NATIONAL SCIENCE CENTRE
POLAND

FREEZE-IN & FREEZE-OUT PRODUCTION OF TWO-COMPONENT DARK SECTORS AT THE PHASE SPACE LEVEL

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Based on:

work (in progress) with **S. Chatterjee**

+ some earlier works with **T. Binder, T. Bringmann, M. Gustafsson, M. Laletin**

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Motivation & Objectives

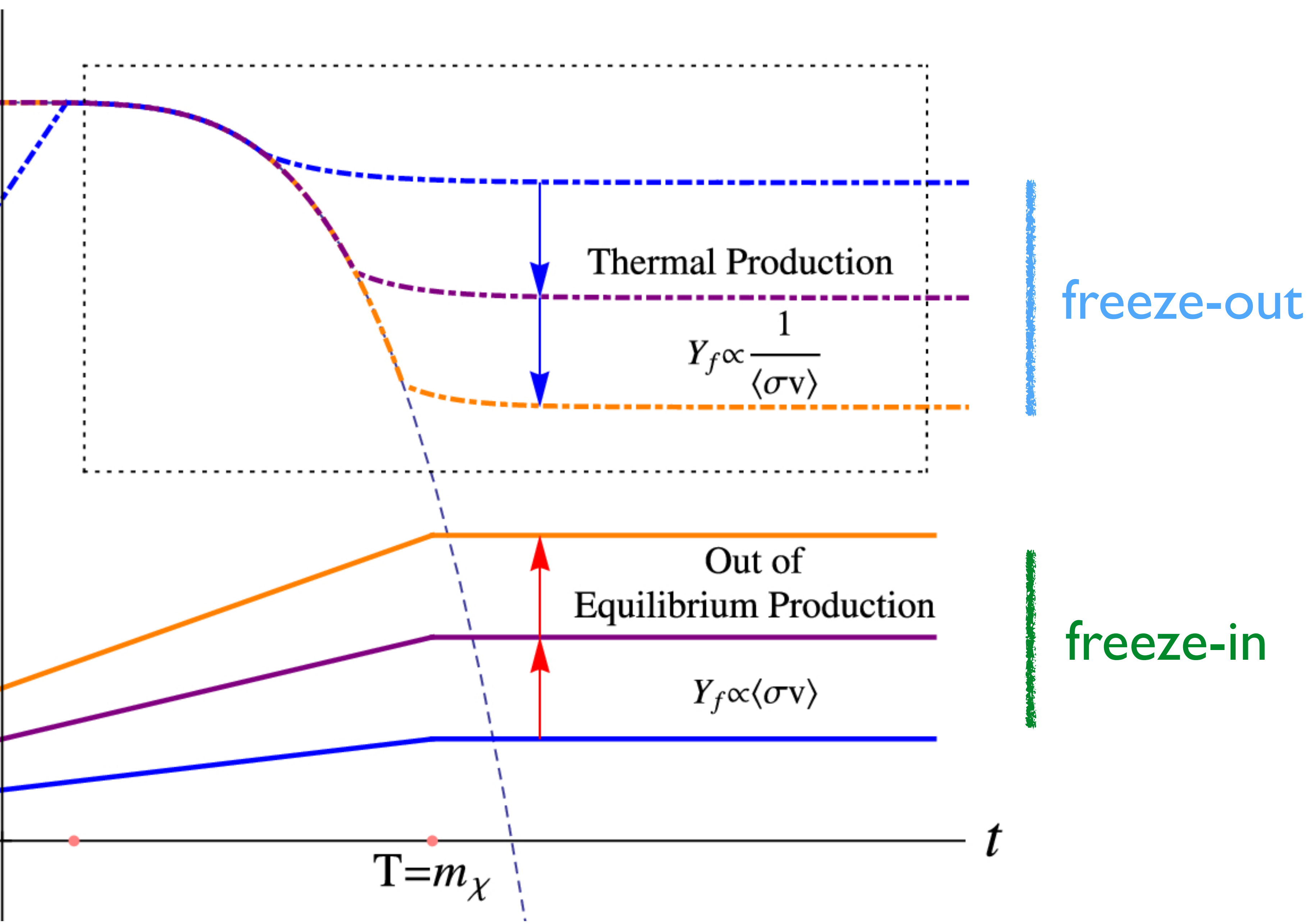
A (step in a) program of describing
Dark Matter production
in systems
departing from local thermal equilibrium

Study of a two concrete theories with detectable DM & non-trivial departures from LTE

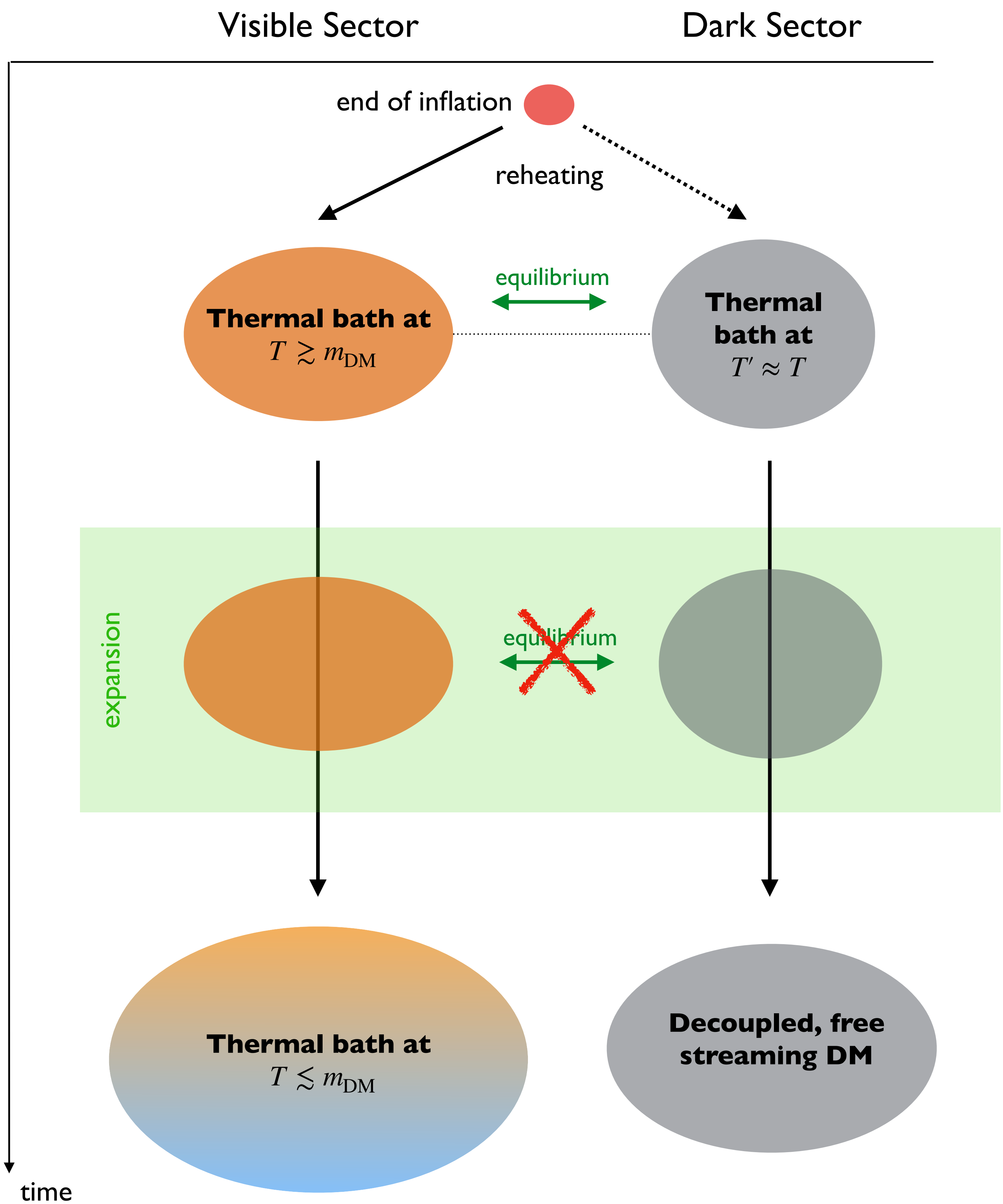
Implementation of freeze-in production & two-component systems in DRAKE2

FREEZE-IN *VS.* FREEZE-OUT

Freeze-in is in a sense the 'opposite' of freeze-out



FREEZE-OUT



I. Natural

Comes out automatically from the expansion of the Universe

Naturally leads to cold DM

II. Predictive

No dependence on initial conditions

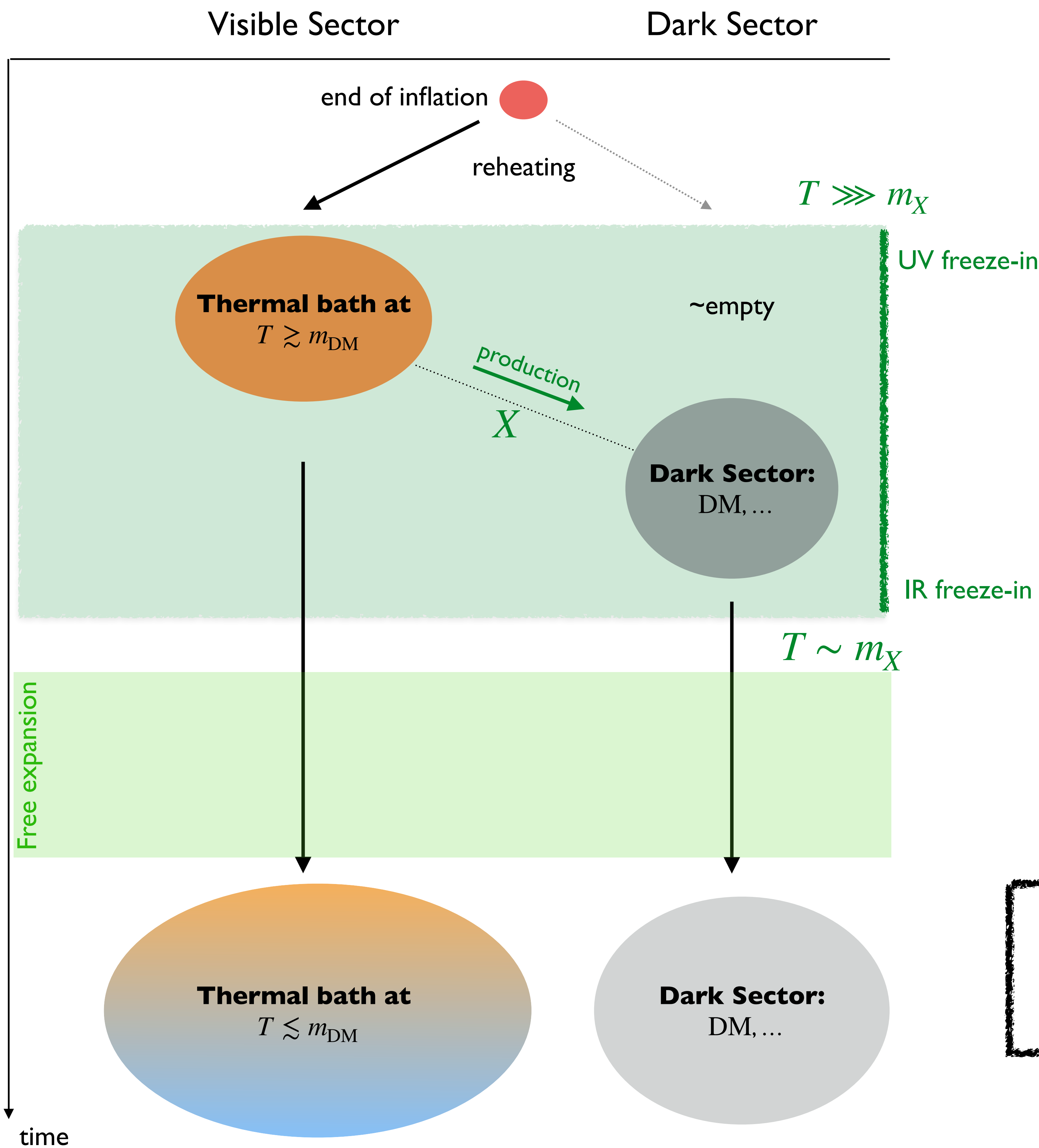
Fixes coupling(s) $\Rightarrow$ signal in DD, ID & LHC

III. It is not optional

Overabundance constraint

To avoid it one needs quite significant deviations from standard cosmology

FREEZE-IN



Freeze-in defined like this is a (very) old idea:

this is a standard production mechanism for e.g. sterile neutrino, gravitino, axino,…

however, old works focused on what now people call UV freeze-in

i.e. dominated by non-renormalizable operators and dependent on $T_{\rm RH}$

Freeze-in = the above mechanism through renormalizable operators (IR freeze-in)

THERMAL RELIC DENSITY

STANDARD APPROACH

Boltzmann equation for $f_\chi(p)$:

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

$\Downarrow$

$$\frac{dn_\chi}{dt} + 3Hn_\chi = -\langle\sigma_{\chi\bar{\chi}\to ij}\sigma_{\rm rel}\rangle^{\rm eq}\left(n_\chi n_{\bar{\chi}} - n_\chi^{\rm eq} n_{\bar{\chi}}^{\rm eq}\right)$$

$\Downarrow$

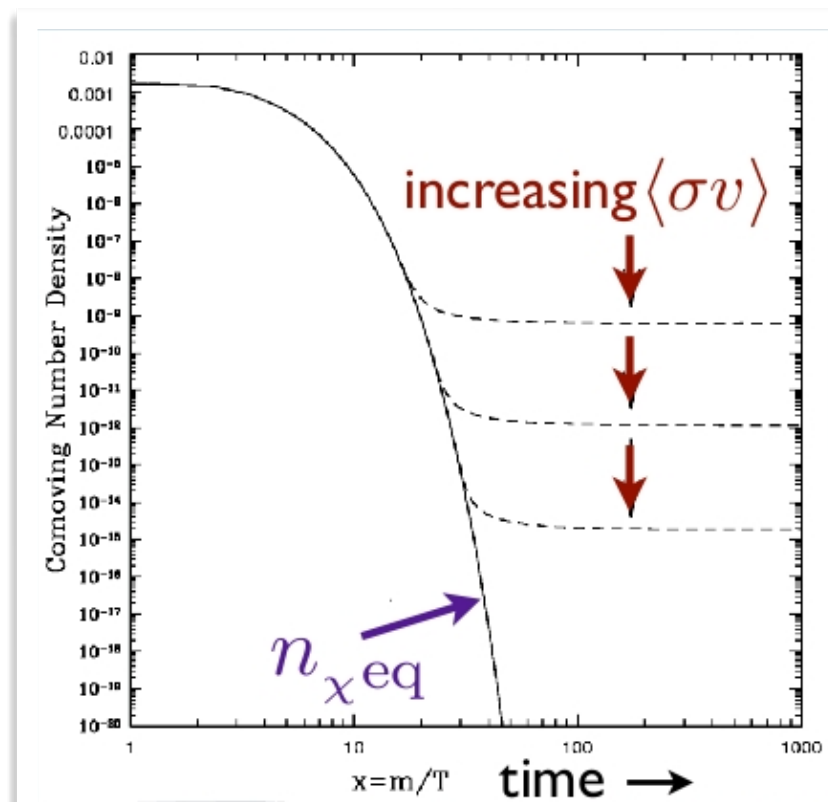


Fig.: Jungman, Kamionkowski & Griest, PR'96

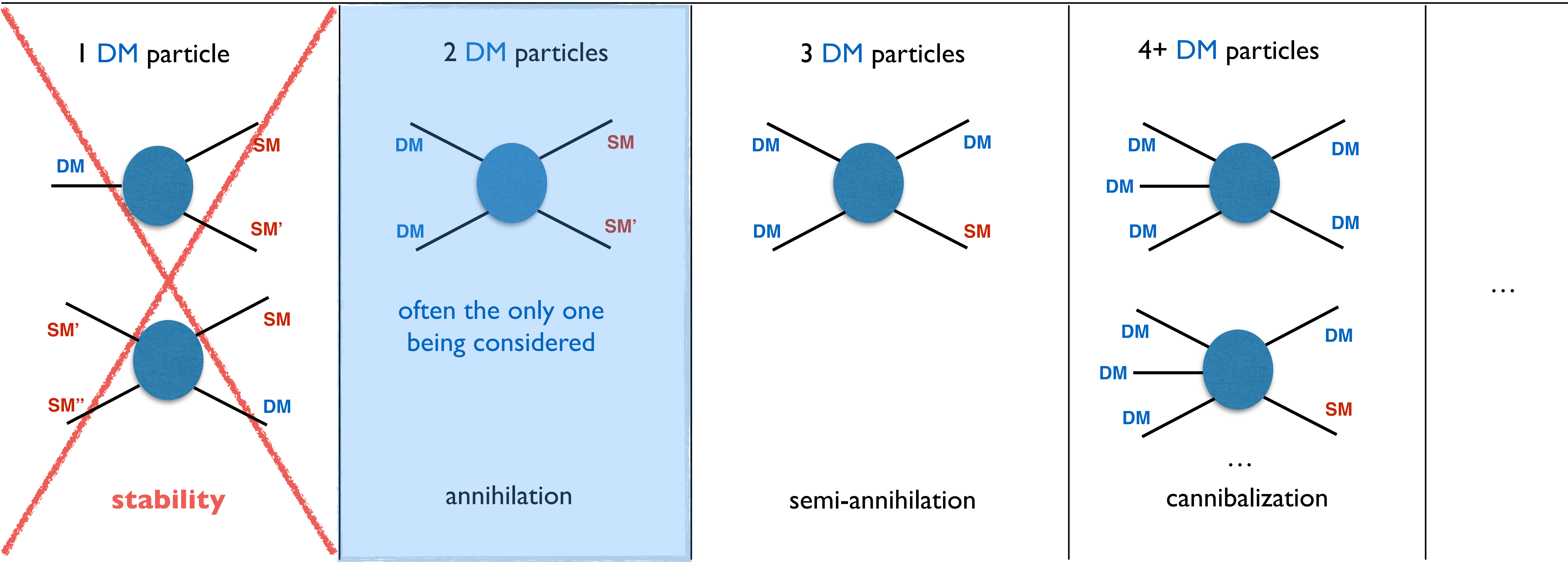
Critical assumption:
kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\rm eq}$$

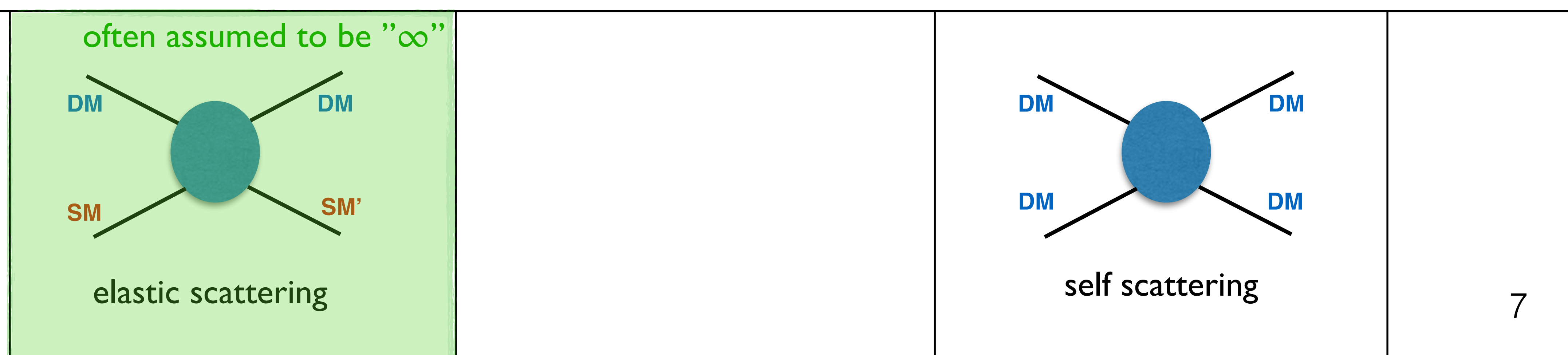
WHAT GOES INTO C IN GENERAL?

