

# SEQUENTIAL FREEZE-IN

## A TALE OF TWO SCALARS

Andrzej Hryczuk



### Based on:

work in progress with **S. Chatterjee**

+ some earlier work with **M. Laletin, T. Binder, T. Bringmann, M. Gustafsson**

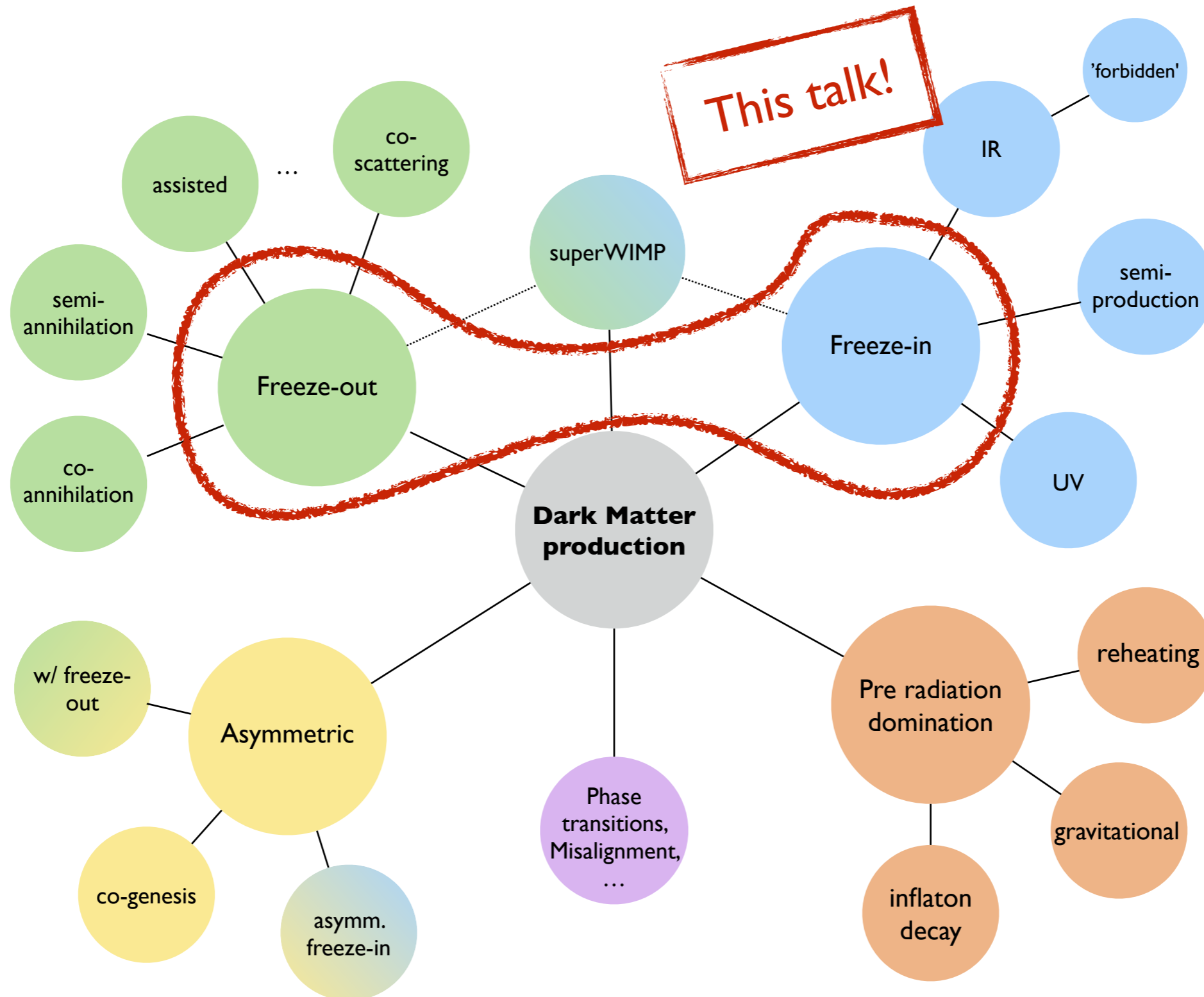
# MOTIVATION & OBJECTIVES

A step in a program of describing  
**Dark Matter production**  
in systems  
departing from local thermal equilibrium

