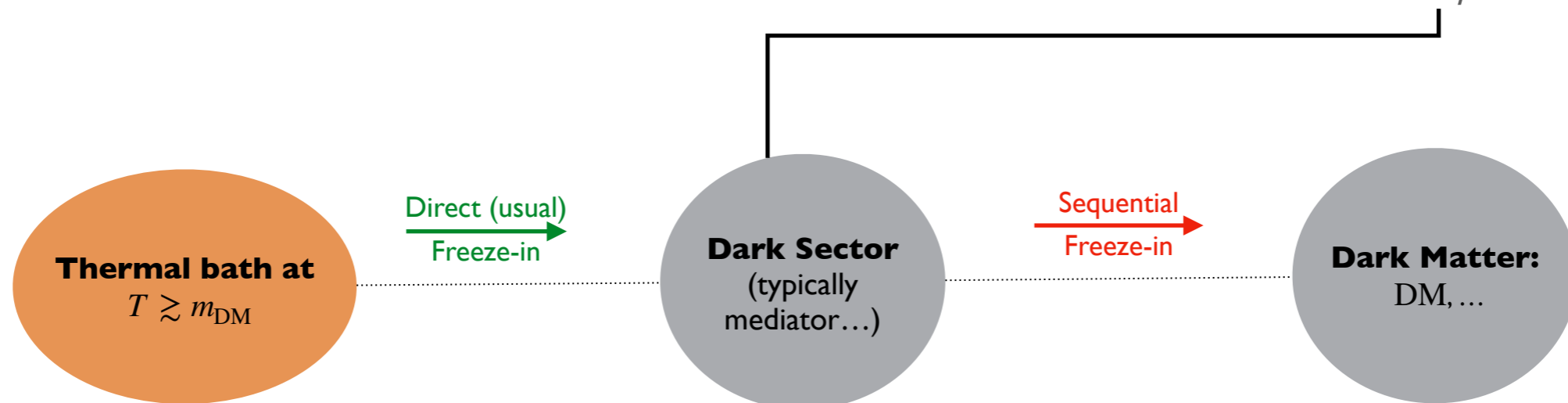
Scan driven towards regions that are covered by any of the experiments

A SECOND LOOK ON Ωh^2

The relic density was the main constraint of the scan. It was obtained by solving the Boltzmann equation for number densities of ϕ and S (nBE) (as e.g. micrOMEGAs or DarkSUSY would)

But wait... isn't relic abundance (*freeze-in or freeze-out*) dependent on the T of the thermal bath it is produced from?

Which temperature is relevant for sequential freeze-in: T_{SM} or T_ϕ ?

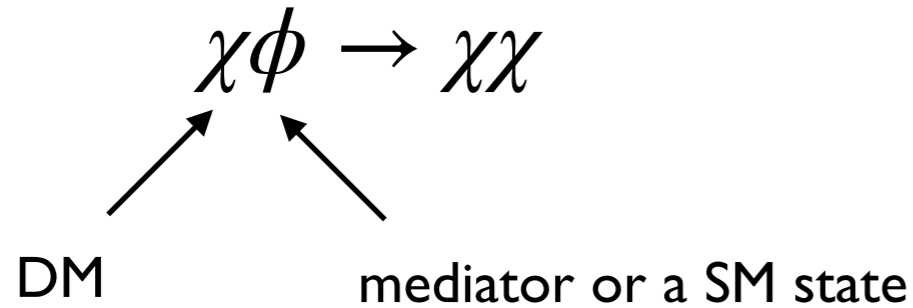


...OK, so it looks like we need to trace T_ϕ as well!

THIS IS REMINISCENT OF...

AH, Laletin 2104.05684
(see also Bringmann et al. 2103.16572)