For now assume a minimal theory of SM + <u>one</u> DM field

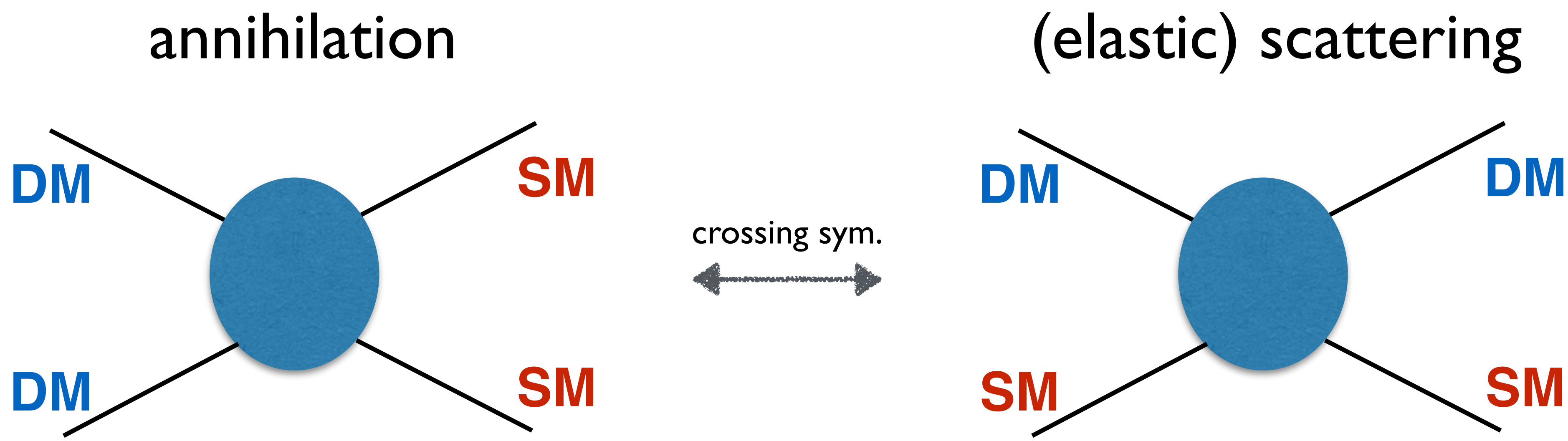
<u>changing</u> processes ⇒ number density



<u>conserving</u> processes ⇒ energy density



FREEZE-OUT *VS.* DECOUPLING



$$\sum_{\text{spins}} \left|\mathcal{M}^{\text{pair}}\right|^2 = F(p_1, p_2, p_1', p_2') \quad \sim \quad \sum_{\text{spins}} \left|\mathcal{M}^{\text{scatt}}\right|^2 = F(k, -k', p', -p)$$

Boltzmann suppression of DM vs. SM $\Rightarrow$ scatterings <u>typically</u> more frequent

dark matter frozen-out but <u>typically</u> still kinetically coupled to the plasma

$$\tau_r(T_{kd}) \equiv N_{coll}/\Gamma_{el} \sim H^{-1}(T_{kd})$$

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Two consequences:

1. During freeze-out (chemical decoupling) <u>typically</u>: $f_\chi \sim a(\mu) f_\chi^{eq}$
2. If kinetic decoupling much, much later: possible impact on the matter power spectrum

i.e. kinetic decoupling can have observable consequences and affect e.g. missing satellites problem

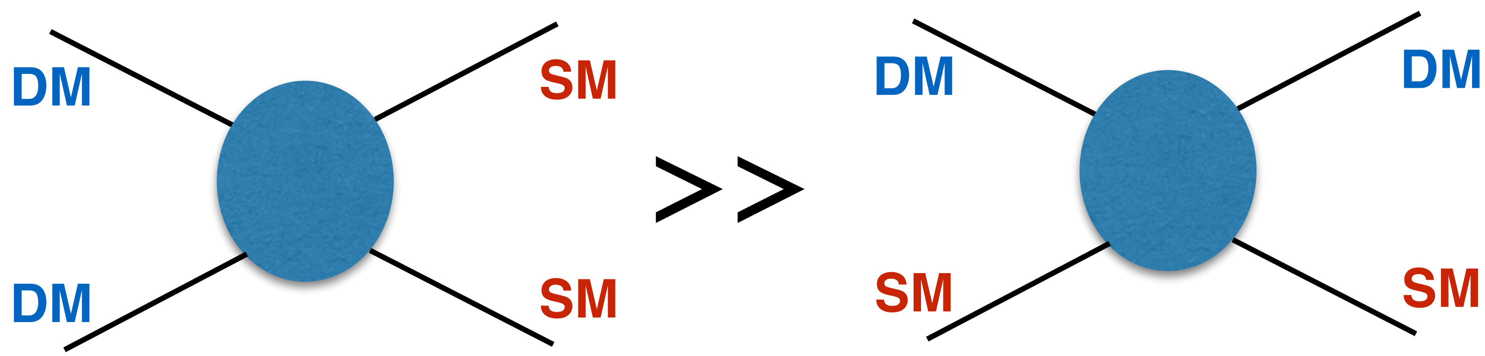
see e.g., Bringmann, Ihle, Karsten, Walia '16

DEPARTURE FROM KINETIC EQUILIBRIUM?

A necessary and sufficient condition: <u>scatterings weaker than annihilation</u>

i.e. rates around freeze-out: $H \sim \Gamma_{\mathrm{ann}} \gtrsim \Gamma_{\mathrm{el}}$

Possibilities:

A) 

e.g., resonant annihilation

B) Boltzmann suppression of SM as strong as for DM

e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure

e.g., semi-annihilation, 3 to 2 models,…

D) Multi-component dark sectors

e.g., additional sources of DM from late decays, …

How to go beyond kinetic equilibrium?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E\left(\partial_t - H\vec{p}\cdot\nabla_{\vec{p}}\right) f_\chi = \mathcal{C}[f_\chi]$$

→ contains both scatterings and annihilations

Two possible approaches:

fBE → solve numerically for full $f_\chi(p)$

- have insight on the distribution
- no constraining assumptions
- numerically challenging
- often an overkill

cBE → consider system of equations for moments of $f_\chi(p)$

0-th moment: n_χ
2-nd moment: T_χ
$\ldots$

- partially analytic/much easier numerically
- manifestly captures all of the relevant physics
- finite range of validity
- no insight on the distribution

PUBLIC TOOL!

Binder, Bringmann, Gustafsson, AH 2103.01944

GOING BEYOND THE STANDARD APPROACH

- Home
- Downloads
- Contact

DRAKE

Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models.
DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium, Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]**

Currently, an user guide can be found in the Appendix A of this reference.
Please cite also quoted other works applying for specific cases.

v1.0 « Click here to download DRAKE

(March 3, 2021)

https://drake.hepforge.org

Applications:

DM relic density for any (user defined) model*

Interplay between chemical and kinetic decoupling

Prediction for the DM phase space distribution

Late kinetic decoupling and impact on cosmology

see e.g., 1202.5456

…

(only) prerequisite: *Wolfram Language* (or *Mathematica*)

*at the moment for a single DM species and w/o co-annihlations… but stay tuned for extensions!

DRAKE2

coming this winter*…

<u>New features:</u>

Two-component dark sectors
(also with potentially unstable states)

Freeze-out & <u>Freeze-in</u>

Automatic model generation
[linking to ***FeynRules etc.***]

<u>Improvements:</u>

Increased efficiency
[e.g. more extended use of compiled functions, parallelisation, matrix formulation]

Updated *user interface*

*hopefully… but already essentially available for sharing, just talk or DM me

Automatic Model Generation

When you have a *.mod* file, e.g. from *FeynRules*, then you need only…

Untitled-6.nb

```
(*****************************************************************)
InitModel[ModFile]
(*****************************************************************)
```

loading generic model file /Users/andrzejhryczuk/Library/Mathematica/Applications/FeynCalc/FeynArts/Models/Lorentz.gen

> $GenericMixing is OFF

generic model {Lorentz} initialized

loading classes model file /Users/andrzejhryczuk/Desktop/DRAKE–v1.6/FeynRules/Models/Sf/Sf/Sf.mod

> 5 particles (incl. antiparticles) in 4 classes

> $CounterTerms are ON

> 6 vertices

classes model {/Users/andrzejhryczuk/Desktop/DRAKE–v1.6/models/../FeynRules/Models/Sf/Sf/Sf} initialized

$$\begin{pmatrix} S(0) & \mathrm{mh} & \{\} & \mathrm{h} \\ S(1) & \mathrm{ms} & \{\} & \mathrm{s} \\ F(0) & \mathrm{mf} & \{\} & \mathrm{f} \\ F(1) & \mathrm{mc} & \{\} & \mathrm{chi} \end{pmatrix}$$

.mod file output

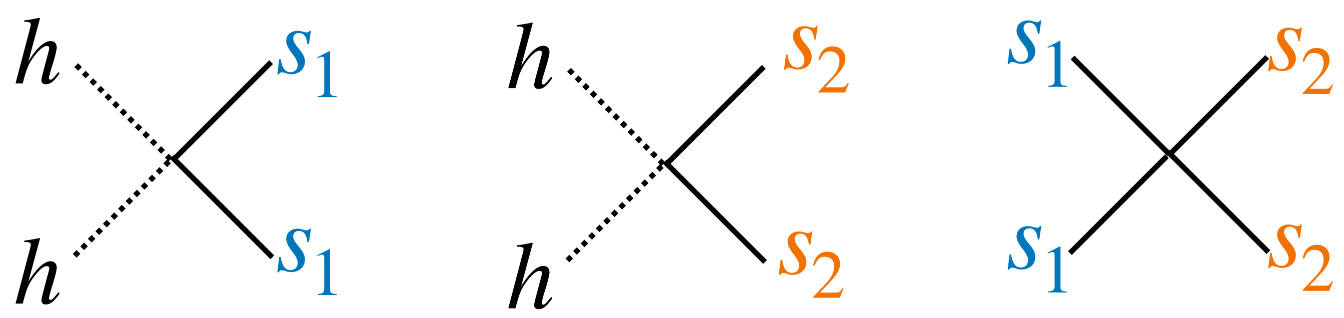
```
(*****************************************************************)
(*{FeynCalc symbol, mass parameter, spin, em.charge, own antiparticle, name, in dark sector?} *)
DrakeParticles["All"] = {
   (* DS particles (for which f(p) is traced *)
   {S[1], ms,  0,   0, True, "S", 1},
   {F[1], mc,  1/2, 0, True, "chi", 1},
   (* SM particles & equilibrium DS particles *)
   {S[0], mh,  0,   0, True, "h", 0},
   {F[0], mf,  1/2, 1, False, "f", 0}
};

(* list of the parameters of the model: masses as in the list above + couplings etc. *)
ModelParameters = {ms, mc, mf, lhs, ls, lhc, lsc};
(*****************************************************************)
```

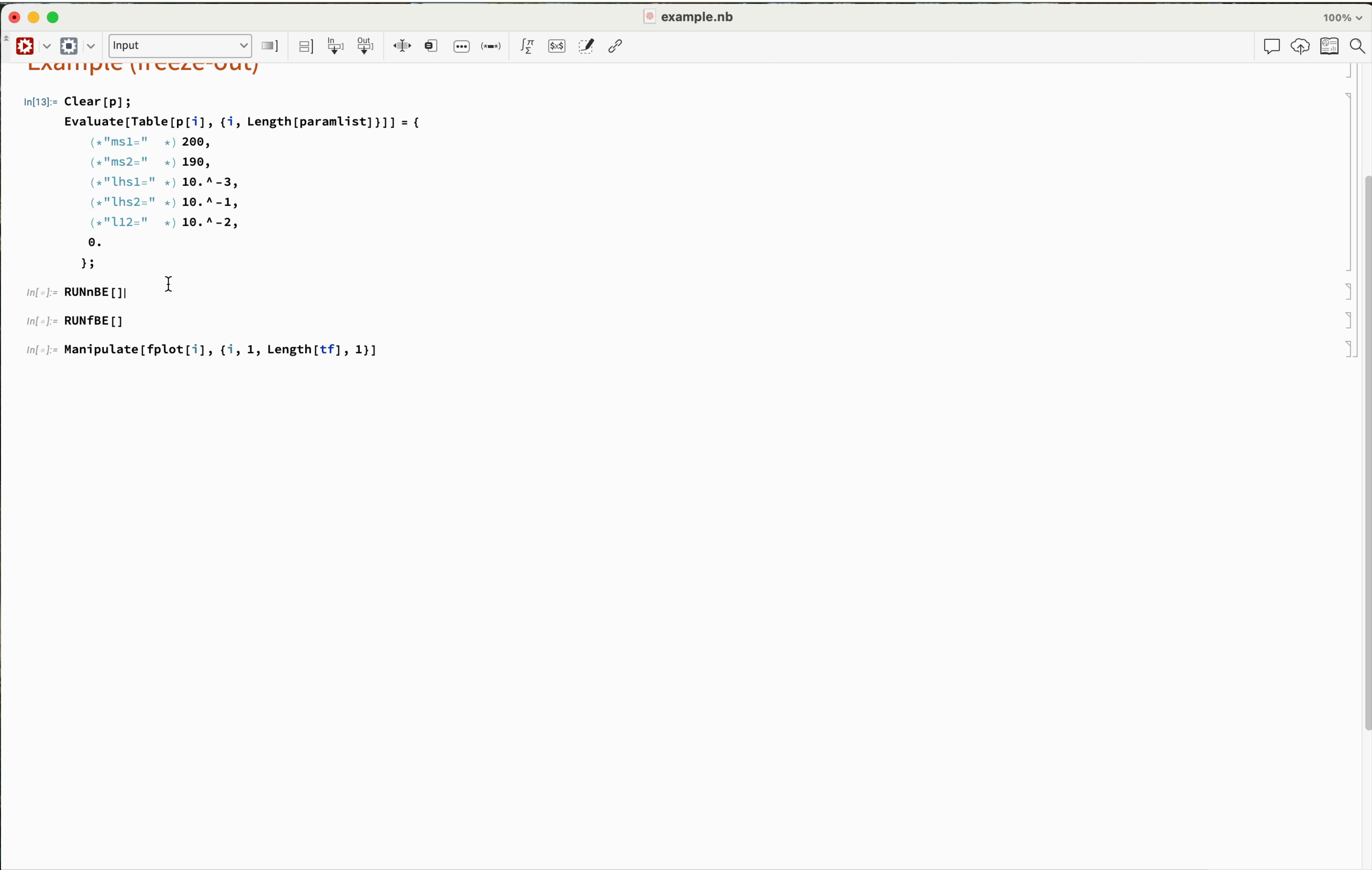
User translated to DRAKE2 input format

EXAMPLE:
RUN EXAMPLE NOTEBOOK

(Toy model of 2 real scalars coupled to the Higgs)



EXAMPLE:
RUN EXAMPLE NOTEBOOK



DRAKE2

UNDER THE HOOD

In its essence: DRAKE $\Leftrightarrow$ BE solvers + UI

Compiled into C (as much as possible)

Parallelisation of rate computations

Vectorisation

$$\frac{\mathrm{d}\vec{\mathbf{f}}}{\mathrm{d}x} = \mathrm{FM}\cdot\vec{\mathbf{f}} + \mathrm{EM}\cdot\vec{\mathbf{f}} + \vec{f}^{eq}(\mathrm{AM}\cdot\vec{f}^{eq}) - \vec{\mathbf{f}}\,(\mathrm{AM}\cdot\vec{\mathbf{f}}) + \alpha\Big[\Big(\big(\hat{A}\,\hat{\mathbf{f}}\big)\cdot\big(\vec{V}\,\vec{\mathbf{f}}\big)\Big)\cdot\vec{V} - \Big(\big(\hat{A}\,\vec{\mathbf{f}}\big)\cdot\vec{V}\Big)\cdot\big(\vec{V}\,\vec{\mathbf{f}}\big)\Big],$$

e.g. Annihilation matrix:

$$(\mathrm{AM}_{\mathtt{i}})_{i,k} \equiv \frac{g_{\chi_{\mathtt{i}}}}{(2\pi^2)x\tilde{H}}\mathrm{d}P\mathrm{d}Q_k^{\chi_{\mathtt{i}}}\,\mathrm{d}q\,W_k\,(p_k^{\chi_{\mathtt{i}}})^2\langle\sigma v_{\mathrm{m}\o\mathrm{l}}\rangle_\theta(p_i^{\chi_{\mathtt{i}}},p_k^{\chi_{\mathtt{i}}}),$$

e.g. Conversion matrix:

$$(\hat{A}_{\chi_1})_{ikj} \equiv \hat{\Theta}^{\chi_1}_{ikj}\hat{\Pi}^{\chi_1}_{ikj}/\,(E_{\chi_1}|\vec{p}_{\chi_1}|)_i\,,$$

Mathematica (or WL script) w/ full benefits

Pre-tabulation of rates

User friendly workflow

Functions for all physical quantities

Easily adjustable settings

Modularity & easy modding

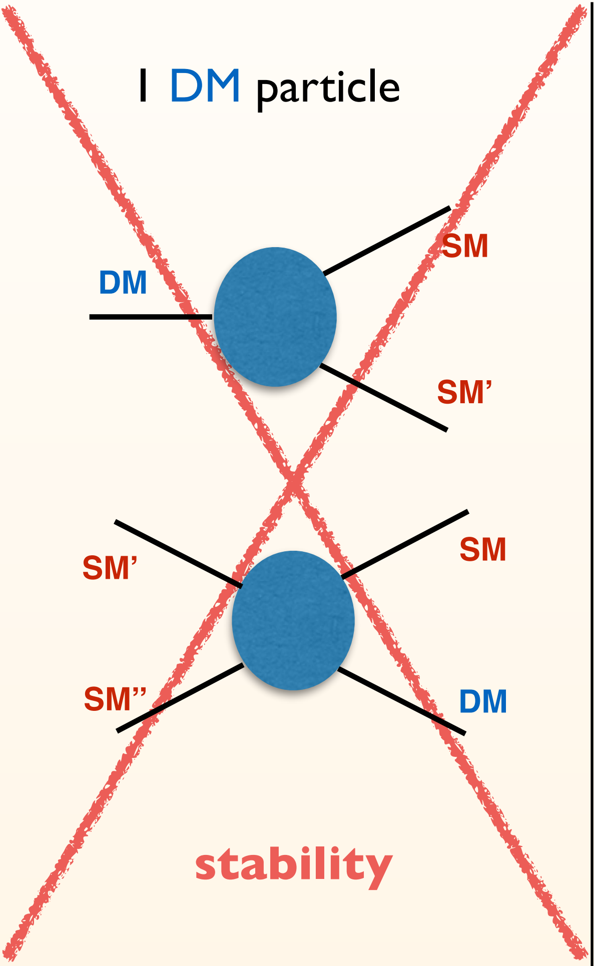
Example A:
Effect of conversion processes

What goes into C in general?