Study of a SM+2 scalars  
theory with **detectable,**  
**frozen-in** DM

Implementation of  
freeze-in production  
in **DRAKE2** code

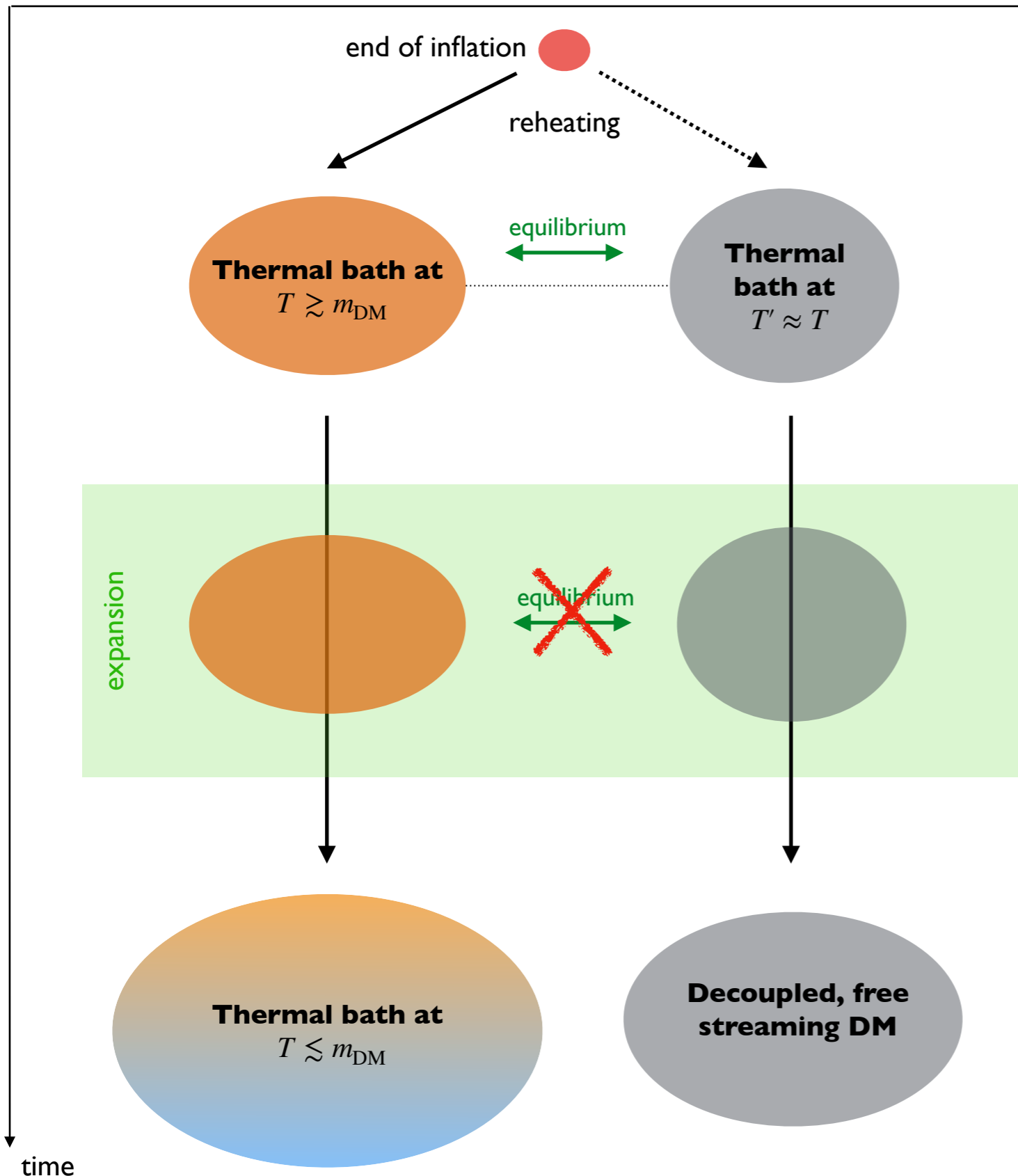
# DARK MATTER ORIGIN



# WHAT IS FREEZE-OUT?

Visible Sector

Dark Sector



## I. Natural

Comes out **automatically** from the expansion of the Universe