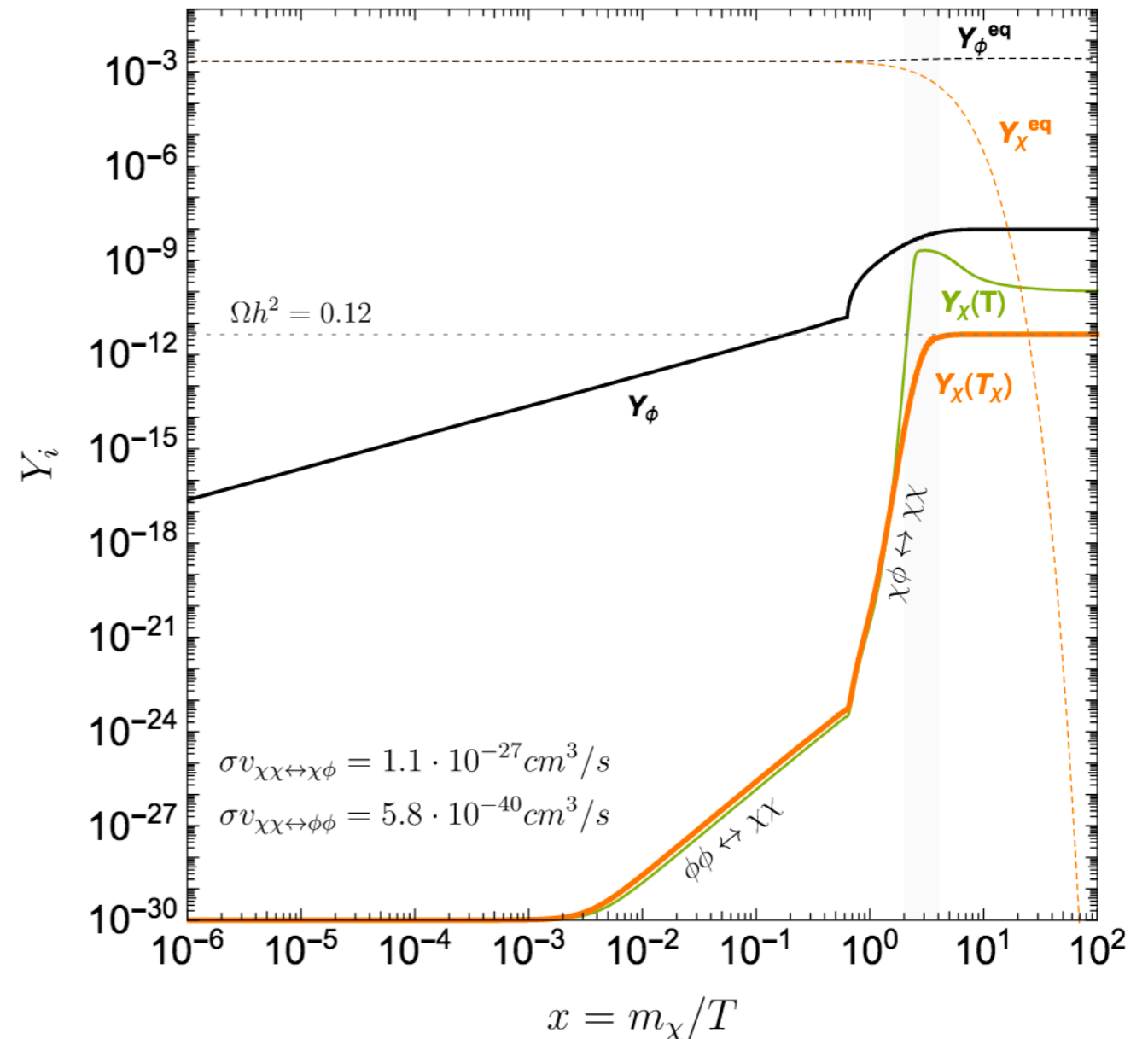
Consider process of production that is the **inverse of semi-annihilation**:



What is different?