For now assume a minimal theory of SM + one DM field

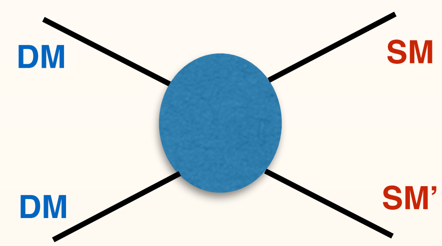
changing processes ⇒ number density

1 DM particle



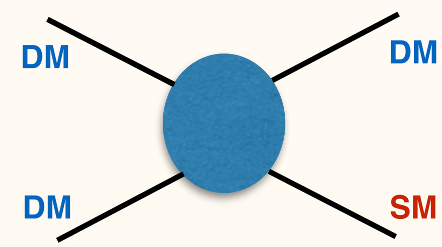
stability

2 DM particles



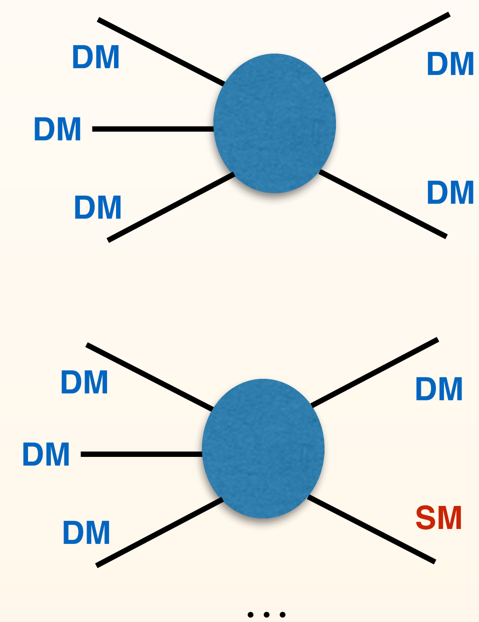
annihilation

3 DM particles



semi-annihilation

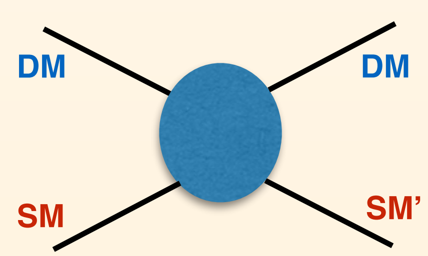
4+ DM particles



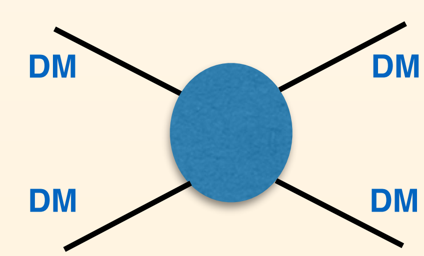
cannibalization

…

conserving processes ⇒ energy density



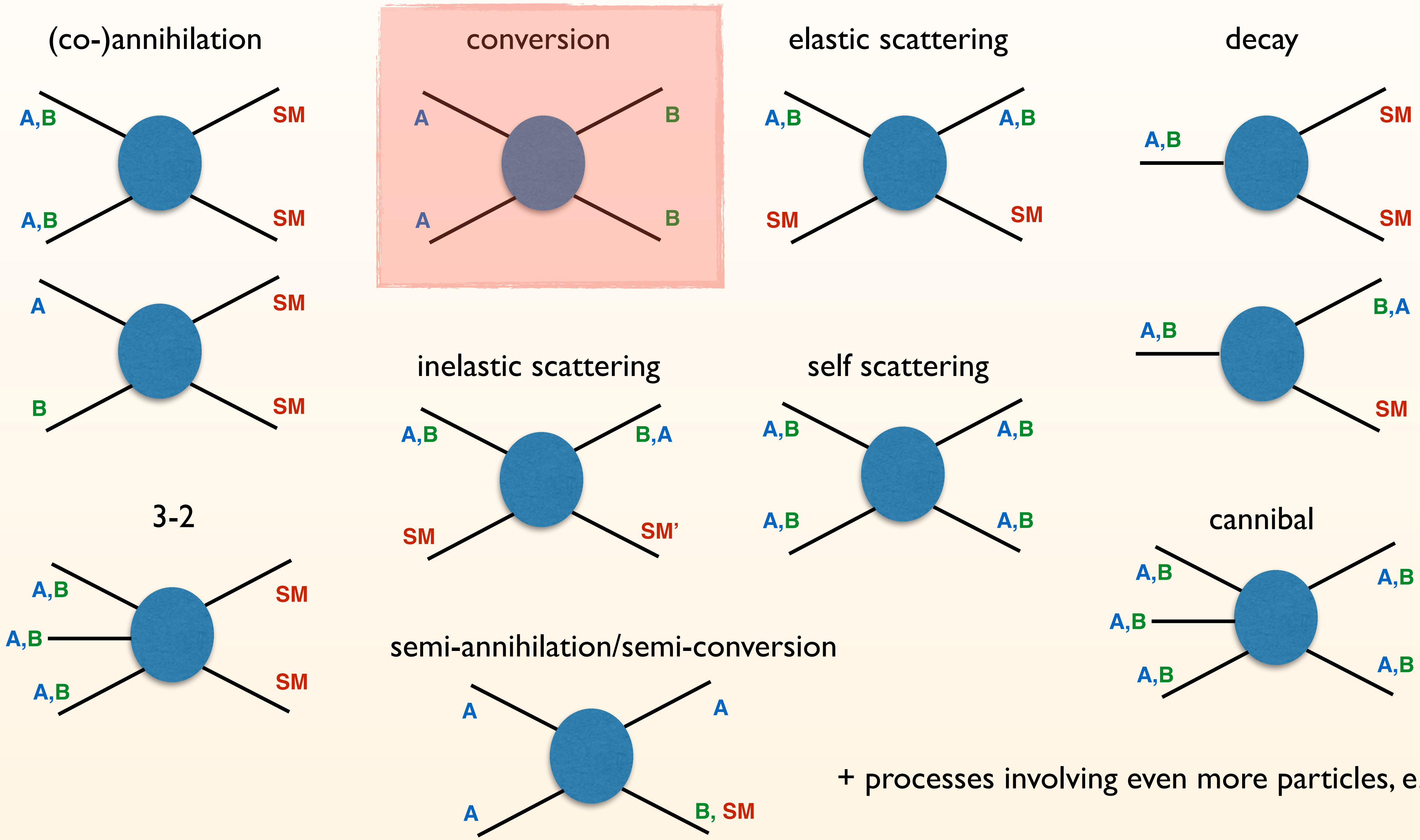
elastic scattering



self scattering

What if a non-minimal scenario?

A,B — two different dark sector states (at least one needs to be stable)

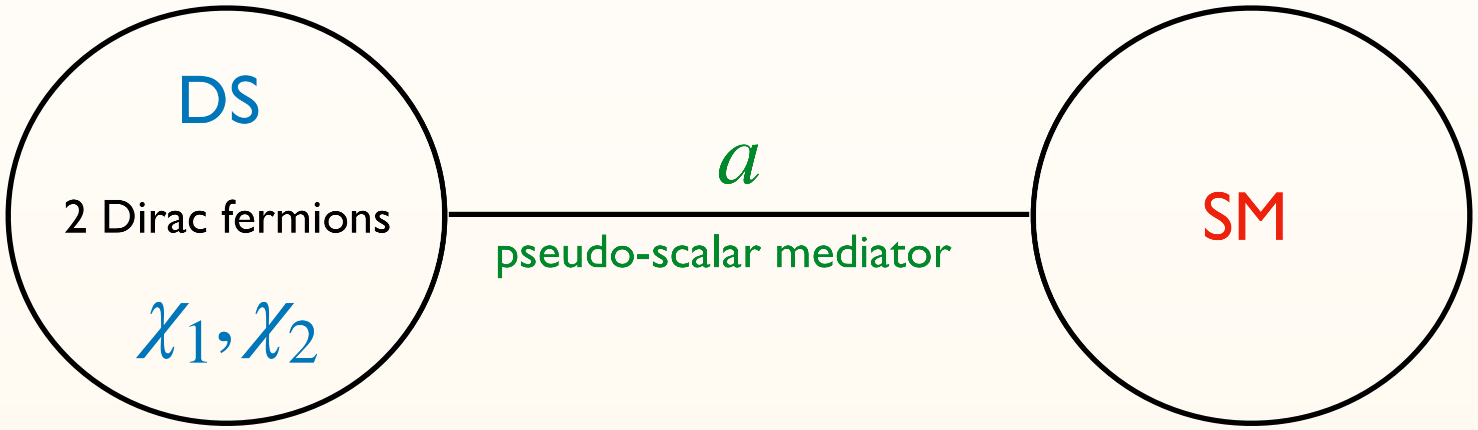


+ processes involving even more particles, e.g. $2 \leftrightarrow 4\ldots$

Note: some of these processes affect not only # density, but also strongly modify the energy distribution of DM particles!

RESULTS: THE MODEL

Let's take one <u>of the simplest</u> two-component DM models:



Parametrically:

$$\sigma_{11\to SM} \sim \sigma_{1SM\to 1SM} \sim \lambda_1^2 \lambda_y^2$$

$$\sigma_{22\to SM} \sim \sigma_{2SM\to 2SM} \sim \lambda_2^2 \lambda_y^2$$

$$\sigma_{11\to 22} \sim \lambda_1^2 \lambda_2^2$$

$$\Downarrow$$

$$\mathcal{L}_{int} = -\sum_{i=1,2} i\lambda_i \, a \, \bar{\chi}_i \gamma^5 \chi_i - i\lambda_y \frac{m_f}{v} a \, \bar{f} \gamma^5 f$$

coupled directly to SM fermions in a MFV way

Varying:

$\lambda_1 \to c\,\lambda_1$
$\lambda_2 \to c\,\lambda_2$
$\lambda_y \to \lambda_y / c$

Keeps everything fixed, <u>except</u> conversions

New fields: χ_1, χ_2, a

New params: m_1, m_2, m_a, $\lambda_1, \lambda_2, \lambda_y$

Main motivation (for models in the literature with pseudo-scalar mediator):

Evasion of the direct detection bounds… while giving strong signal in indirect detection, in particular for explaining the Galactic Centre excess

(see e.g. „Coy DM")

C. Boehm et al. 1401.6485,

Motivational Plot

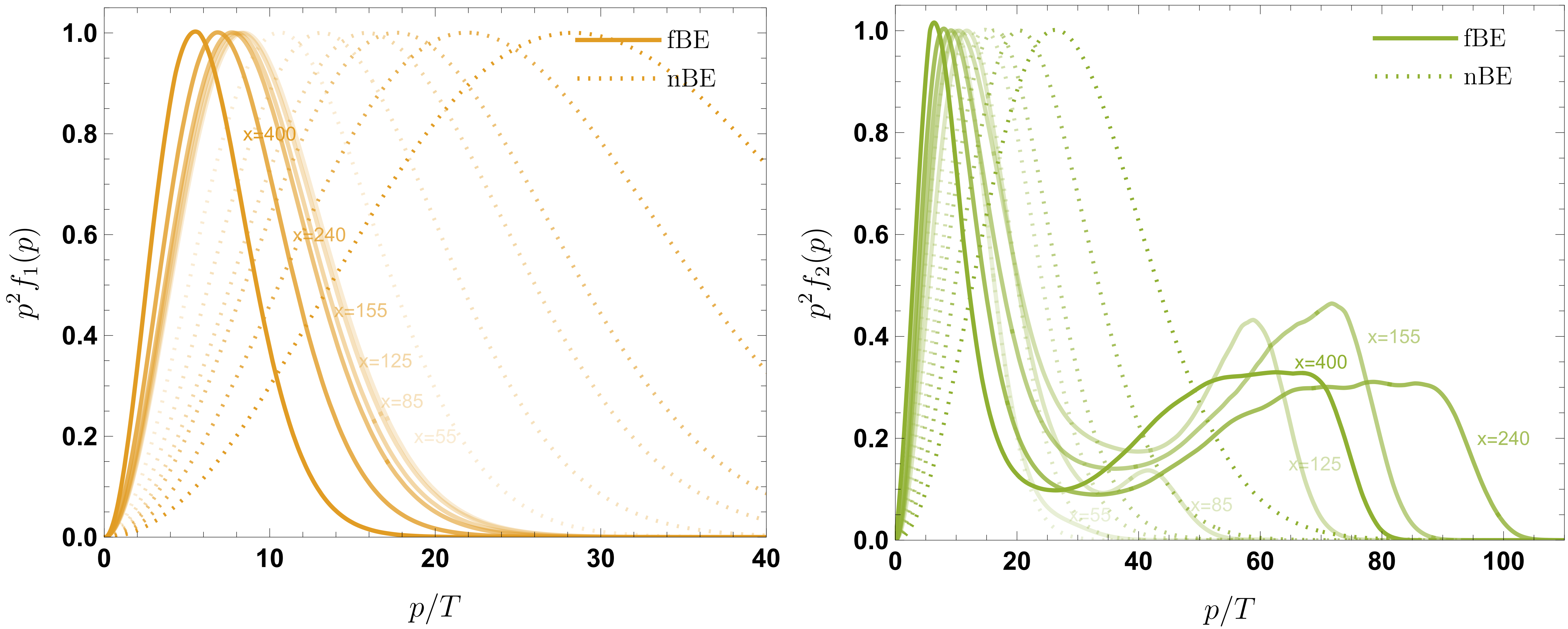
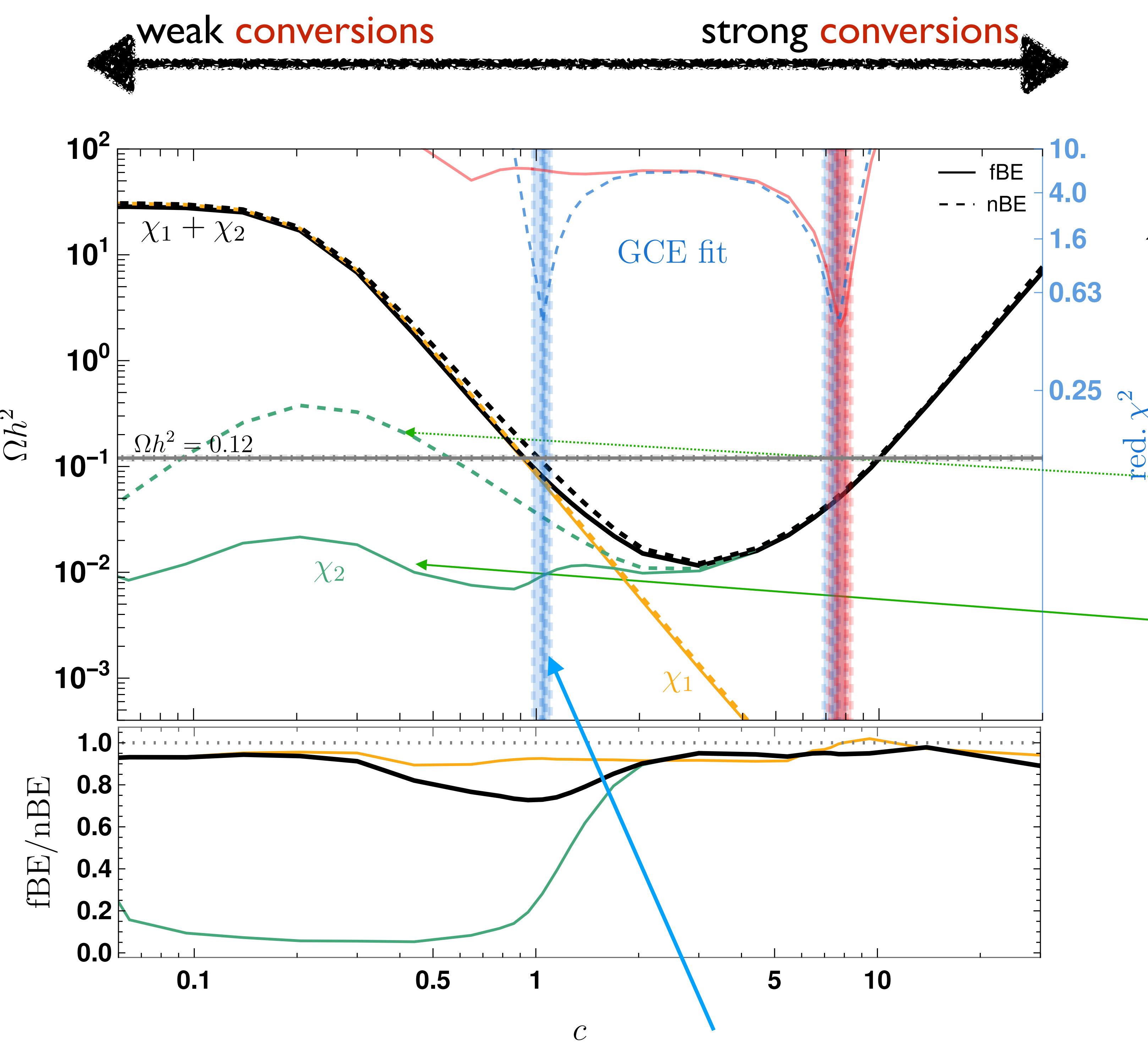


Figure 1: Time snapshots of the evolution of the normalised momentum distributions for χ_1 (left) and χ_2 (right) in p/T, for the benchmark point showcasing the interplay of conversions and resonant annihilations with early kinetic decoupling. The solid lines show the particle distribution functions f_{χ_i} while the dotted lines show the corresponding equilibrium distributions.