Naturally leads to **cold DM**

## II. Predictive

No dependence on **initial conditions**

**Fixes coupling(s)**  $\Rightarrow$  signal in DD, ID & LHC

## III. It is not optional

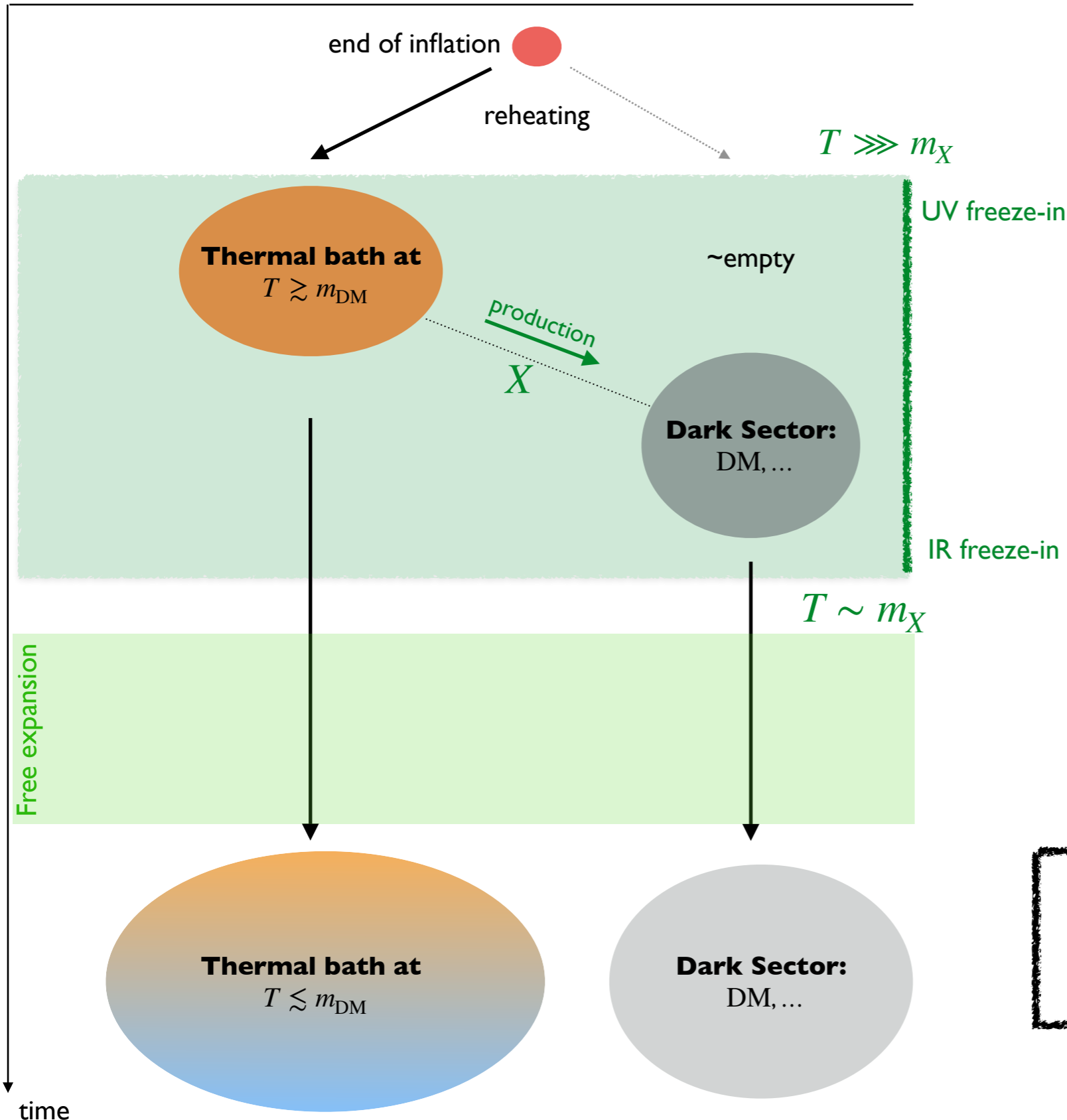
**Overabundance** constraint

To avoid it one needs **quite significant deviations** from standard cosmology