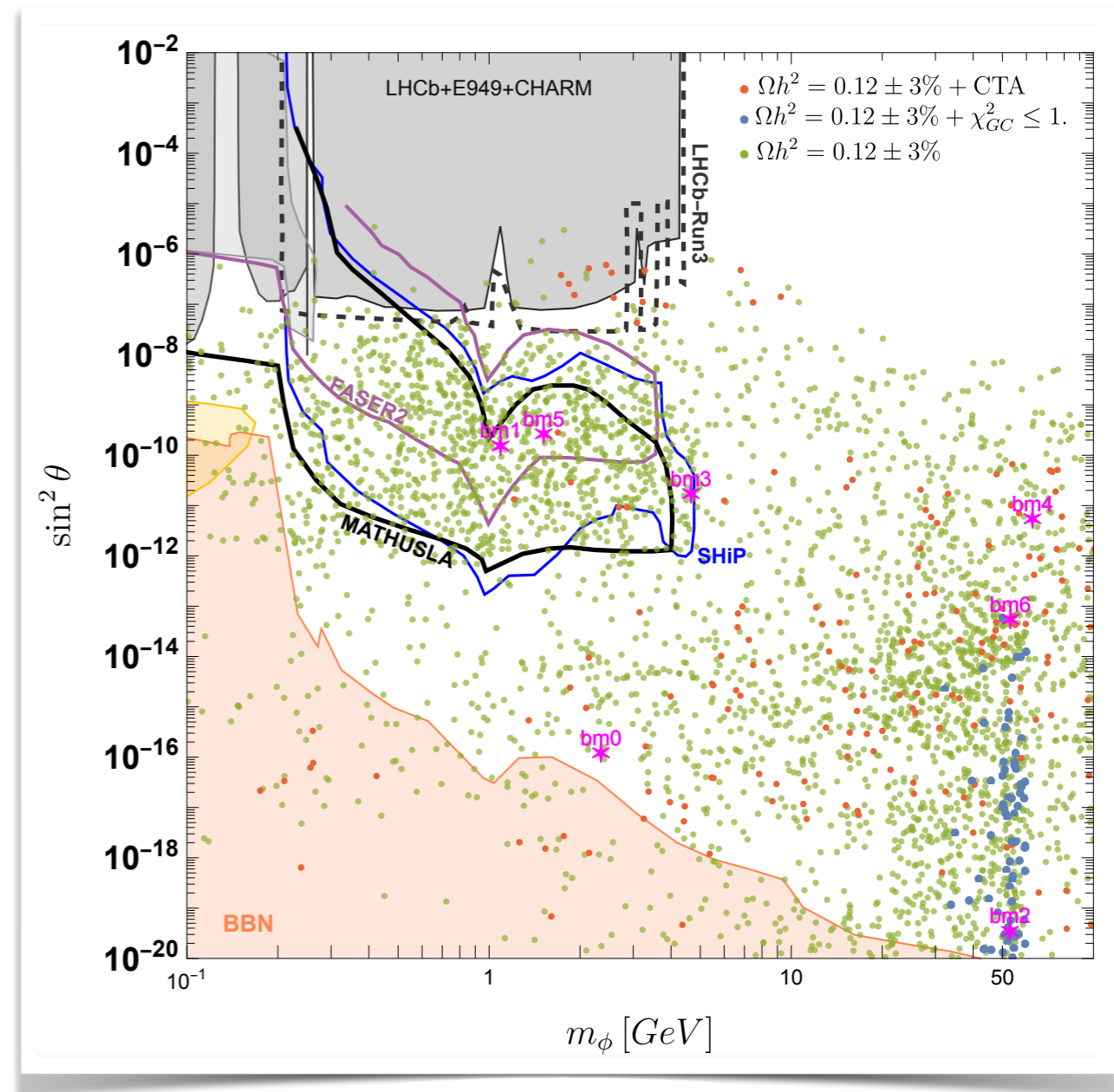
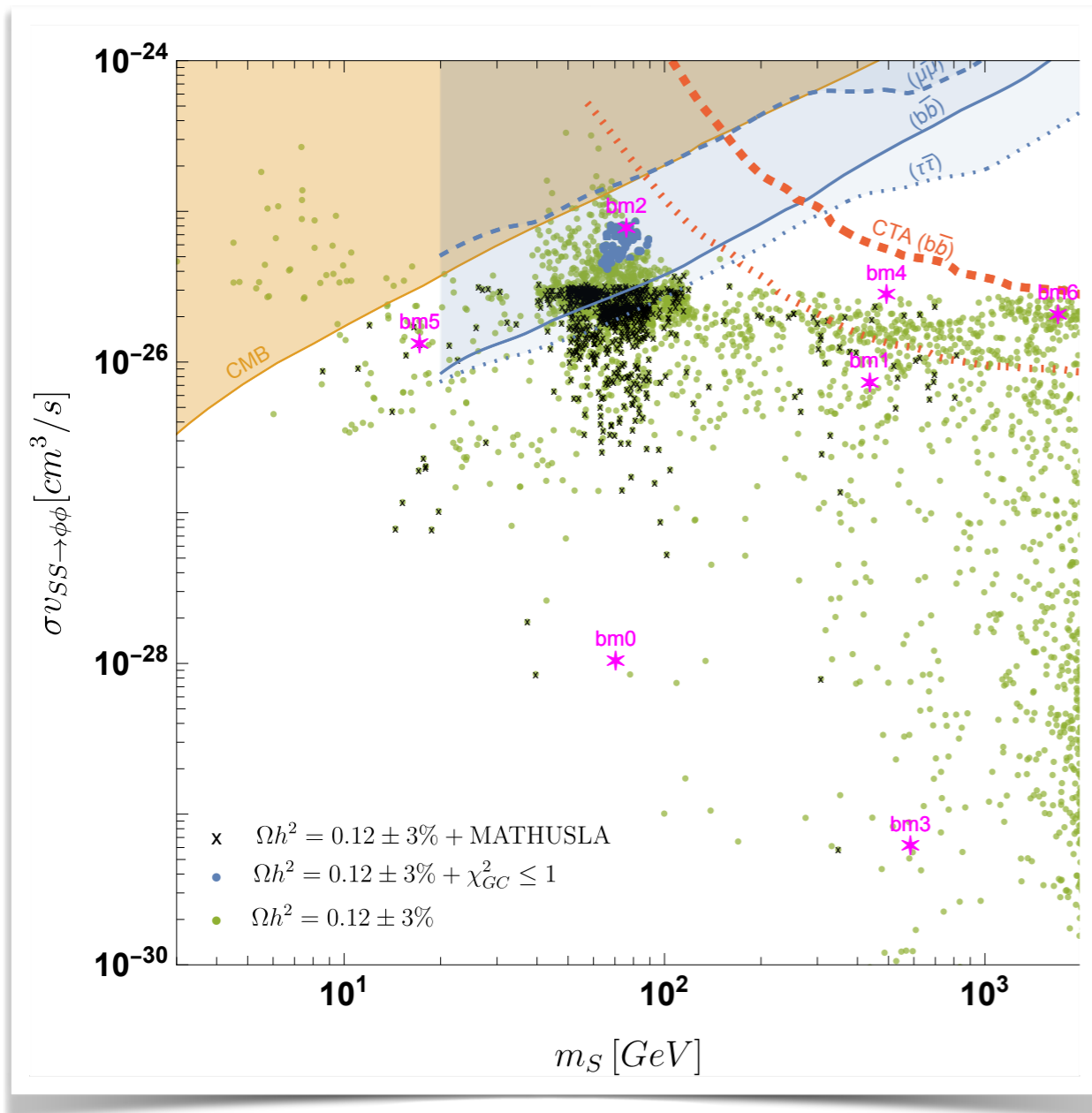
(from the decay/annihilation freeze-in)