Results: conversion impact

Varying: $\lambda_1 \to c\,\lambda_1$ $\lambda_2 \to c\,\lambda_2$ $\lambda_y \to \lambda_y/c$ Only conversions change!



total relic abundance

only number density (nBE) — *as in micrOMEGAs*

phase space calculation (fBE)

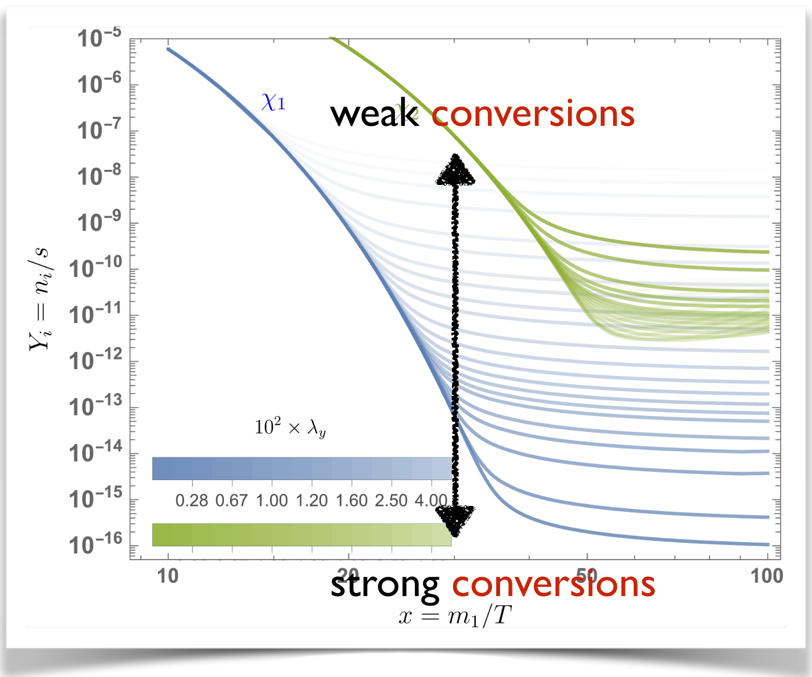
This region for GCE disappears!

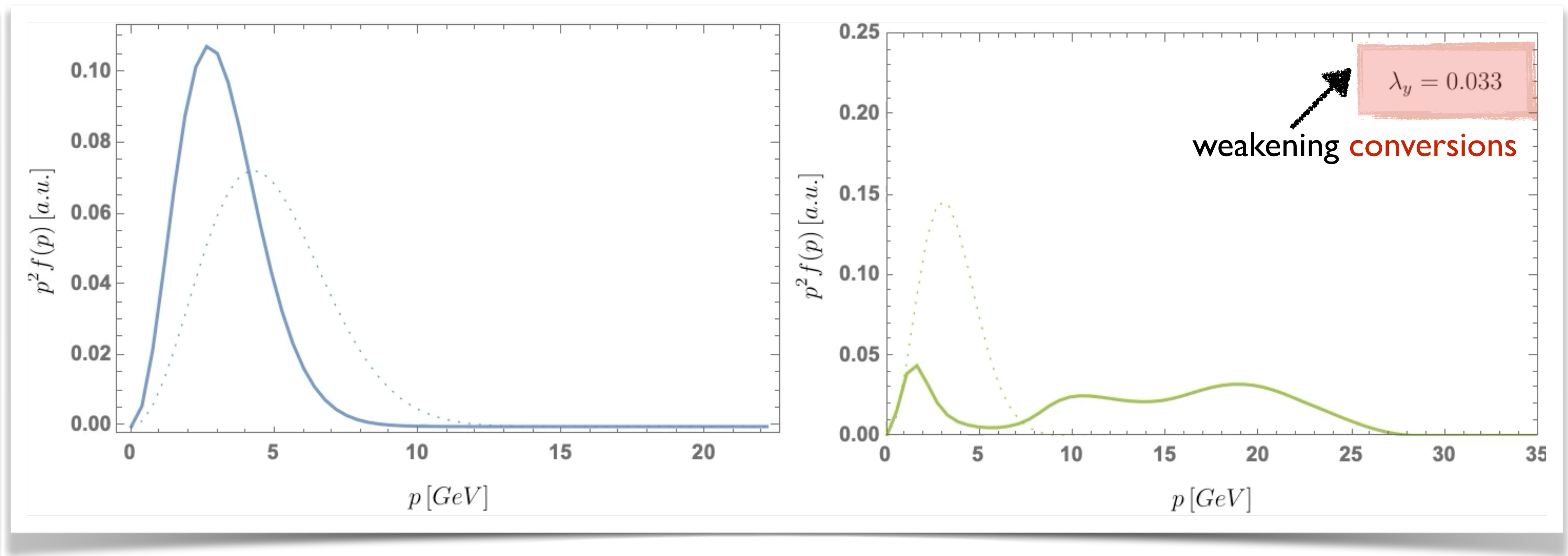
Weak conversions lead to larger discrepancy between nBE and fBE calculations!

cf. Kamada et al. 2111.06808

RESULTS: CONVERSION IMPACT

weaker conversions $\Rightarrow$ less depletion of χ_1 $\Rightarrow$ around χ_2 freeze-out more χ_1 in the plasma $\Rightarrow$ larger distortion of thermal shape





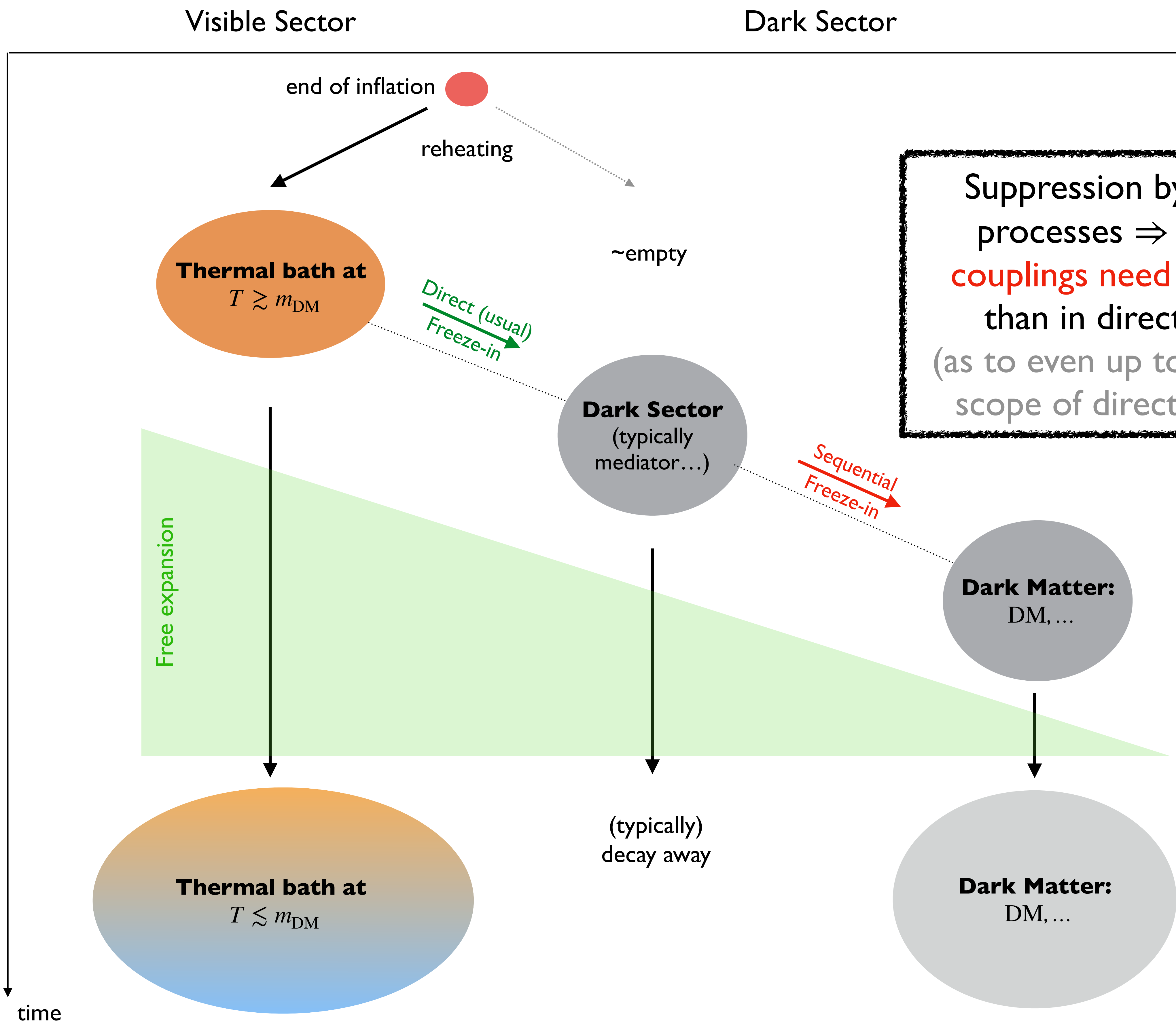
Conversions are ubiquitous in multicomponent models… but not the only processes affecting the distributions:

- **decays** of heavier to lighter dark sector states — A.H. & Laletin 2204.07078 (see also Beauchesne & Chiang 2401.03657)
- inelastic scatterings
- semi-annihilations — A.H. & Laletin 2104.05684
- cannibal $(3 \leftrightarrow 2)$ — Cervantes & A.H. 2407.12104

bits useful for next DRAKE release

Example B:
(Sequential) freeze-in

What is Sequential Freeze-in?



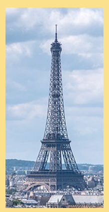
Suppression by two weak processes ⇒ respective couplings need to be larger than in direct freeze-in (as to even up to being within scope of direct detection!)

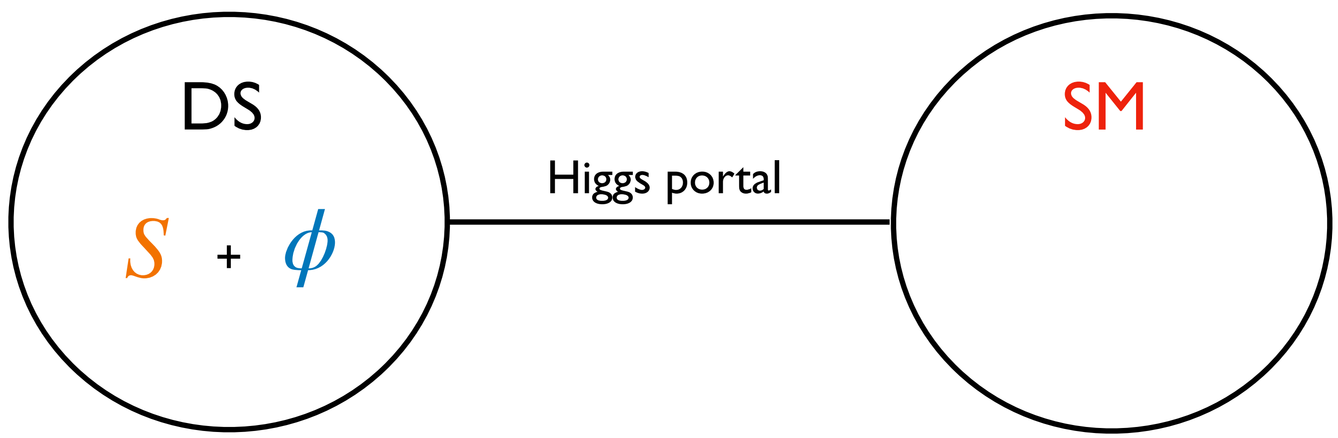
Hambye *et al.* 1908.09864
Belanger *et al.* 2005.06294

…but can also be a secluded dark sector

A Tale of Two Scalars

Postulate two new scalars (singlets w.r.t SM gauge group):

|  |	S	$\mathbb{Z}_2$-symmetric	stable	dark matter	feeble int. with SM
					$\wedge$
|  |	ϕ	~~$\mathbb{Z}_2$~~ explicitly broken	unstable	"mediator"	feeble int. with SM



$$V \supset -A\phi H^\dagger H - \frac{\lambda_{h\phi}}{2}\phi^2 H^\dagger H - \frac{\lambda_{Sh}}{2} S^2 H^\dagger H - \frac{1}{4}\lambda_{S\phi} S^2 \phi^2$$

mediator-Higgs DM-Higgs DM-mediator

mediator-Higgs mixing

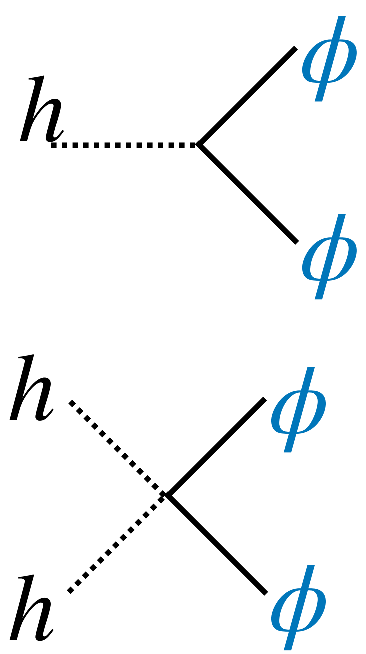
$$\sin\theta = \frac{Av}{m_h^2 - m_\phi^2}\left(1 - \frac{\lambda_{h\phi} v^2}{2m_\phi^2}\right)$$

Such models are not unheard of. Most similar in the literature:

…; Wang, Han '14; Claude, Godfrey '21; …

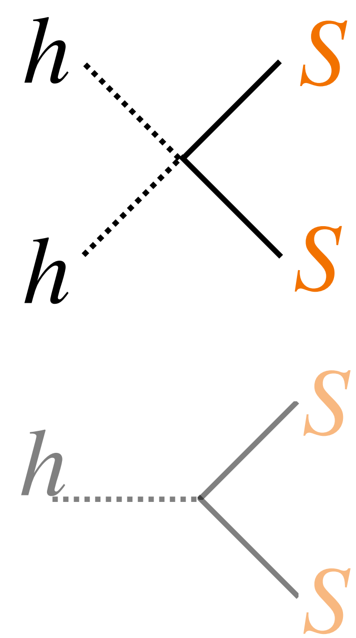
A Tale of Two Scalars

mediator freeze-in:



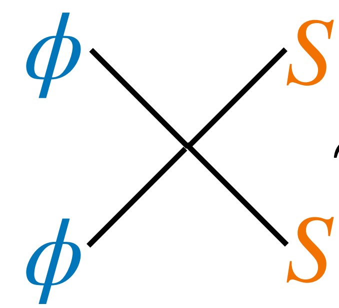
$\sim \lambda_{h\phi}$

DM freeze-in:



$\sim \lambda_{Sh}$

sequential freeze-in:



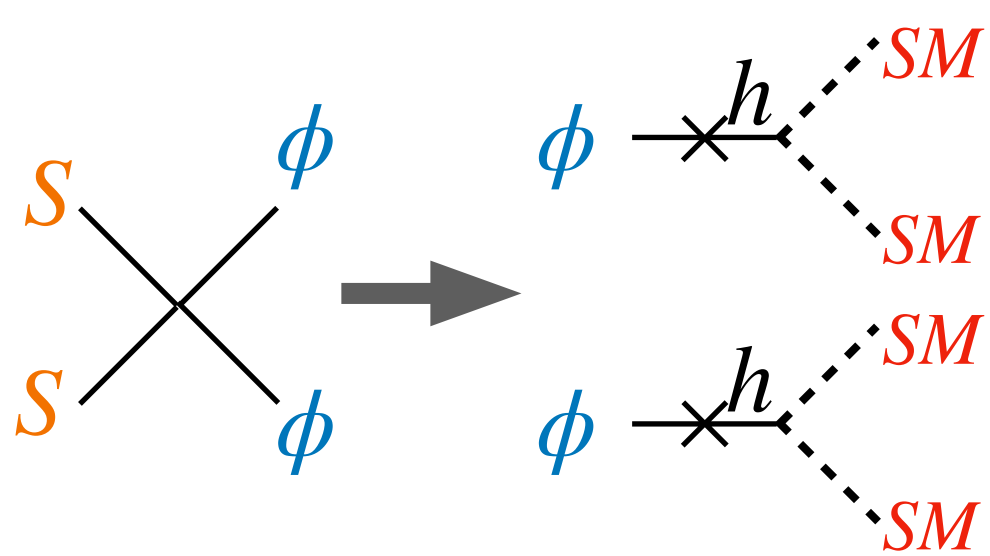
$\sim \lambda_{S\phi}$

Typical hierarchy:

$\lambda_{S\phi} \ggg \lambda_{h\phi} \gg \lambda_{Sh}$

Freeze-out-like — Freeze-in-like

Indirect detection through a cascade decay (iff $m_S > m_\phi$):



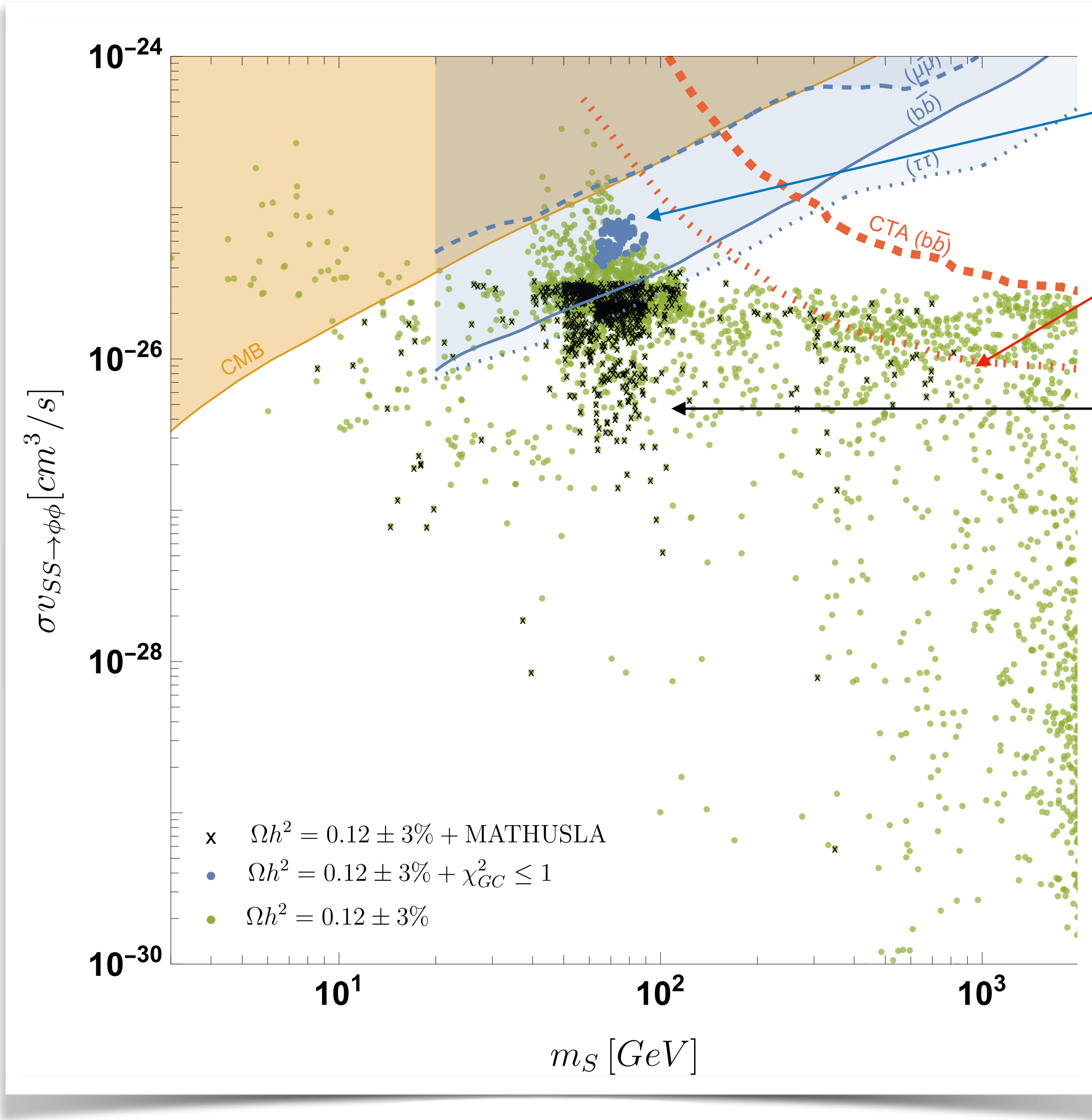
ID signal = requirement of sub-threshold sequential freeze-in