# WHAT IS FREEZE-IN?

Visible Sector

Dark Sector



Freeze-in defined like this is a (very) old idea:

this is a standard production mechanism for e.g. **sterile neutrino, gravitino, axino, ...**

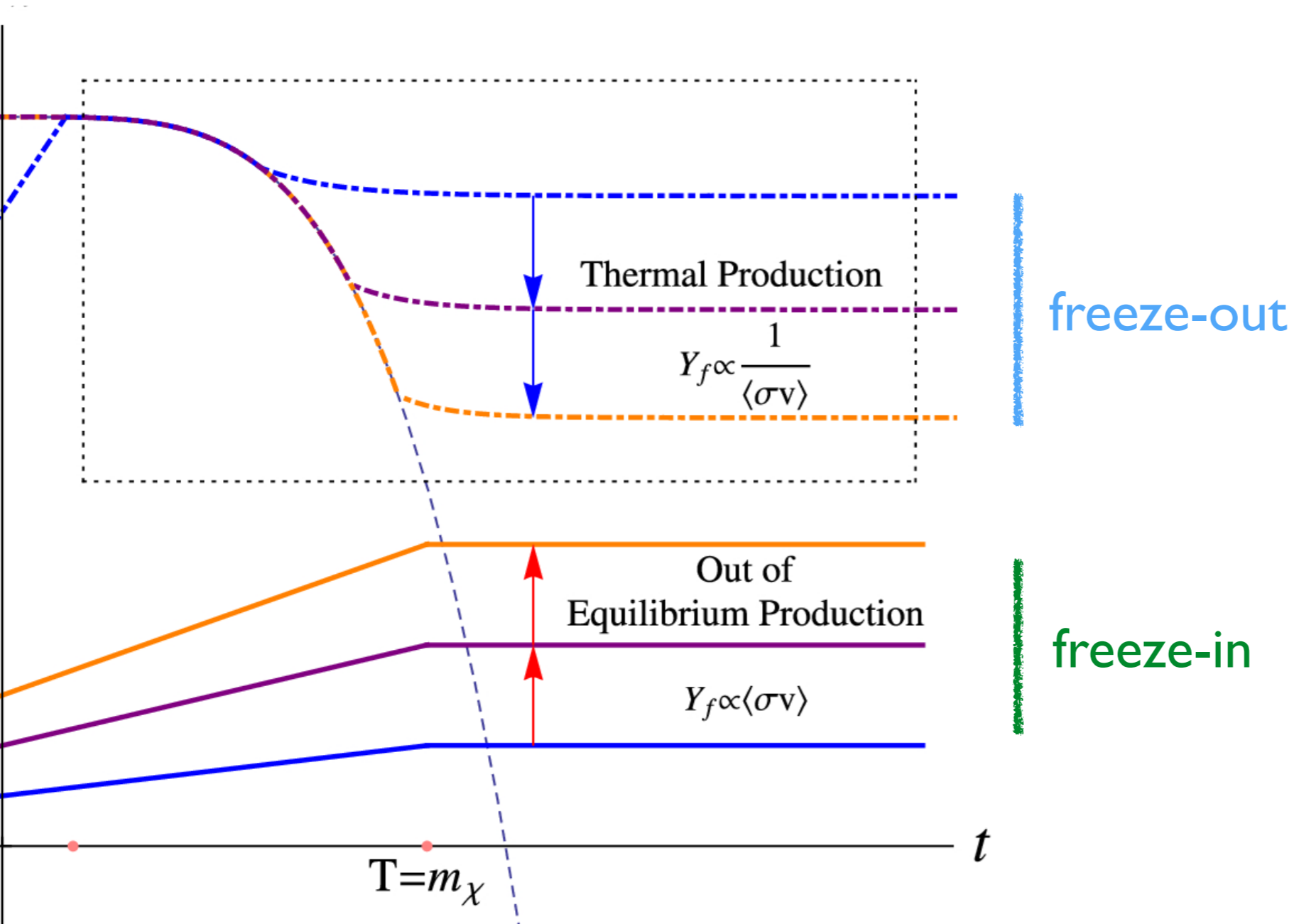
however, old works focused on what now people call **UV freeze-in**