- The production rate is **proportional to the DM density**. (Smaller initial abundance \rightarrow larger cross section...)
- **Semi-production** modifies the energy of DM particles in a non-trivial way, so the **temperature evolution can affect the relic density**



MCMC SCAN & BENCHMARKS

Points that give correct relic abundance at nBE level, projected onto indirect detection and forward physics searches planes



BENCHMARKS: SUMMARY

Name	m_ϕ	m_S	θ	$\lambda_{h\phi}$	λ_{hS}	$\lambda_{S\phi}$	$(\Omega h^2)_{\text{nBE}}$	$(\Omega h^2)_{\text{cBE}}$	change [%]	description
BM0	2.35	70.4	1.09×10^{-8}	1.67×10^{-13}	5.98×10^{-11}	0.00298	0.113	0.110	-1.96	direct FI
BM1	1.09	438.	1.24×10^{-5}	3.56×10^{-11}	3.72×10^{-13}	0.155	0.124	0.0205	-83.5	seq. FI/dark FO + MATHUSLA
BM2	53.0	76.1	1.87×10^{-10}	3.51×10^{-7}	1.96×10^{-11}	0.104	0.115	0.0199	-82.7	dark FO + best GCE fit
BM3	4.66	586.	4.15×10^{-6}	8.62×10^{-11}	4.32×10^{-15}	0.00603	0.0971	0.000883	-99.1	seq. FI
BM4	63.0	494.	2.34×10^{-6}	1.08×10^{-15}	2.70×10^{-6}	0.344	0.0902	0.0503	-44.2	dark FO/co-decay + CTA
BM5	1.52	17.2	1.62×10^{-5}	1.30×10^{-9}	4.46×10^{-9}	0.00823	0.110	0.0555	-49.5	co-decay + MATHUSLA
BM6	53.2	1.69×10^3	2.33×10^{-7}	5.14×10^{-8}	1.16×10^{-7}	1.01	0.119	0.0571	-51.9	dark FO + CTA

The model's parameter space spans over various production modes:

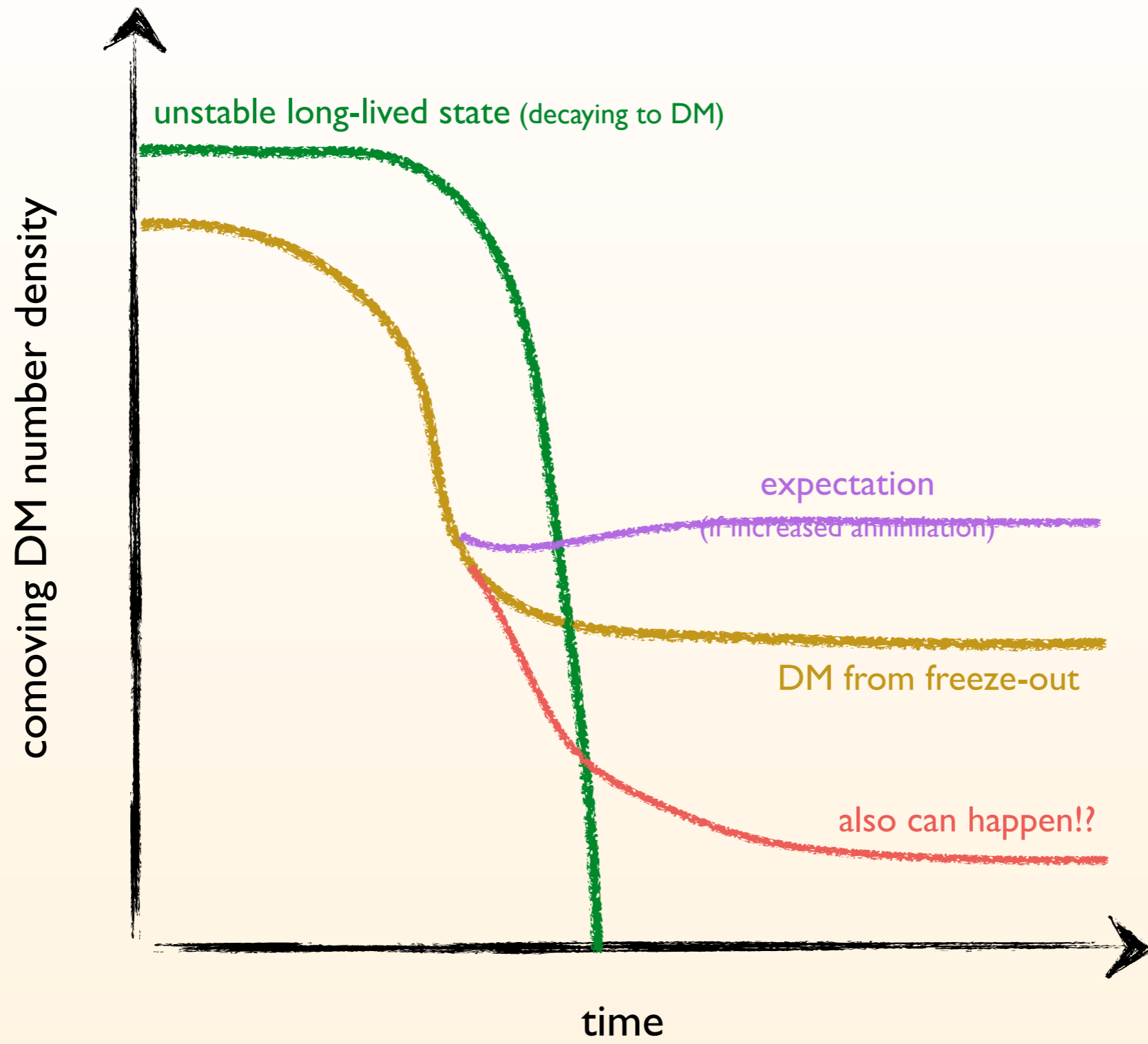
- direct & sequential freeze-in
- dark freeze-out
- co-decaying
- (and mixtures of these)

Effect of performing calculation at cBE level: from $\sim \mathcal{O}(1\%)$ to > 100

EXAMPLE D: WHEN ADDITIONAL INFLUX OF DM ARRIVES

Sudden injection of more DM particles **distorts** $f_\chi(p)$
(e.g. from a decay or annihilation of other states)