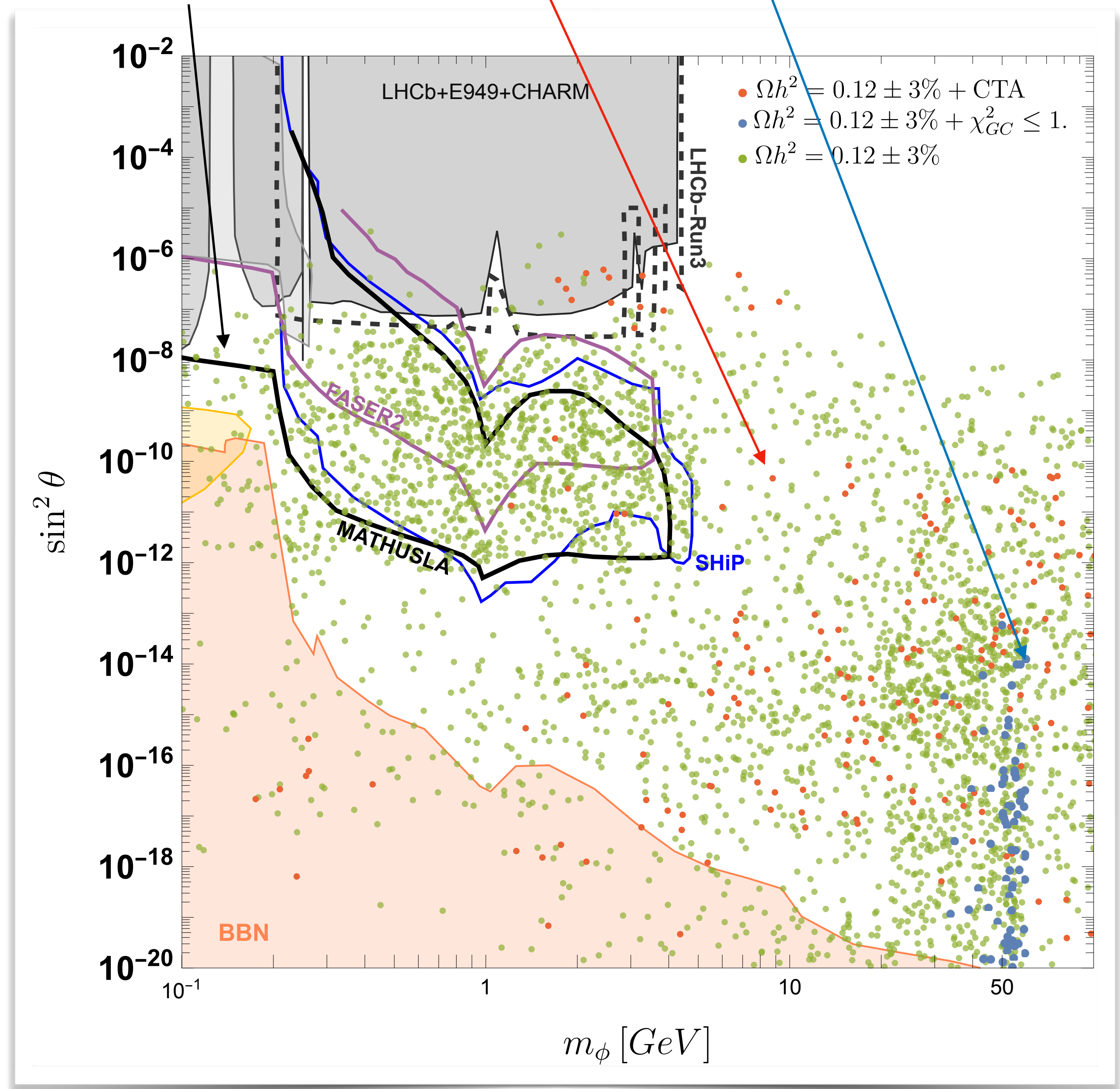
Scan Results: ID and forward physics

Points giving good fit to GCE

Points within (optimistic) reach of CTA

within reach of MATHUSLA





<u>All points satisfy relic density constraint</u>

Scan driven towards regions that are covered by any of the experiments

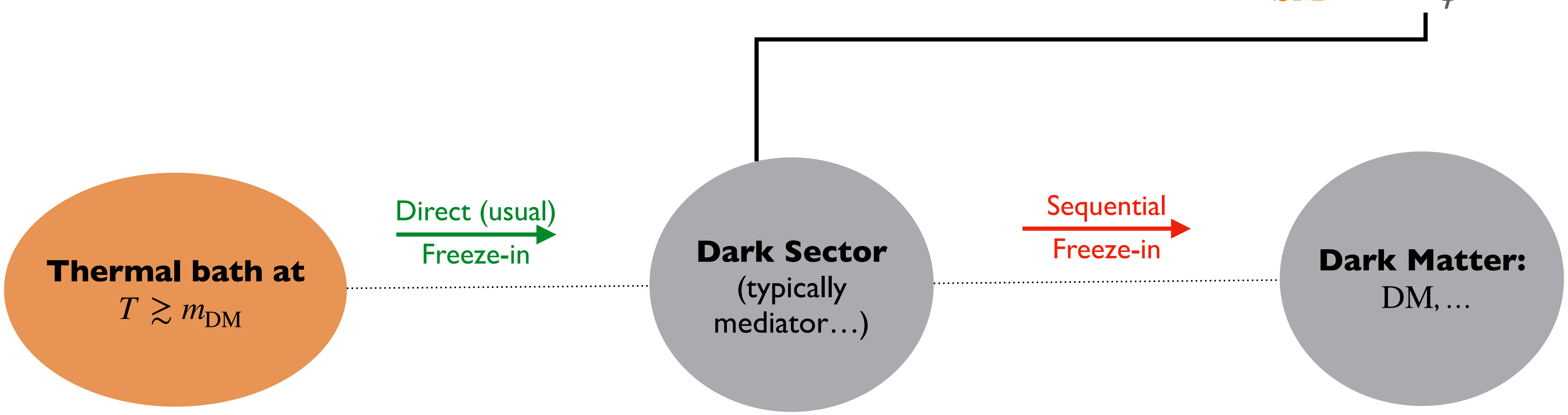
A SECOND LOOK ON Ωh^2

The relic density was the main constraint of the scan. It was obtained by solving the Boltzmann equation for number densities of ϕ and S (nBE)
(as e.g. micrOMEGAs or DarkSUSY would)

But wait… isn't relic abundance *(freeze-in or freeze-out)* dependent on the T of the thermal bath it is produced from?

$\Downarrow$

Which temperature is relevant for sequential freeze-in: T_{SM} or T_ϕ?



…OK, so it looks like we need to trace T_ϕ as well!

THIS IS REMINISCENT OF…

AH, Laletin 2104.05684
(see also Bringmann et al. 2103.16572)

Consider process of production that is the inverse of semi-annihilation:

$$\chi\phi \to \chi\chi$$

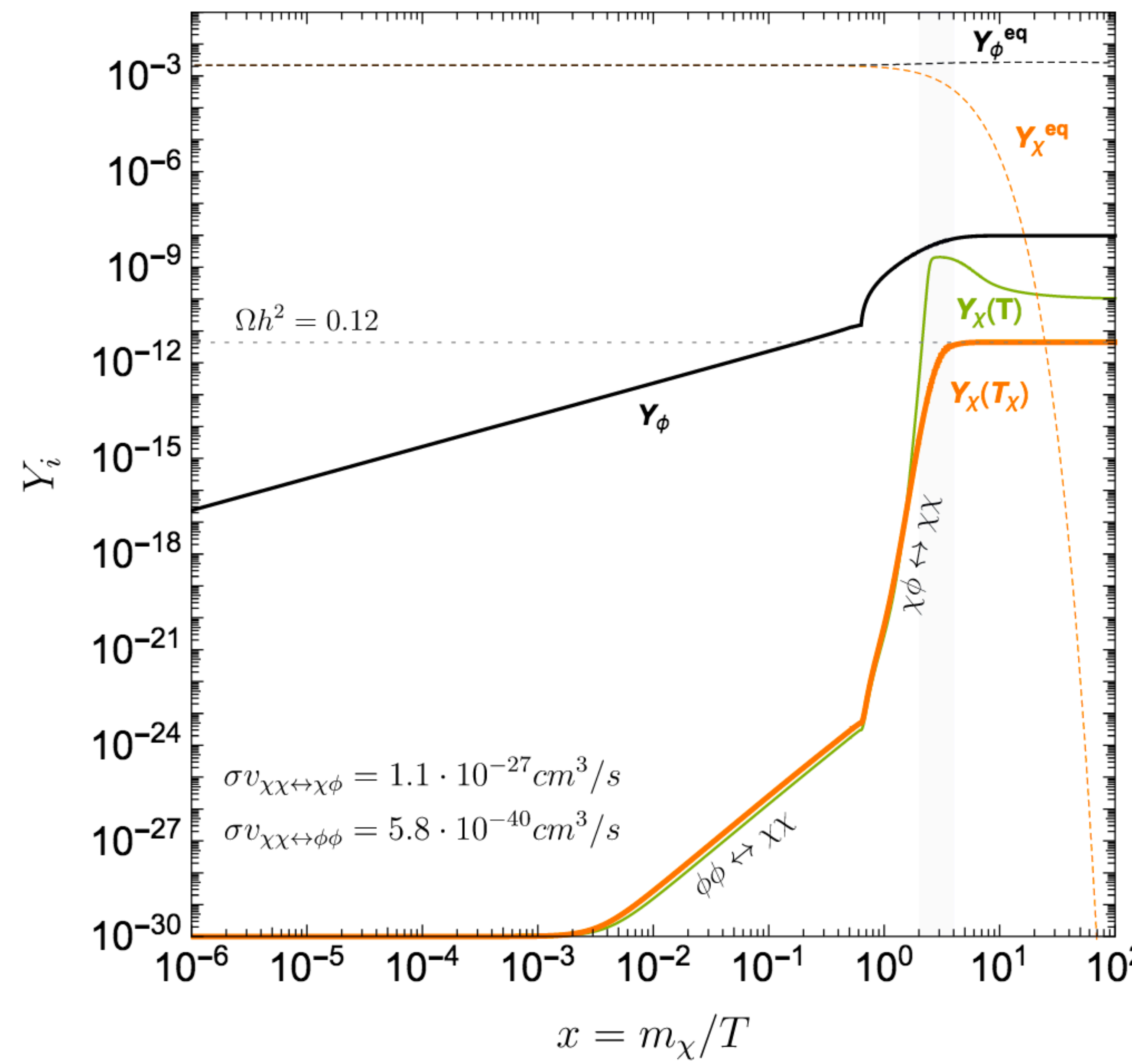
DM

mediator or a SM state

What is different?
(from the decay/annihilation freeze-in)

- The production rate is proportional to the DM density. (Smaller initial abundance → larger cross section…)

- Semi-production modifies the energy of DM particles in a non-trivial way, so the temperature evolution can affect the relic density



System of cBE for Y_i and T_i

This we obtain through equations for the 0th and 2nd moment of the BE:

$$\frac{Y_i'}{Y_i} = \frac{m_i}{x\tilde{H}} C_i^0 \,,$$
$$\frac{y_i'}{y_i} = \frac{m_i}{x\tilde{H}} C_i^2 - \frac{Y_i'}{Y_i} + \frac{H}{x\tilde{H}} \frac{\langle p^4/E_i^3 \rangle}{3T_i}$$

where $y \equiv \frac{m_\chi T_\chi}{s^{2/3}}$ is a parameter that describes

the 'temperature' $T_\chi \equiv \frac{g_\chi}{3n_\chi} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{p^2}{E} f_\chi(p)$

The collision term is also given by its moments:

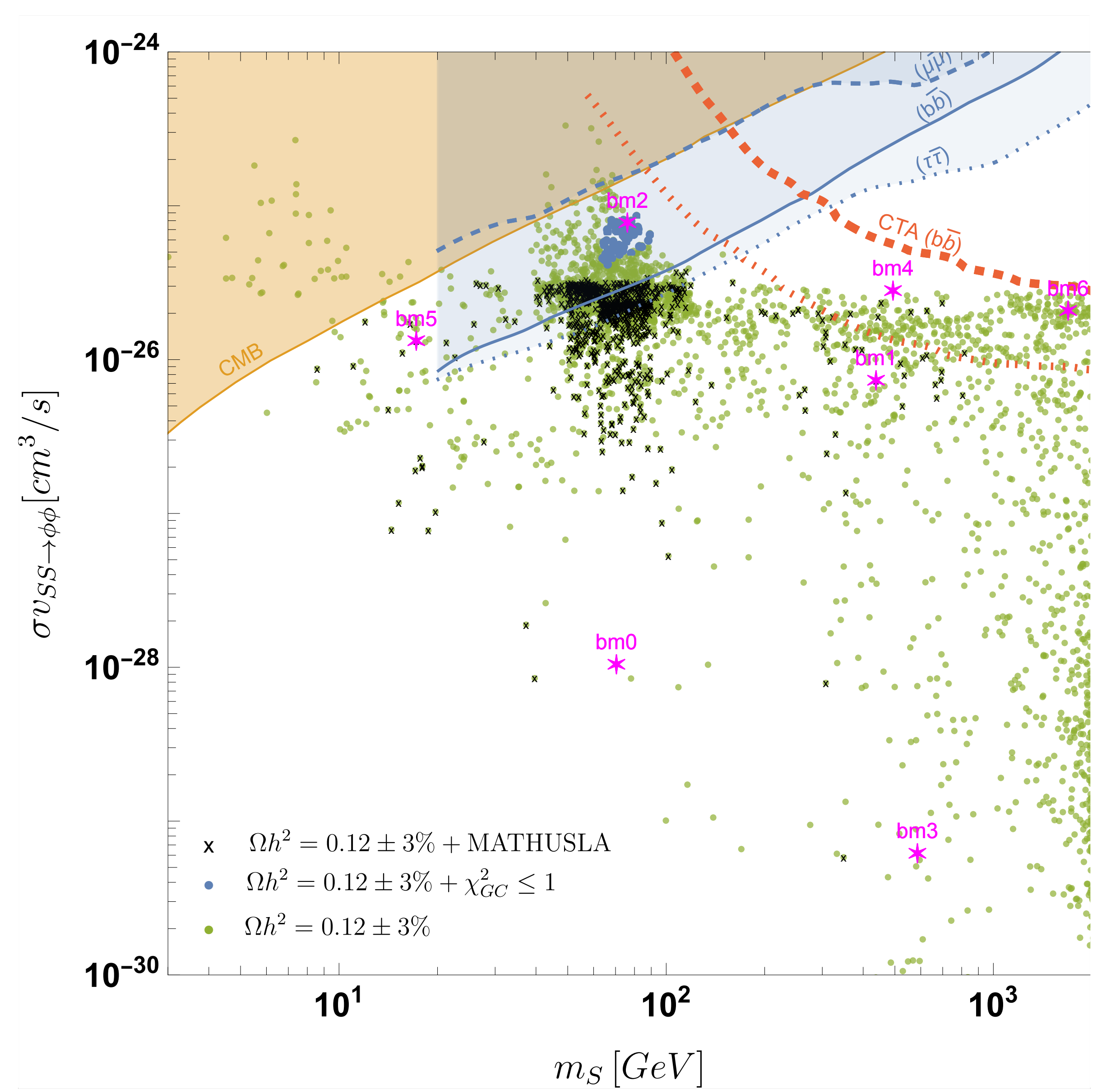
contains all scatterings and production/annihilation processes

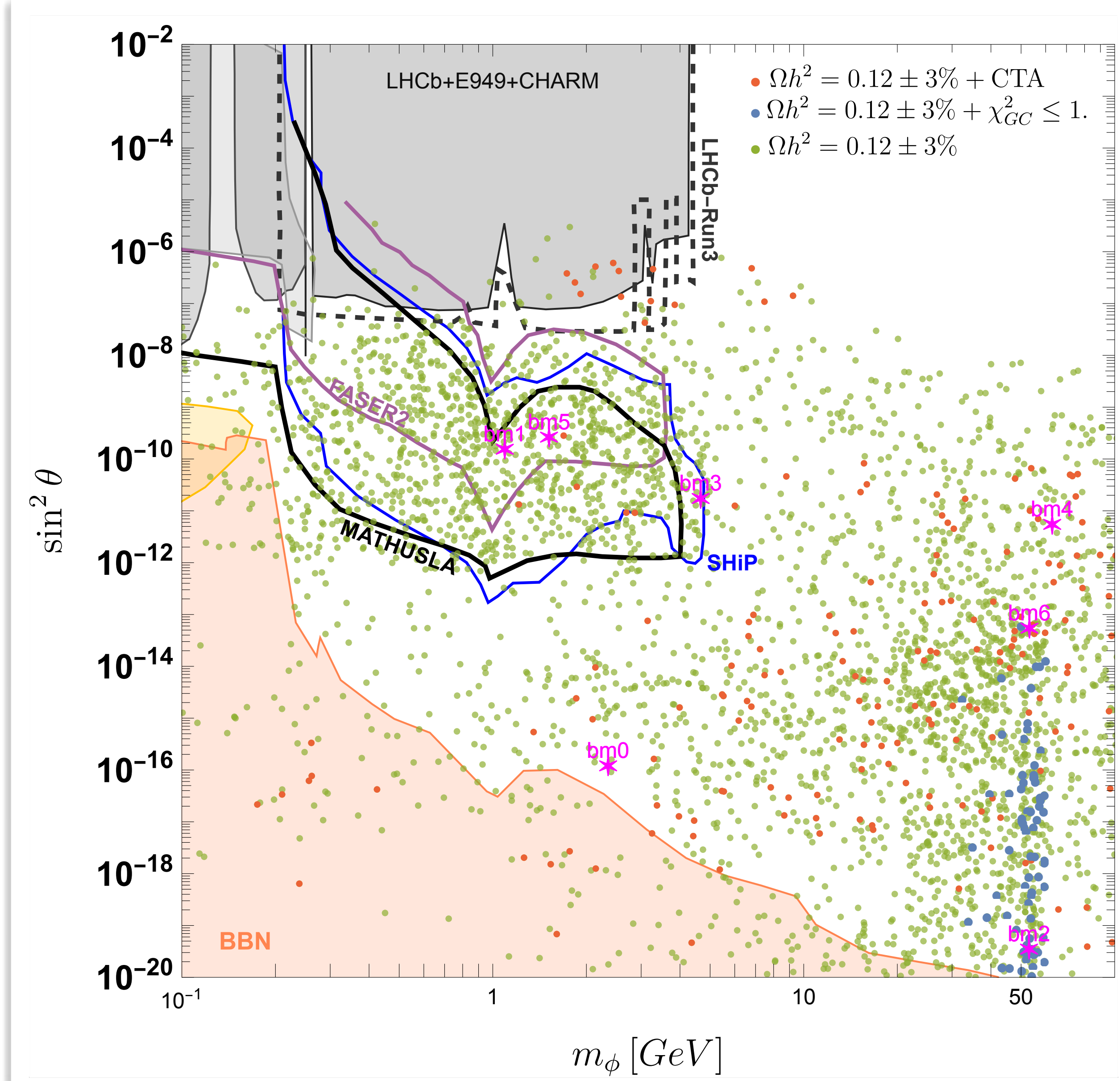
$$C_i^0 \equiv \frac{g_i}{m_i n_i} \int \frac{d^3 p}{(2\pi)^3 E_i} C[f_i] \,, \qquad C_i^2 \equiv \frac{g_i}{3 m_i n_i T_i} \int \frac{d^3 p}{(2\pi)^3 E_i} \frac{p^2}{E_i} C[f_i]$$