i.e. dominated by **non-renormalizable operators** and dependent on  $T_{\text{RH}}$

**Freeze-in** = the above mechanism through renormalizable operators (**IR freeze-in**)

# FREEZE-IN vs. FREEZE-OUT

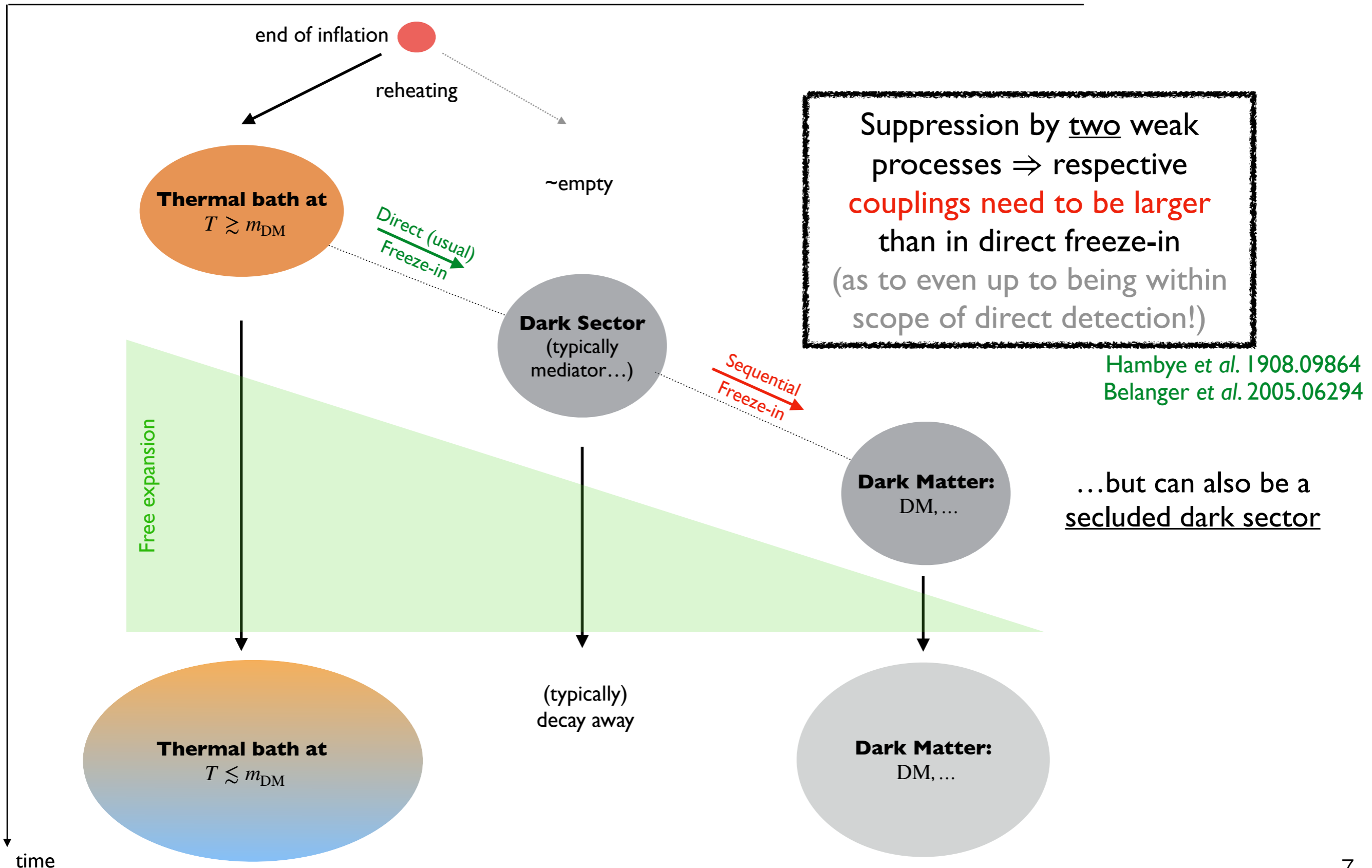
Freeze-in is in a sense the 'opposite' of freeze-out