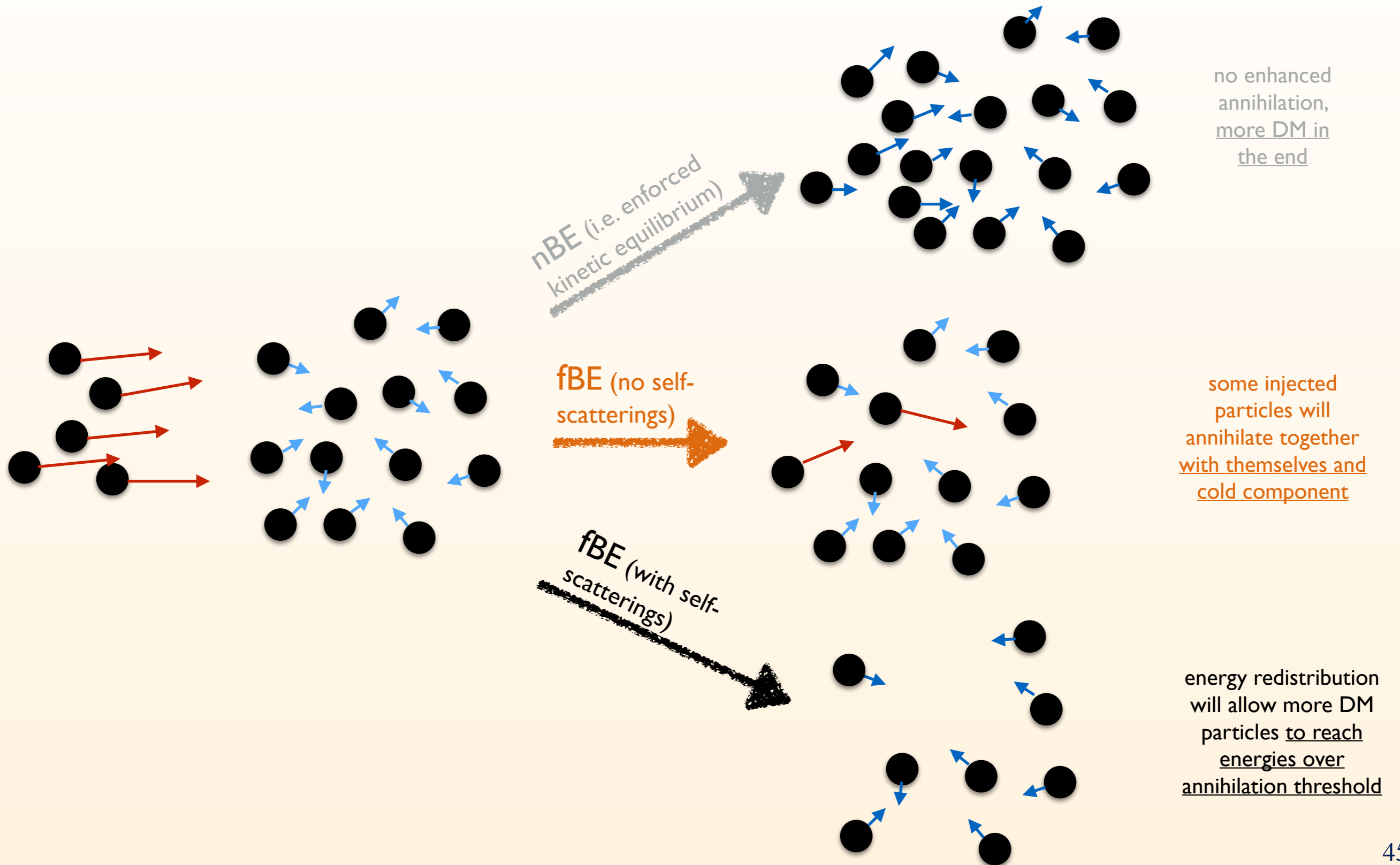
- this can **modify the annihilation rate** (if still active)
- how does the **thermalization** due to elastic scatterings happen?



1) DM produced via:

- 1st component from **thermal freeze-out**
- 2nd component from **a decay $\phi \rightarrow \bar{\chi}\chi$**

2) DM annihilation has a **threshold**
 e.g. $\chi\bar{\chi} \rightarrow f\bar{f}$ with $m_\chi \lesssim m_f$



EXAMPLE EVOLUTION

1) DM produced via:

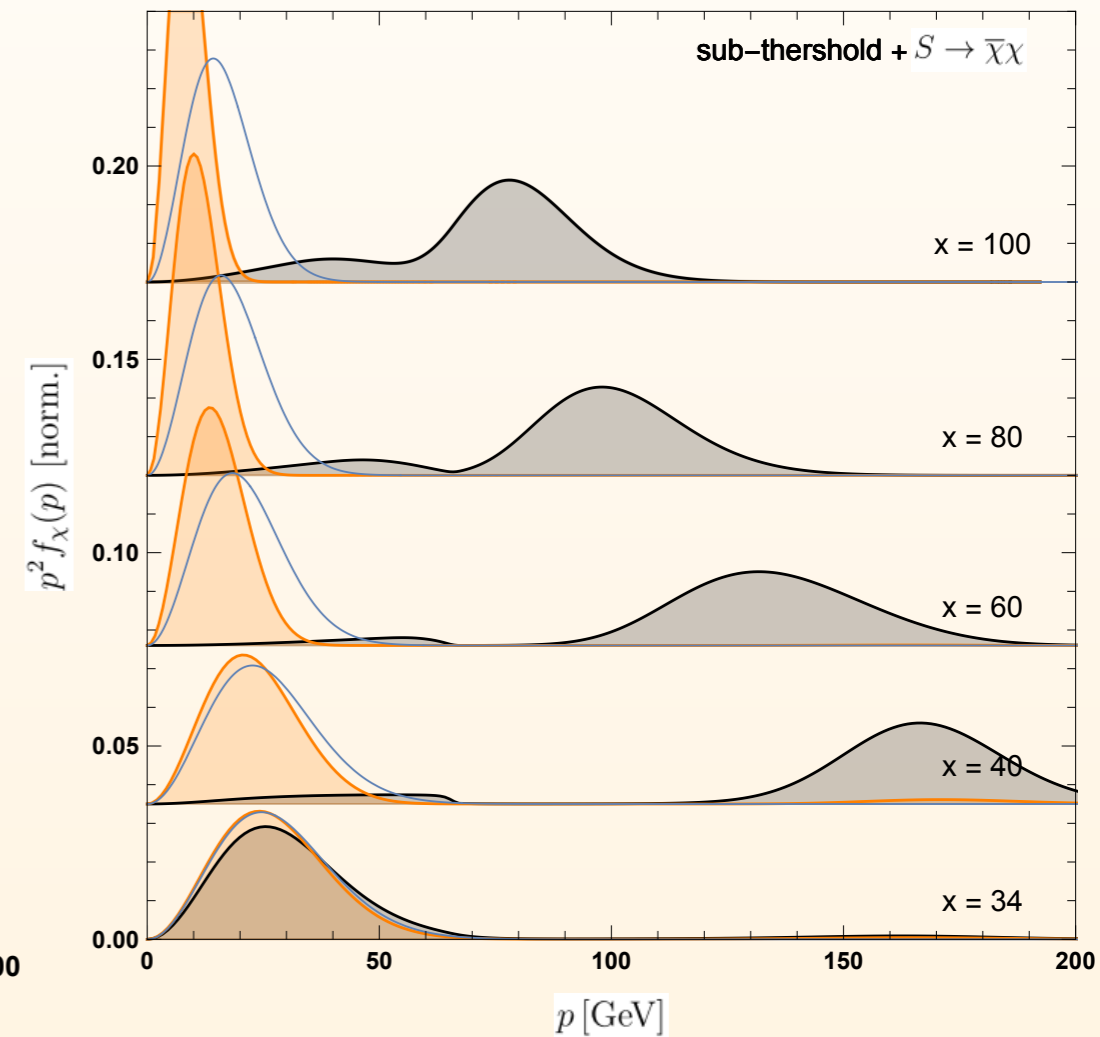
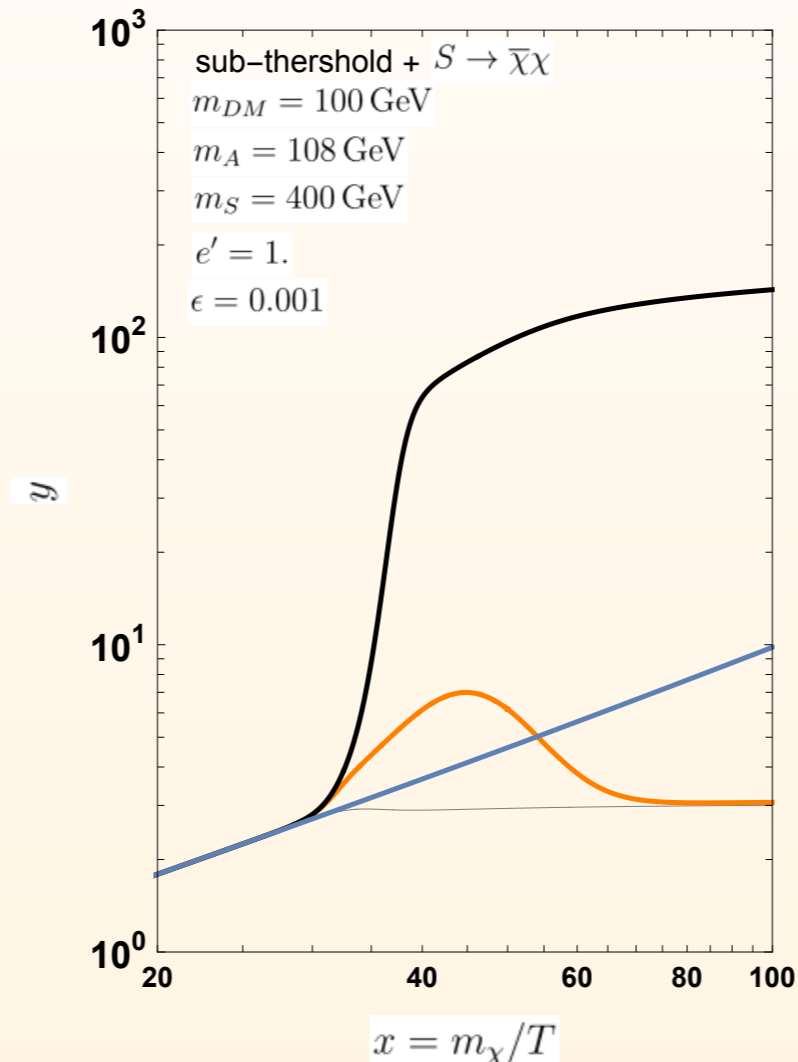
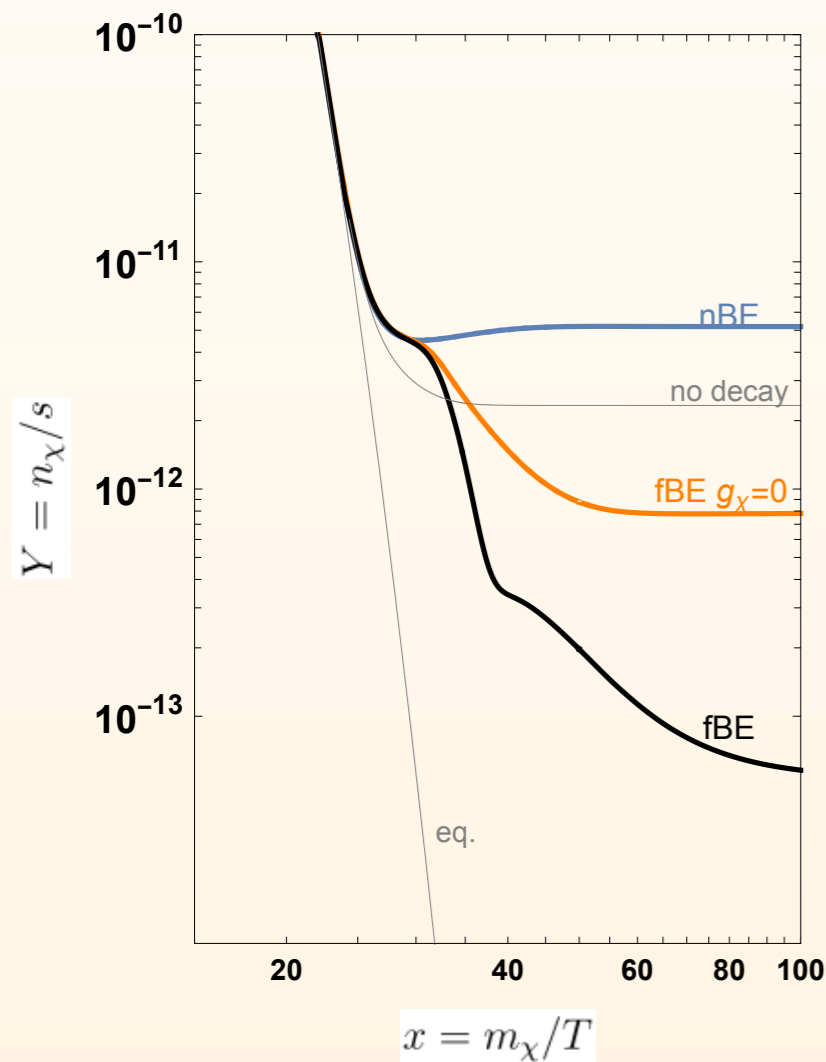
- 1st component from **thermal freeze-out**
- 2nd component from **a decay $\phi \rightarrow \bar{\chi}\chi$**