In our model we got 4 equations for: Y_S, T_S, Y_ϕ, T_ϕ

Implementation of such capability [together with fBE system, giving also evolution of the $f(p)$] is a part of update in the new version of DRAKE2

BENCHMARKS





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BM0	2.35	70.4	1.09×10^{-8}	1.67×10^{-13}	5.98×10^{-11}	0.00298	0.113	0.110	−1.96	direct FI
BM1	1.09	438.	1.24×10^{-5}	3.56×10^{-11}	3.72×10^{-13}	0.155	0.124	0.0205	−83.5	seq. FI/dark FO + MATHUSLA
BM2	53.0	76.1	1.87×10^{-10}	3.51×10^{-7}	1.96×10^{-11}	0.104	0.115	0.0199	−82.7	dark FO + best GCE fit
BM3	4.66	586.	4.15×10^{-6}	8.62×10^{-11}	4.32×10^{-15}	0.00603	0.0971	0.000883	−99.1	seq. FI
BM4	63.0	494.	2.34×10^{-6}	1.08×10^{-15}	2.70×10^{-6}	0.344	0.0902	0.0503	−44.2	dark FO/co−decay + CTA
BM5	1.52	17.2	1.62×10^{-5}	1.30×10^{-9}	4.46×10^{-9}	0.00823	0.110	0.0555	−49.5	co−decay + MATHUSLA
BM6	53.2	1.69×10^{3}	2.33×10^{-7}	5.14×10^{-8}	1.16×10^{-7}	1.01	0.119	0.0571	−51.9	dark FO + CTA

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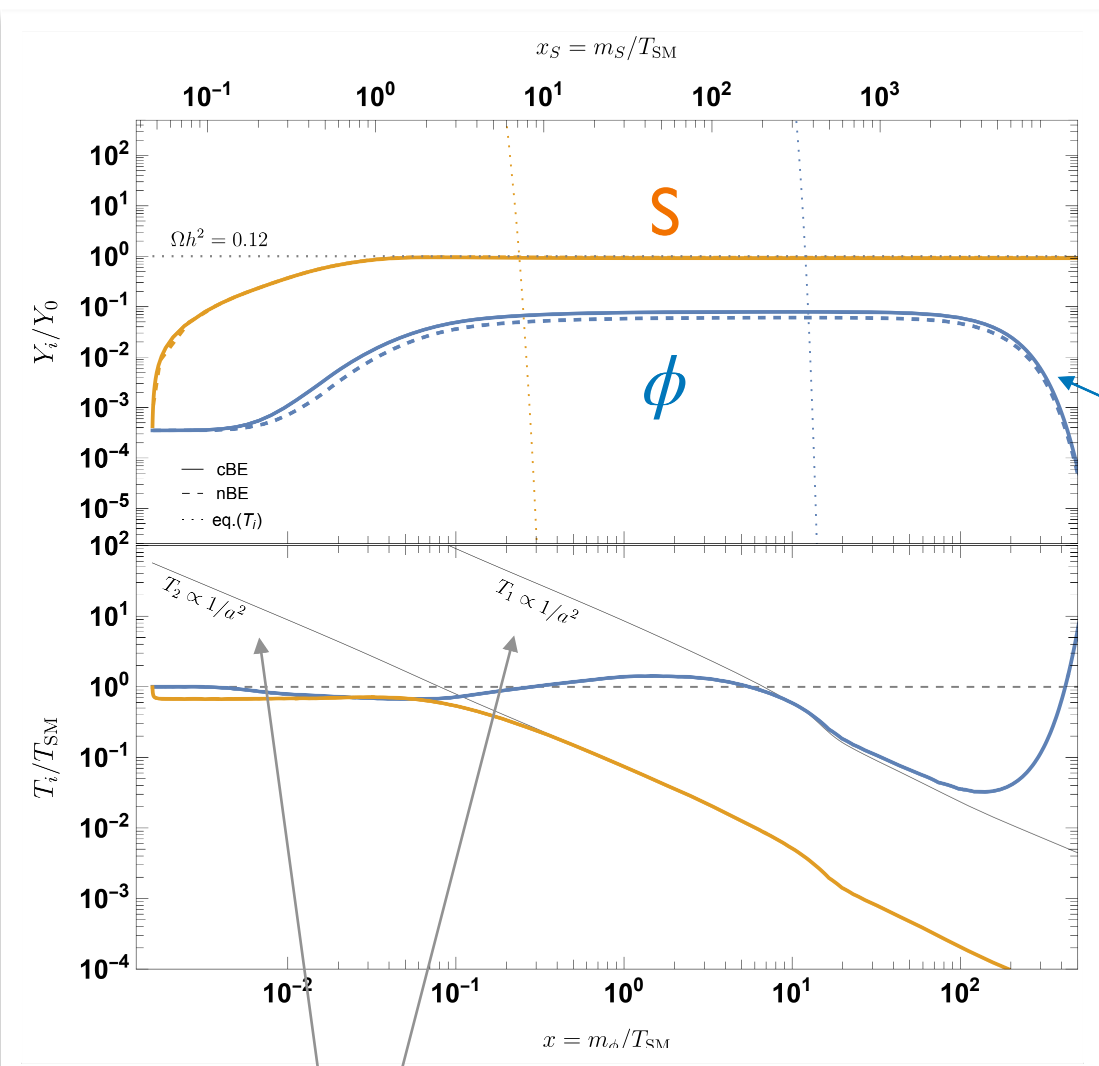

Free scaling after departure from LTE

yield (abundance)

Ratio of S and ϕ temperatures to the SM plasma one

Simple point to keep in mind as a baseline

Relatively small $\lambda_{S\phi}$ means both S and ϕ evolve separately

In the end ϕ decays

Very mild cBE effect

BENCHMARKS

Name	m_ϕ	m_S	θ	$\lambda_{h\phi}$	λ_{hS}	$\lambda_{S\phi}$	$(\Omega h^2)_{\rm nBE}$	$(\Omega h^2)_{\rm cBE}$	change [%]	description
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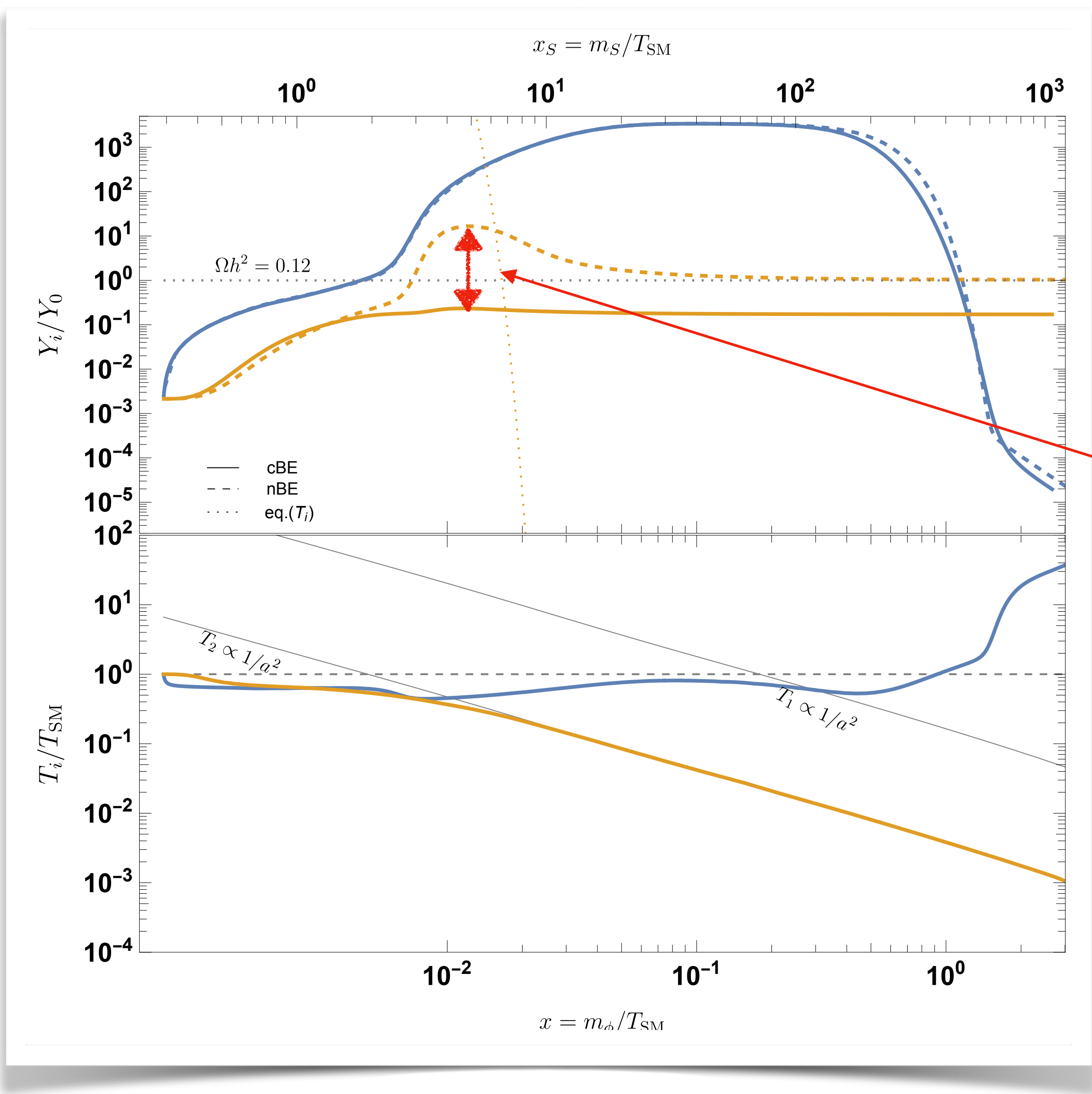

Hierarchy of $\lambda_{h\phi} \gg \lambda_{hS}$ and $m_S \gg m_\phi$ means freeze-in is sequential, followed by (mild) annihilation due to large $\lambda_{S\phi}$

Large change due to cBE:
lower T_ϕ + large threshold from ϕ to S suppresses sequential freeze-in!

This point lies within reach of MATHUSLA, SHiP and FASER2

BENCHMARKS

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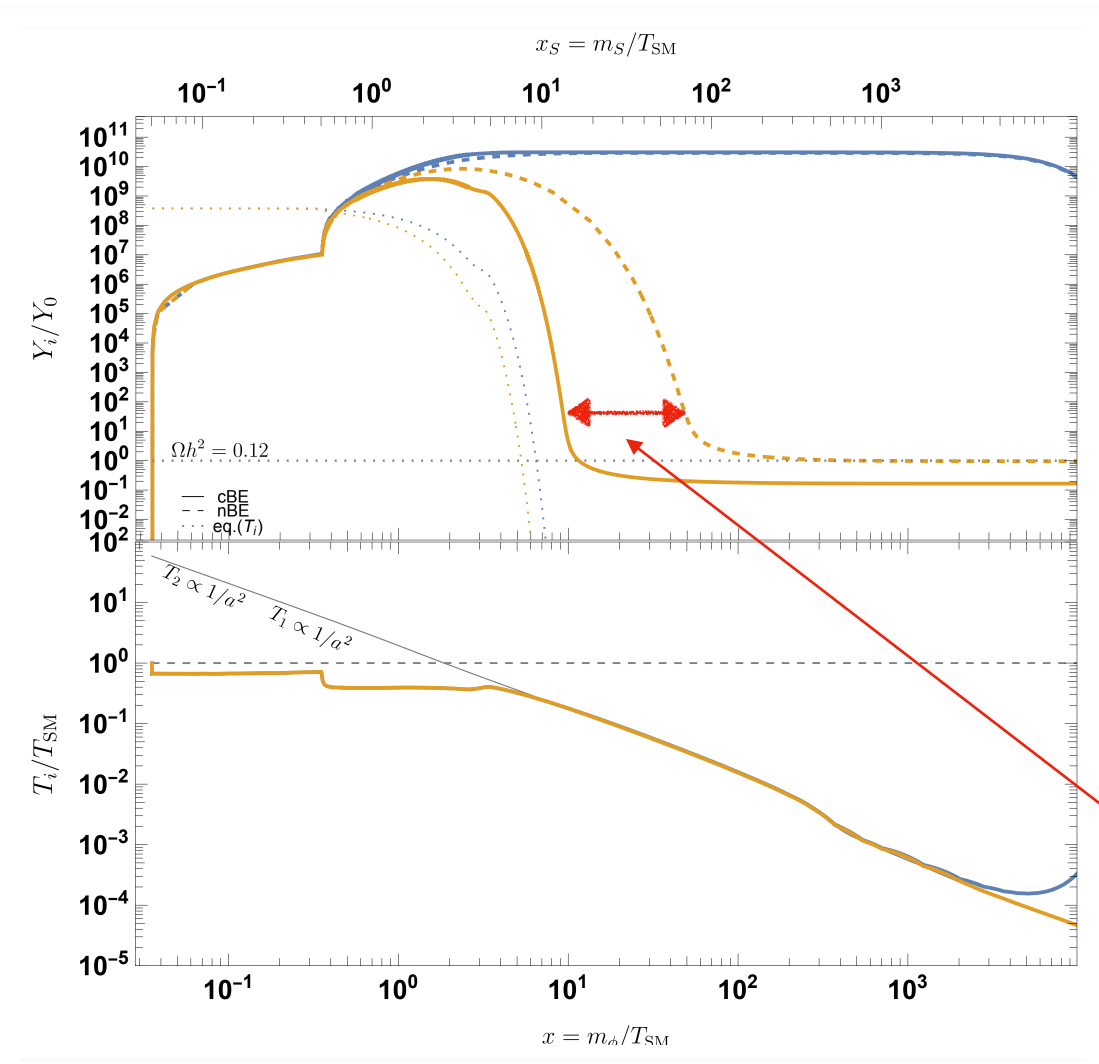
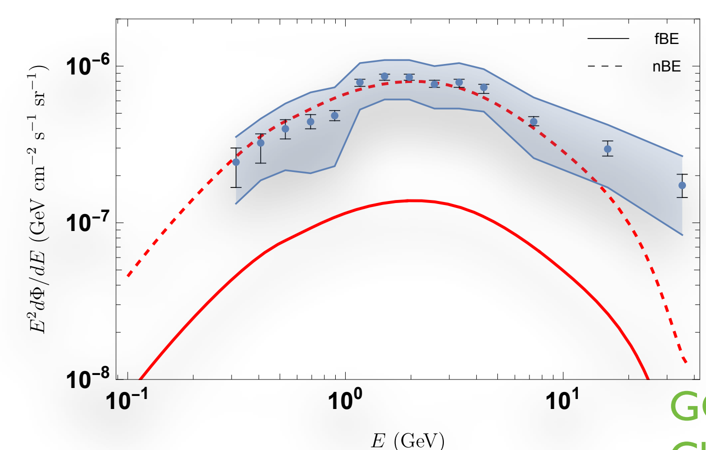


Best fit point to the GCE found in the scan:



GCE analysis from Cholis et al. '22

Mostly dark freeze-out from a thermal bath with $T_S \approx T_\phi < T_{SM}$

change in Ωh^2 due sooner freeze-out

BENCHMARKS

Name	m_ϕ	m_S	θ	$\lambda_{h\phi}$	λ_{hS}	$\lambda_{S\phi}$	$(\Omega h^2)_{\rm nBE}$	$(\Omega h^2)_{\rm cBE}$	change [%]	description
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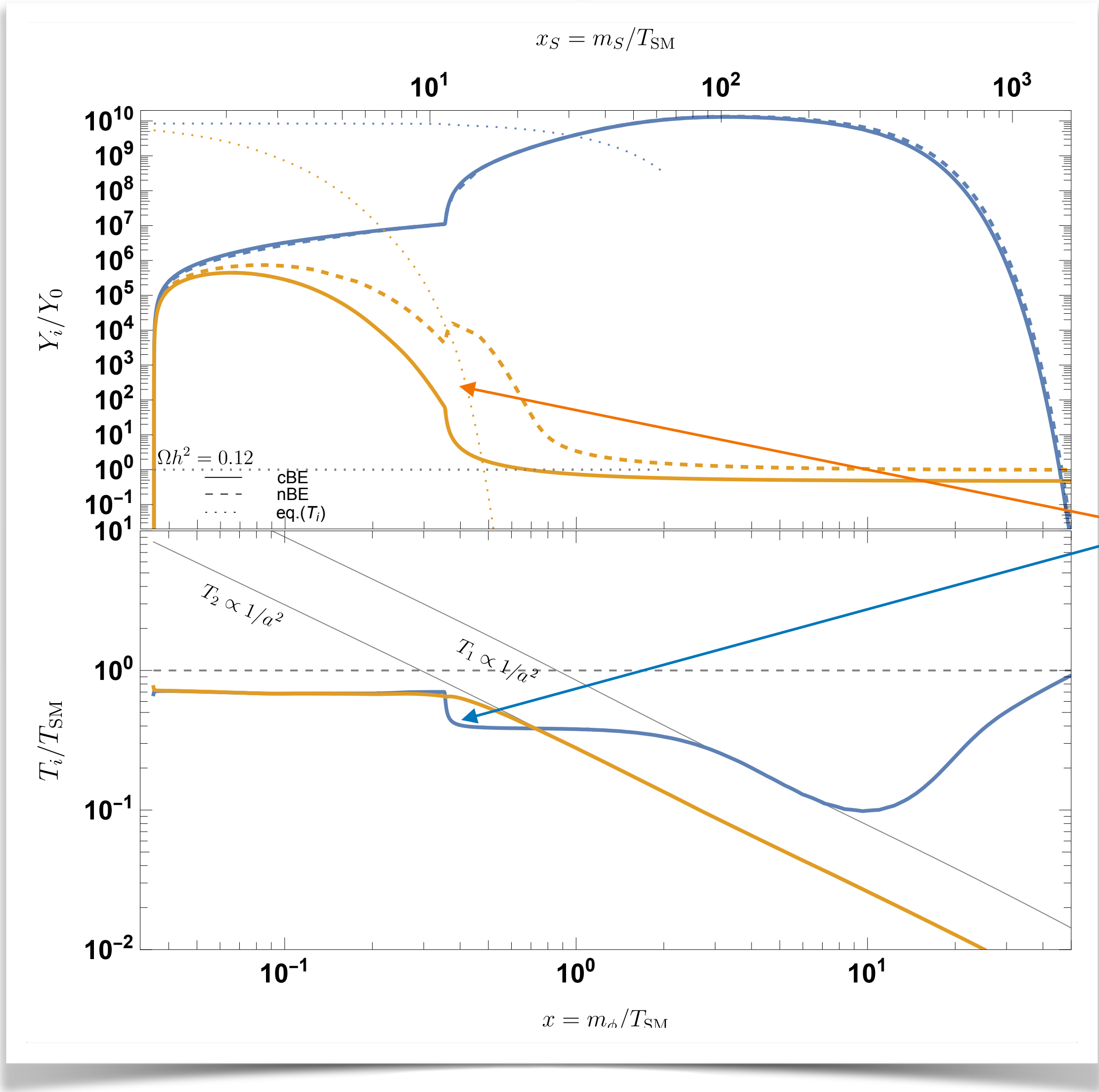