# WHAT IS SEQUENTIAL FREEZE-IN?



Visible Sector

Dark Sector

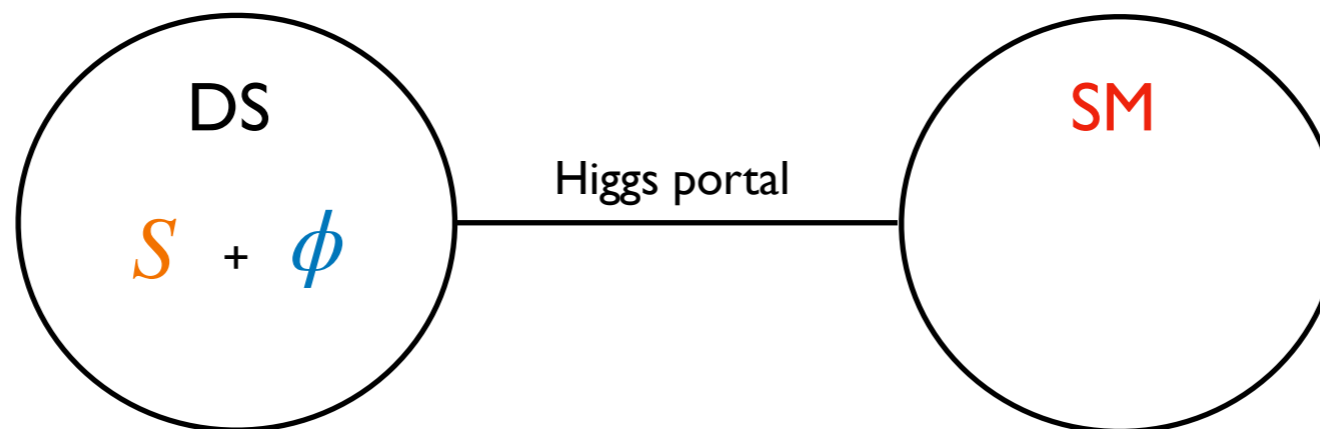


# A TALE OF TWO SCALARS

Postulate two new scalars (singlets w.r.t SM gauge group):

	$S$	$\mathbb{Z}_2$ -symmetric	stable	dark matter	feeble int. with SM
	$\phi$	<del><math>\mathbb{Z}_2</math></del> explicitly broken	unstable	"mediator"	feeble int. with SM

^



$$V \supset -A\phi H^\dagger H - \frac{\lambda_{h\phi}}{2}\phi^2 H^\dagger H - \frac{\lambda_{Sh}}{2}S^2 H^\dagger H - \frac{1}{4}\lambda_{S\phi}S^2\phi^2$$