2) DM annihilation has a **threshold**
 e.g. $\chi\bar{\chi} \rightarrow f\bar{f}$ with $m_\chi \lesssim m_f$

$Y \sim$ number density

$y \sim$ temperature

$p^2 f(p) \sim$ momentum distribution



OTHER EXAMPLES...

Sequential freeze-in thus adds to the list of scenarios where **departure from LTE needs to be considered**:

Annihilation through a (narrow) resonance

Duch, Grządkowski '17; Binder, Bringmann, Gustafsson, A.H '17; Abe '21; Ala-Mattinen et al '22

Sub-threshold (e.g. forbidden DM)

Binder, Bringmann, Gustafsson, A.H 2103.01944; Liu et al '23; Aboubrahim et al. '23

Semi-annihilation and production

Kamada et al. '18; Cai, Spray '18; Hektor, AH & Kannike '19; AH & Laletin 2104.05684

Cannibal DM (freeze-out or freeze-in)

Herba et al '18; Cervantes & AH 2407.12104; Bernal, Cervantes, Deka, AH 2506.09155

Sommerfeld enhanced annihilation

Feng et al '10; Binder, Bringmann, Gustafsson, A.H 2103.01944

Two-component dark sectors (e.g. conversion-driven or co-decaying)

Beauchesne & Chiang 2401.03657; Chatterjee & AH 2502.08725

Freeze-out/freeze-in intermediate regime

Du et al. '22

SuperWIMP, WDM and Lyman- α limits

Decant et al. '22; AH & Laletin 2204.07078

...

CONCLUSIONS

1. Cannibal self-cooling can make DM lighter

2. Production in multicomponent dark sectors proceeds in a T -dependent way. This can alter the naive predictions by **more than an order of magnitude**.

3. In recent years a **significant progress** in refining the relic density calculations in **DRAKE2**  to include **multicomponent case** & **freeze-in**

Thank you!

BACKUP