Finally, a point within reach of CTA

Notice impact of h decay after EPWT:

(as $m_\phi \sim m_h/2$ it lowers T_ϕ)

this cooling suppresses $\phi\phi \to SS$ while annihilation $SS \to \phi\phi$ can proceed

BENCHMARKS: SUMMARY

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The model's parameter space spans over various production modes:
- direct & sequential freeze-in
- dark freeze-out
- co-decaying
- (and mixtures of these)

Effect of performing calculation at cBE level: from $\sim \mathcal{O}(1\%)$ to > 100

OTHER EXAMPLES…

Sequential freeze-in thus adds to the list of scenarios where departure from LTE needs to be considered:

Annihilation through a (narrow) resonance

Duch, Grządkowski '17; Binder, Bringmann, Gustafsson, A.H '17; Abe '21; Ala-Mattinen et al .'22

Sub-threshold (e.g. forbidden DM)

Binder, Bringmann, Gustafsson, A.H 2103.01944; Liu et al '23; Aboubrahim et al. '23

Semi-annihilation and production

Kamada et al. '18; Cai, Spray '18; Hektor, AH & Kannike '19; AH & Laletin 2104.05684

Cannibal DM (freeze-out *or* freeze-in)

Herba et al '18; Cervantes & AH 2407.12104; Bernal, Cervantes, Deka, AH 2506.09155

Sommerfeld enhanced annihilation

Feng et al '10; Binder, Bringmann, Gustafsson, A.H 2103.01944

Two-component dark sectors (e.g. conversion-driven or co-decaying)

Beauchesne & Chiang 2401.03657; Chatterjee & AH 2502.08725

Freeze-out/freeze-in intermediate regime

Du et al. '22

SuperWIMP, WDM and Lyman-α limits

Decant et al. '22; AH & Laletin 2204.07078

…

CONCLUSIONS

1. Freeze-in in multicomponent dark sectors (like sequential freeze-in) proceeds in a $T-$dependent way. This can alter the naive predictions by more than an order of magnitude. This is another example of importance of non-equilibration in dark matter production (as seen in some freeze-out scenarios)

2. A simple two scalar model with feeble couplings to SM can provide interesting phenomenology with cross correlation of ID & forward physics experiments

3. In recent years a significant progress in refining the relic density calculations in DRAKE2 to include multicomponent case & freeze-in

Thank you!

OTHER EXAMPLES

EXAMPLE D:
WHEN ADDITIONAL INFLUX OF DM ARRIVES

Sudden injection of more DM particles distorts $f_\chi(p)$
(e.g. from a decay or annihilation of other states)

- this can modify the annihilation rate (if still active)
- how does the thermalization due to elastic scatterings happen?

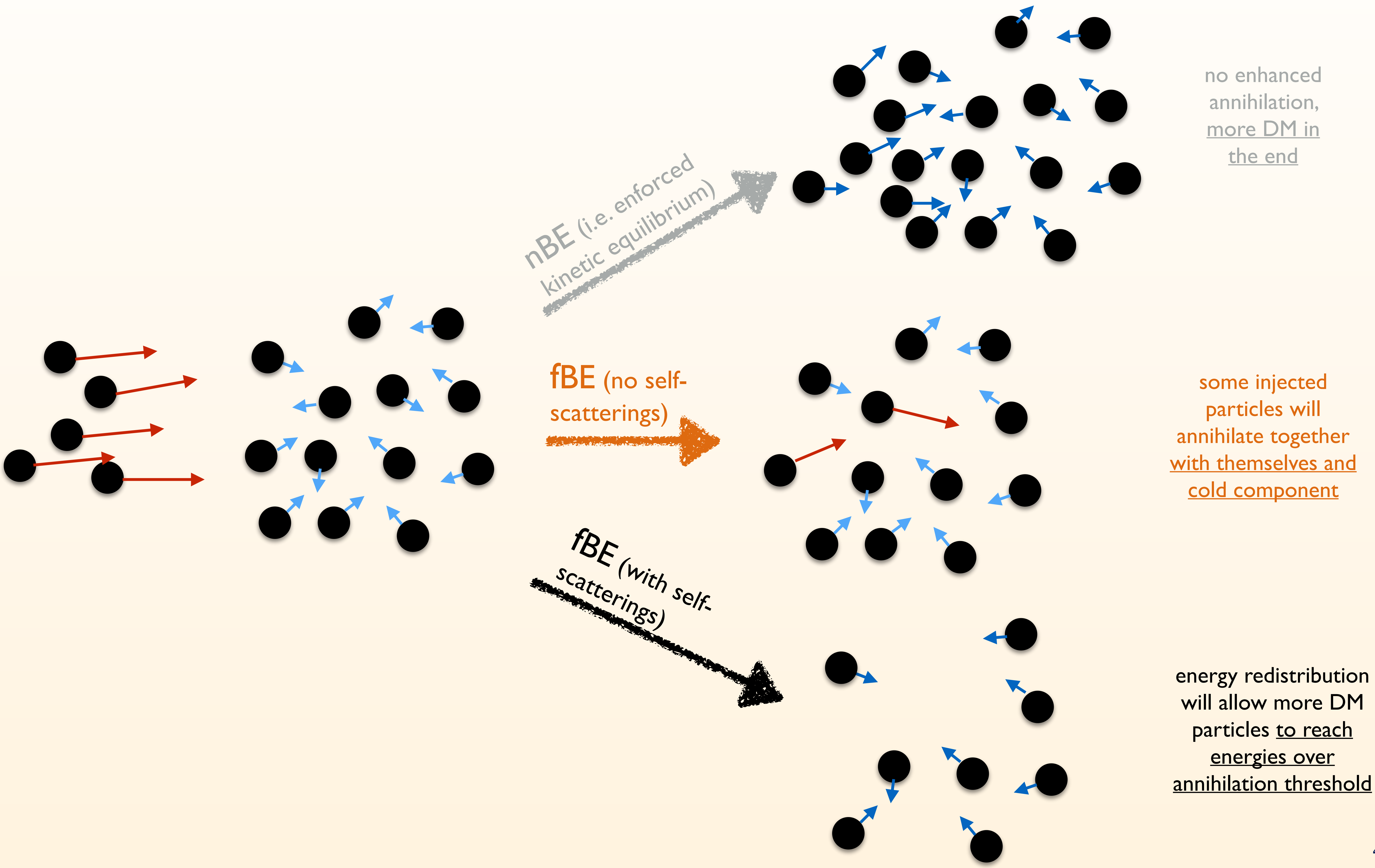

unstable long-lived state (decaying to DM)
comoving DM number density
expectation
(if increased annihilation)
DM from freeze-out
also can happen!?
time

1) DM produced via:

- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

2) DM annihilation has a threshold

e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$



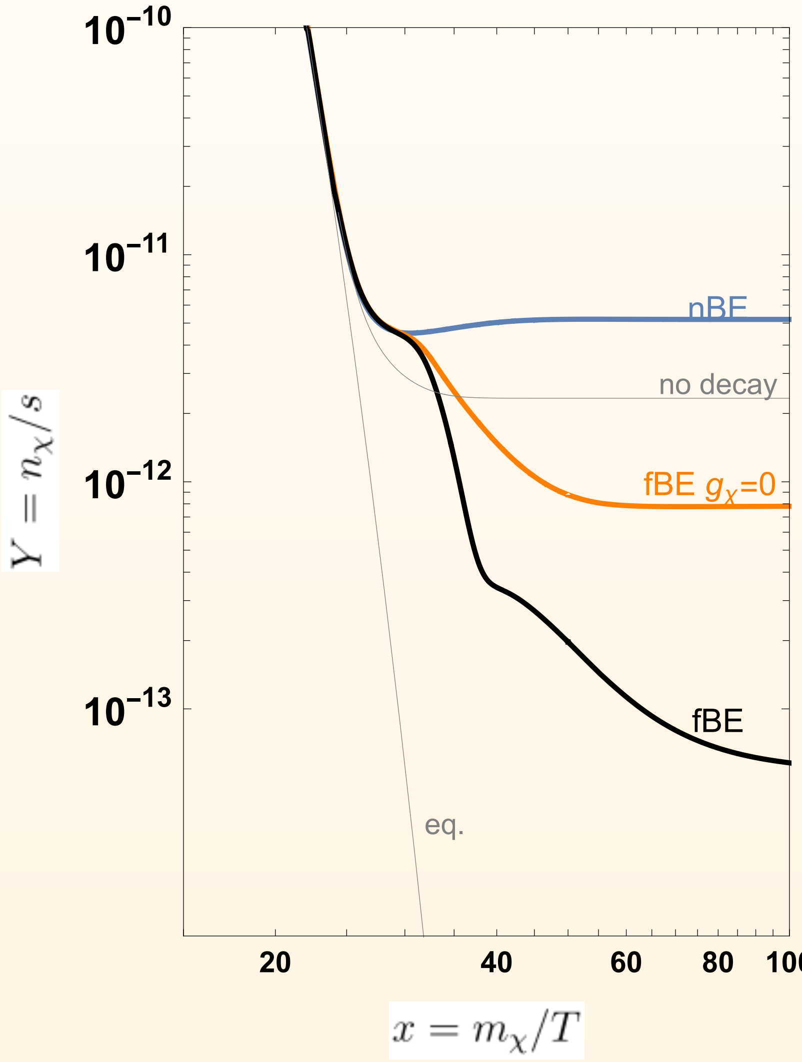
AH, Laletin 2204.07078

EXAMPLE EVOLUTION

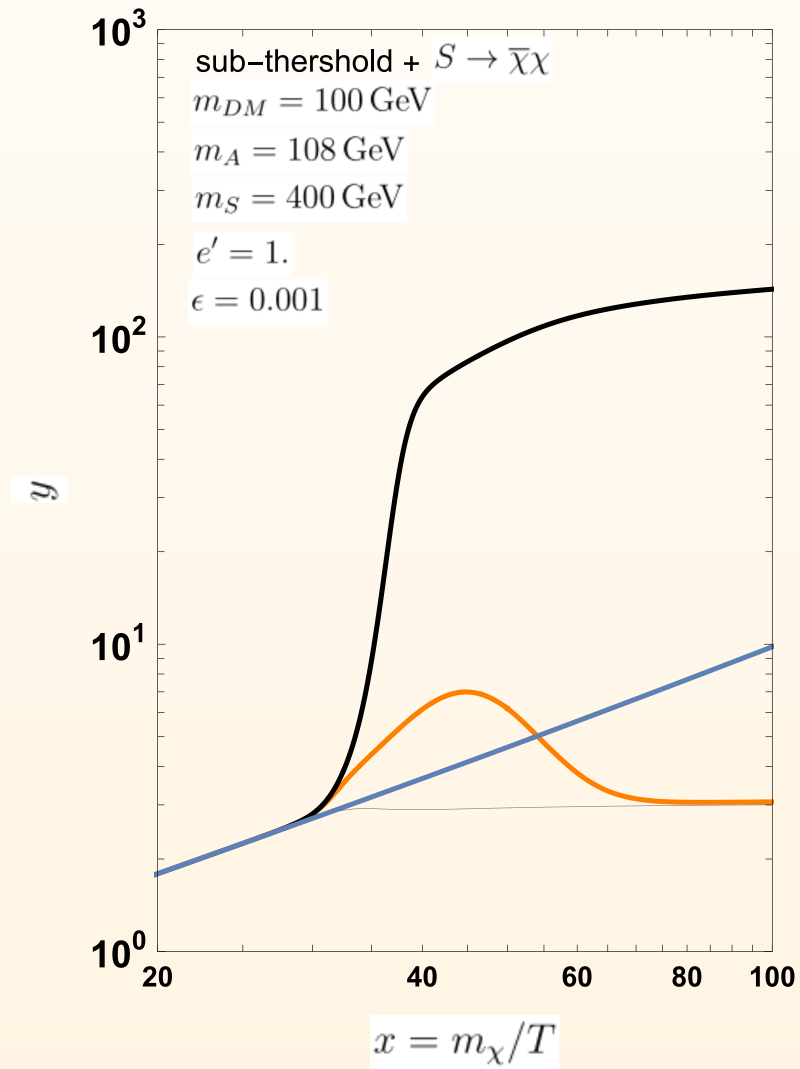
1) DM produced via:
- 1st component from thermal freeze-out
- 2nd component from a decay $\phi \to \bar{\chi}\chi$

2) DM annihilation has a threshold
e.g. $\chi\bar{\chi} \to f\bar{f}$ with $m_\chi \lesssim m_f$

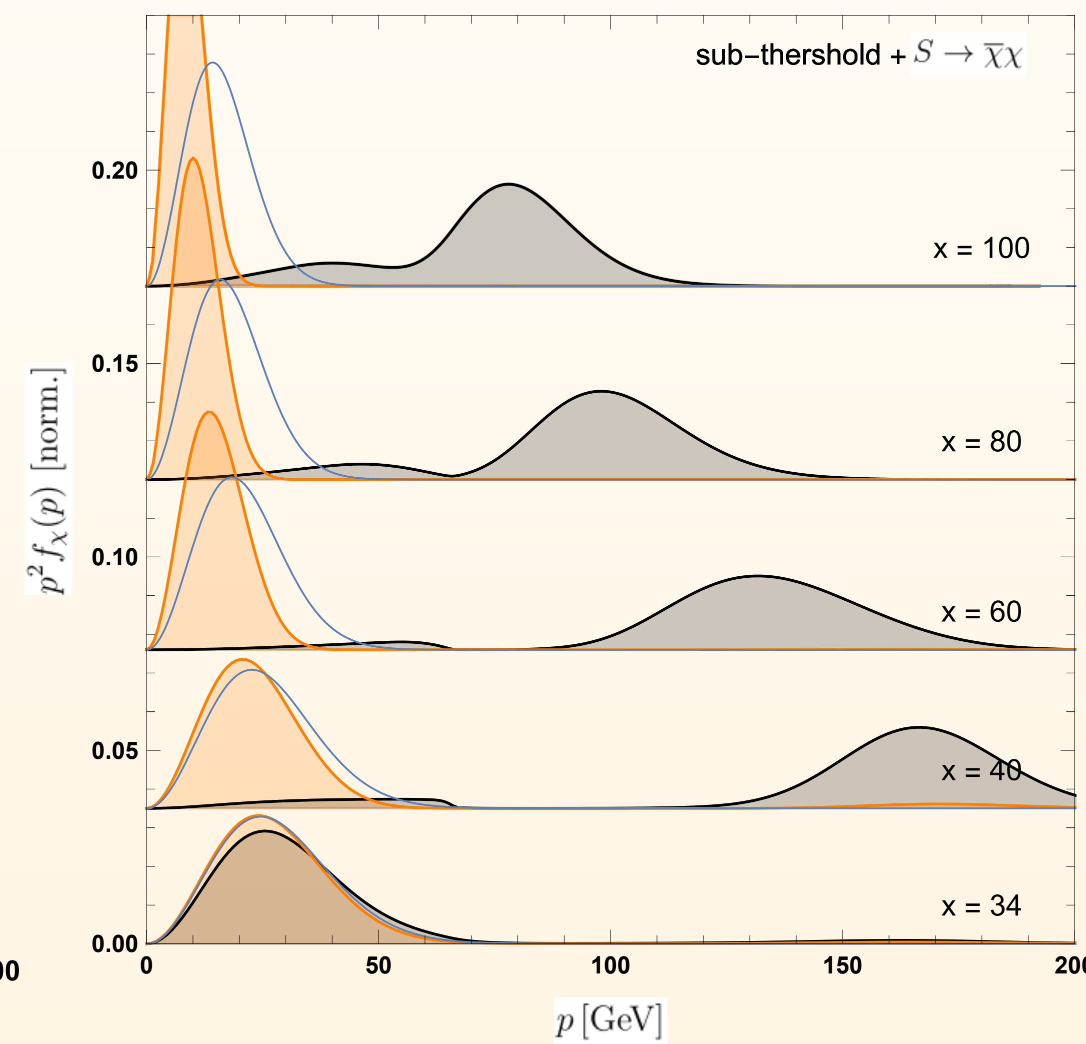
$Y \sim$ number density



$y \sim$ temperature



$p^2 f(p) \sim$ momentum distribution



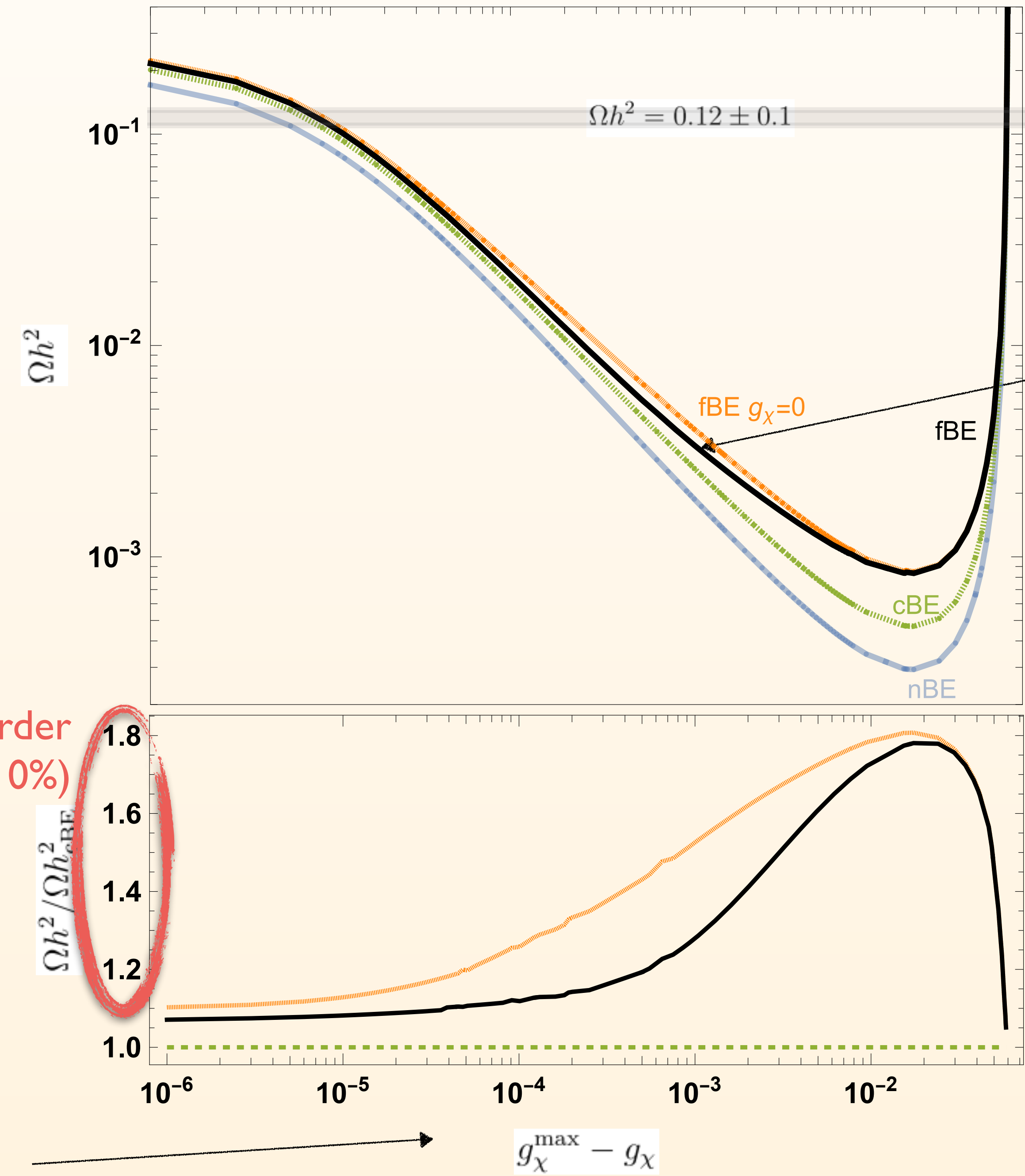
CBE vs. FBE

WHICH IS MORE ACCURATE?! **A.H. & M. Laletin** 2204.07078

They correspond to the opposite limits of self-interaction strengths:

- very efficient - **cBE**
- inefficient - f**BE**

Which limit is closer to reality depends on the model, but it seems that fBE is typically more accurate, unless self-scattering is tuned up, e.g:



difference of order O(10%)

black line gives the result including self-scattering processes! (being between pure fBE and cBE)

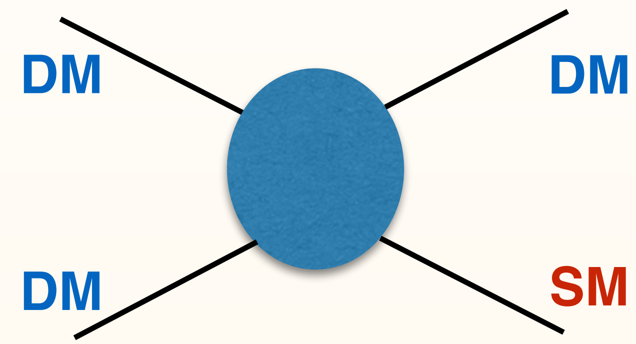
coupling to the mediator; governs self-scatterings

SEMI-ANNIHILATION

DARK MATTER SEMI-ANNIHILATION
AND ITS SIMPLEST REALIZATION

DM is a thermal relic but with freeze-out governed by the semi-annihilation process

D'Eramo, Thaler '10; …



Z_3 complex scalar singlet:

$$V = \mu_H^2 |H|^2 + \lambda_H |H|^4 + \mu_S^2 |S|^2 + \lambda_S |S|^4 + \lambda_{SH} |S|^2 |H|^2 + \frac{\mu_3}{2}(S^3 + S^{\dagger 3}).$$

just above the Higgs threshold semi-annihilation dominant!

Belanger, Kannike, Pukhov, Raidal '13

very weak elastic scatterings → semi-annihilation by itself does not equilibrate DM

↓

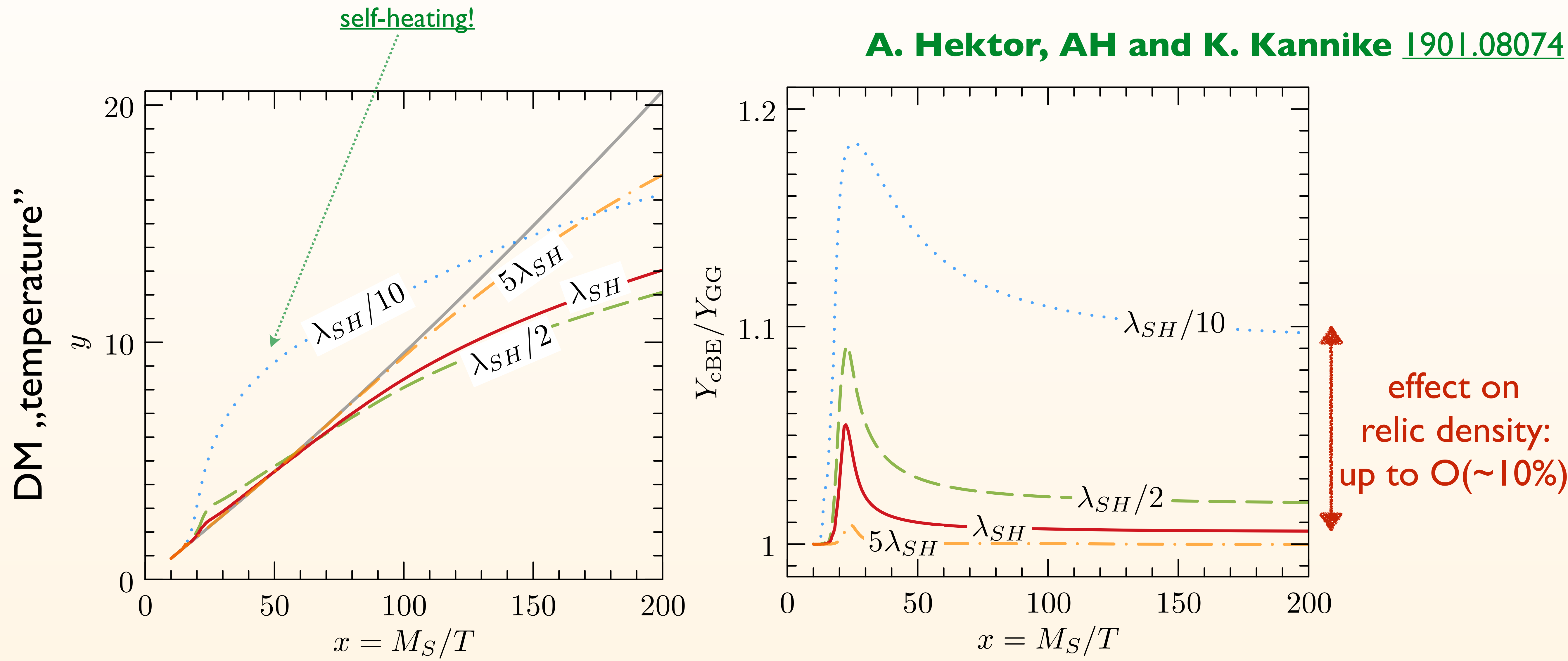
but rather leads to self-heating! → **implications for ID**

Kamada et al. 1707.09238

see also Cai, Spray 1807.00832

Semi-Annihilation
Example effect on early KD on relic density

A. Hektor, AH and K. Kannike 1901.08074



Note: here the final effect is relatively mild (though still larger than the observational error), but only because in the simplest model the velocity dependence of annihilation is mild as well…

*calculated with the (modified) coupled BEs method

Less simple example

Inert doublet model H_1, H_2 an with additional scalar singlet S:

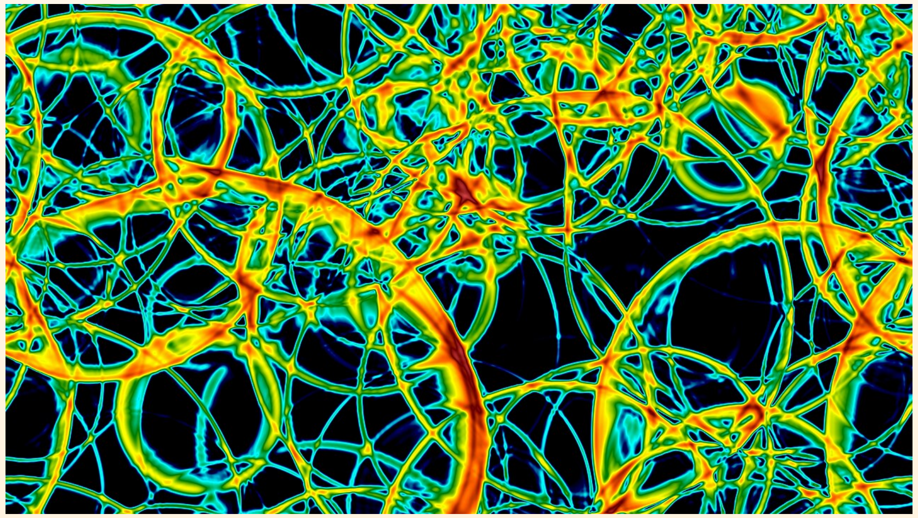
$$\mathbb{Z}_3 \qquad H_1 \to H_1,\ S \to \omega S,\ H_2 \to \omega H_2 \qquad \omega^3 = 1$$

SM Higgs — *Classical* Inert Doublet Model — *Classical* Scalar Singlet Model (**Z_2**)

$$V = \mu_1^2|H_1|^2 + \lambda_1|H_1|^4 + \mu_2^2|H_2|^2 + \lambda_2|H_2|^4 + \mu_S^2|S|^2 + \lambda_S|S|^4$$
$$+ \lambda_{S1}|S|^2|H_1|^2 + \lambda_{S2}|S|^2|H_2|^2 + \lambda_3|H_1|^2|H_2|^2 + \lambda_4(H_1^\dagger H_2)(H_2^\dagger H_1)$$
$$+ \frac{\mu_S''}{2}(S^3 + S^{\dagger 3}) + \frac{\lambda_{S12}}{2}(S^2 H_1^\dagger H_2 + S^{\dagger 2} H_2^\dagger H_1) + \frac{\mu_{SH}}{2}(S H_2^\dagger H_1 + S^\dagger H_1^\dagger H_2)$$

Z_3 mixing terms

Such a scalar potential allows
for FOPT ⇒ nucleation of bubbles
& stochastic GW background

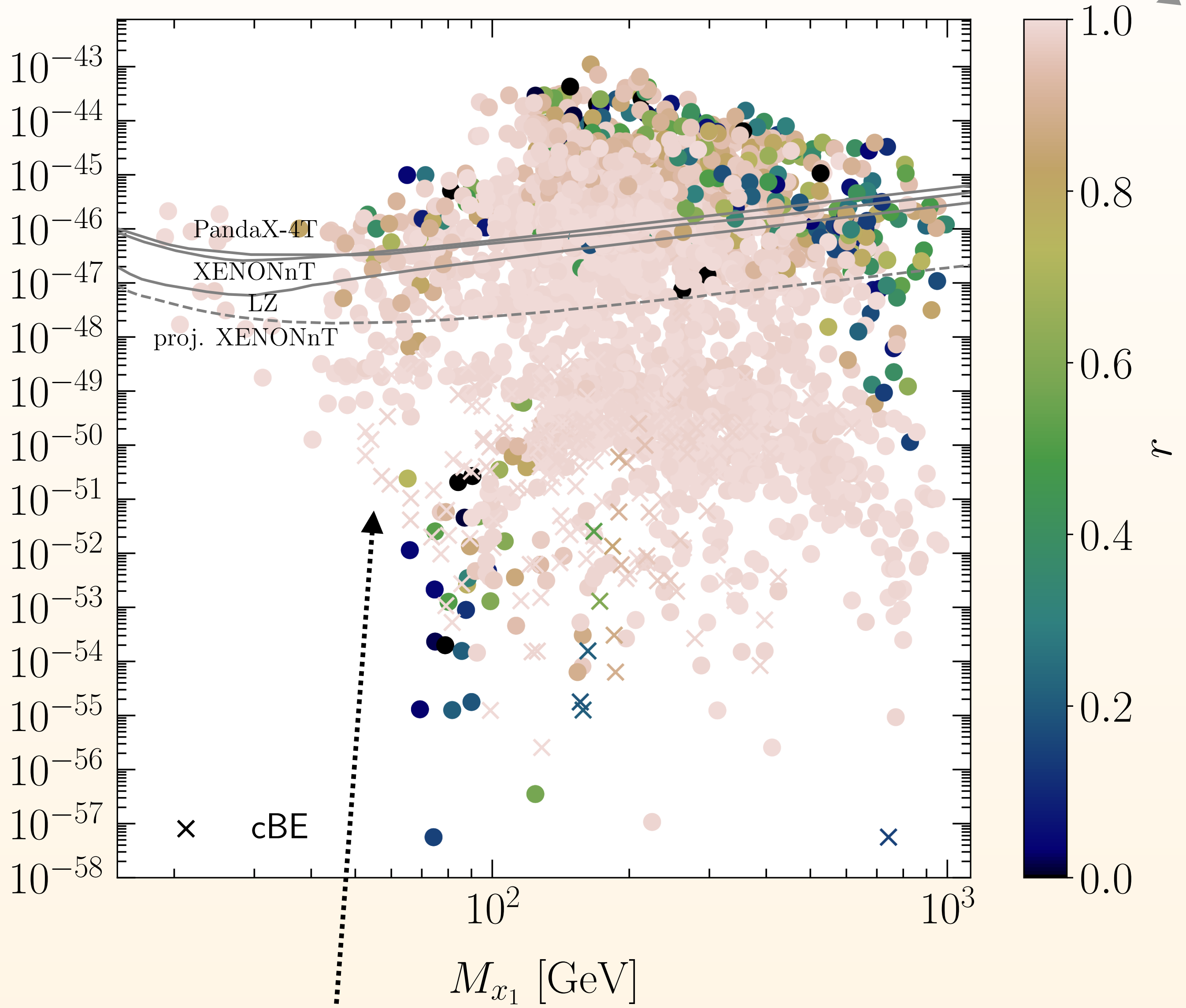


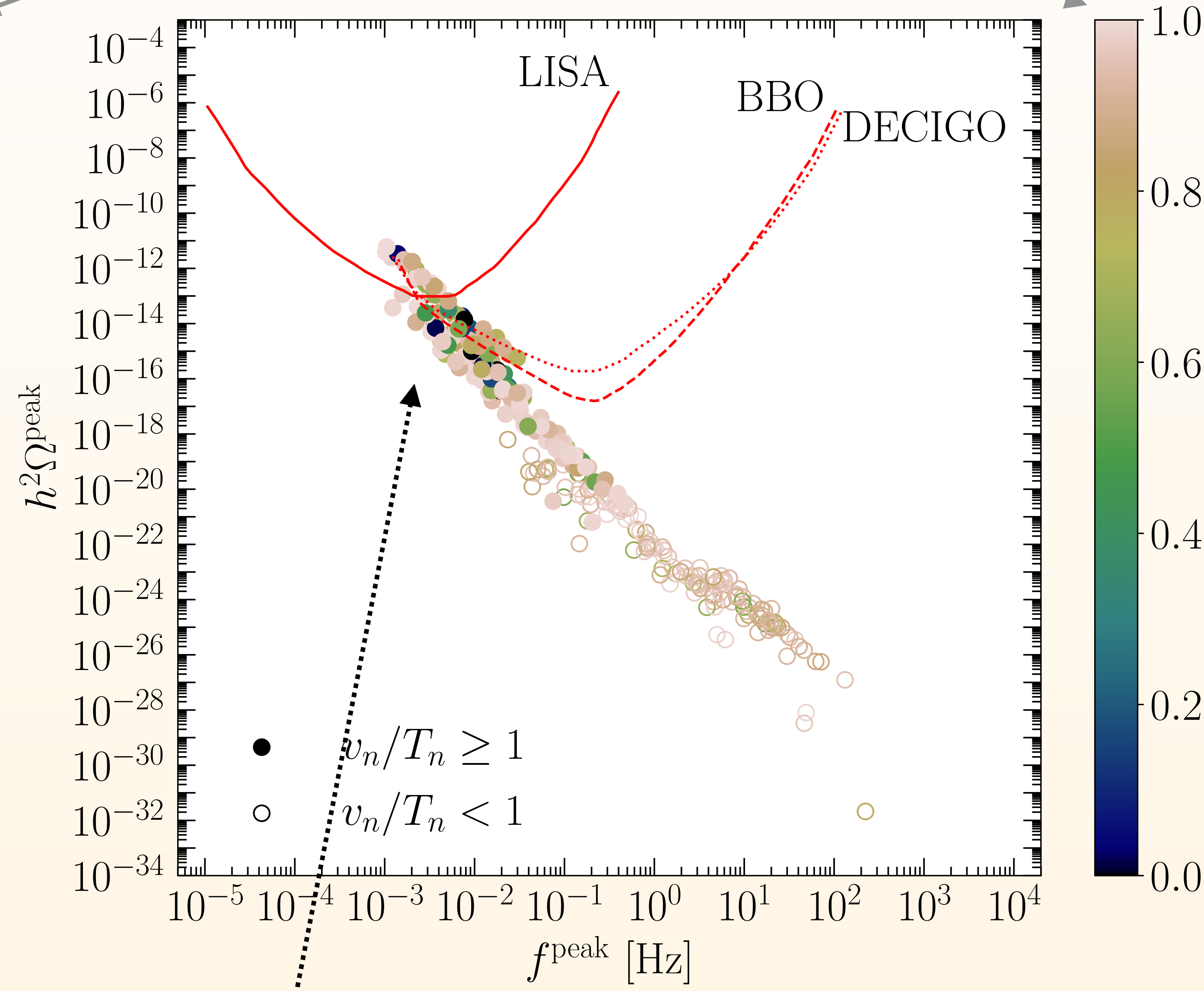
Credit: D. Weir

N. Benincasa, A.H, K. Kannike & M. Laletin 2312.04627

SCAN RESULTS

color: strength of semi-annihilation vs. annihilation





Significant fraction of points has early kinetic decoupling

Some (small) portion of the allowed parameter space will be detectable with future GW instruments