mediator-Higgs
DM-Higgs
DM-mediator

mediator-Higgs mixing

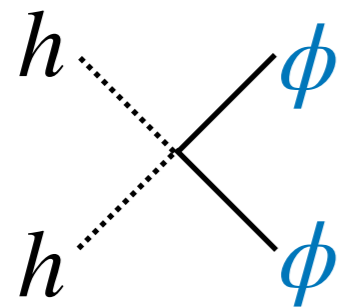
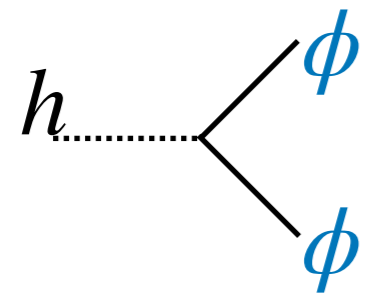
$$\sin \theta = \frac{Av}{m_h^2 - m_\phi^2} \left( 1 - \frac{\lambda_{h\phi} v^2}{2m_\phi^2} \right) \quad \mathbf{8}$$

Such models are not unheard of. Most similar in the literature:

...; Wang, Han '14; Claude, Godfrey '21; ...

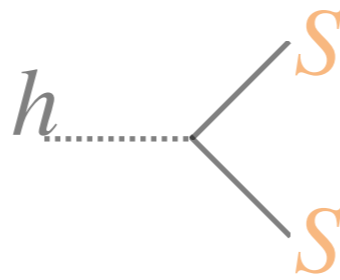
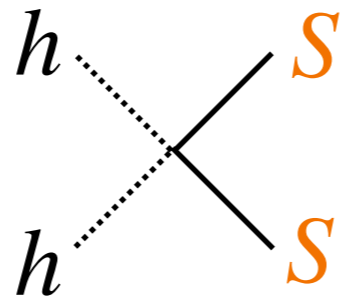
# A TALE OF TWO SCALARS

mediator freeze-in:



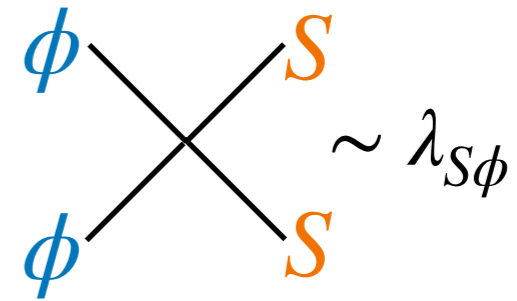
$$\sim \lambda_{h\phi}$$

DM freeze-in:



$$\sim \lambda_{Sh}$$

sequential freeze-in:

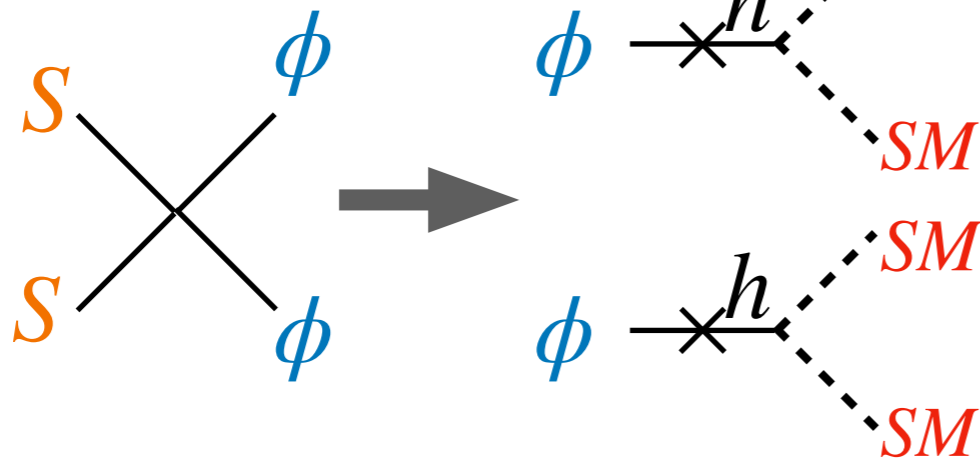


Typical hierarchy:

$$\boxed{\lambda_{S\phi}} \gg \gg \boxed{\lambda_{h\phi} \gg \lambda_{Sh}}$$

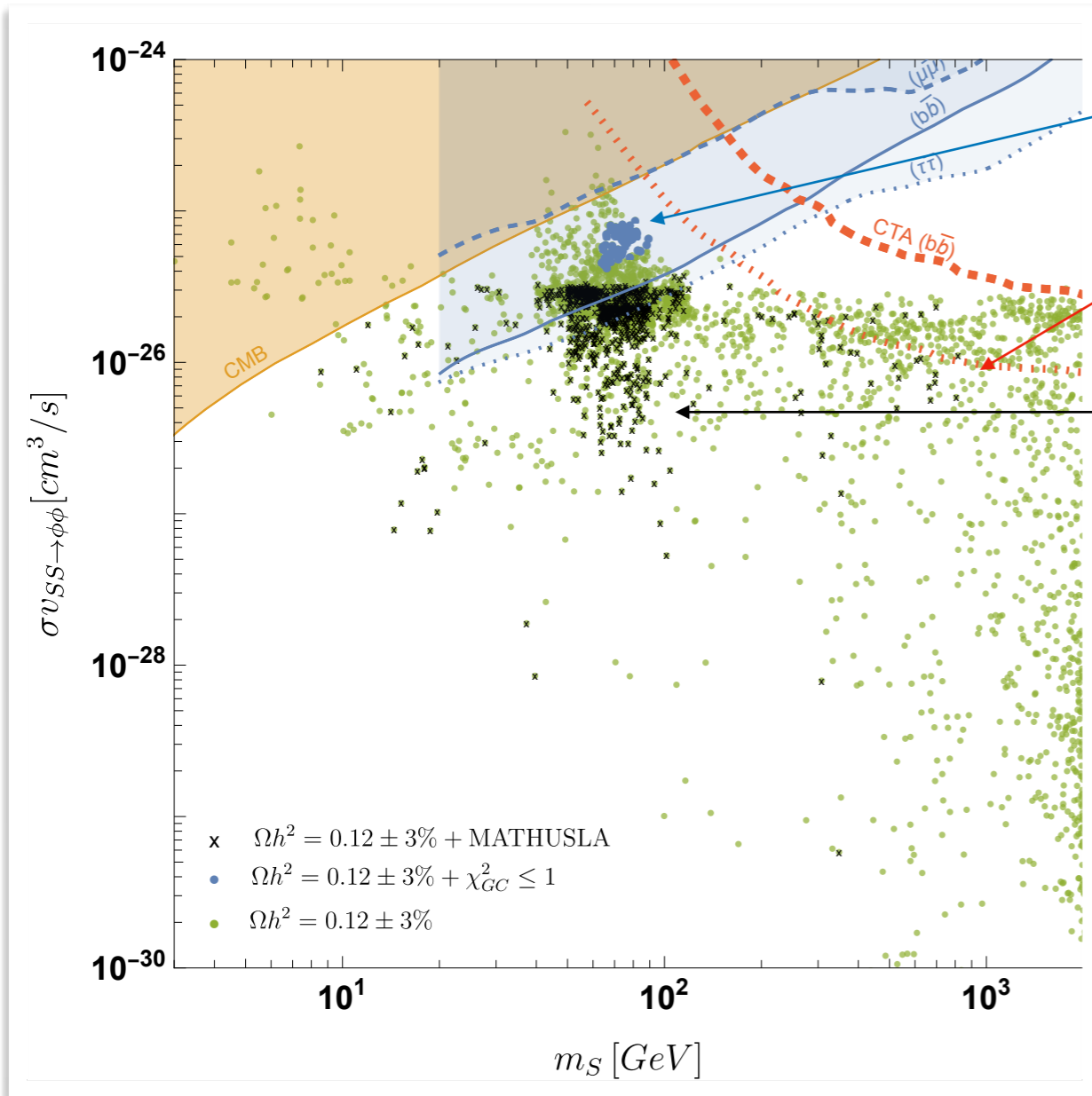
Freeze-out-like Freeze-in-like

Indirect detection through a cascade decay (iff  $m_S > m_\phi$ ):



ID signal = requirement of sub-threshold sequential freeze-in

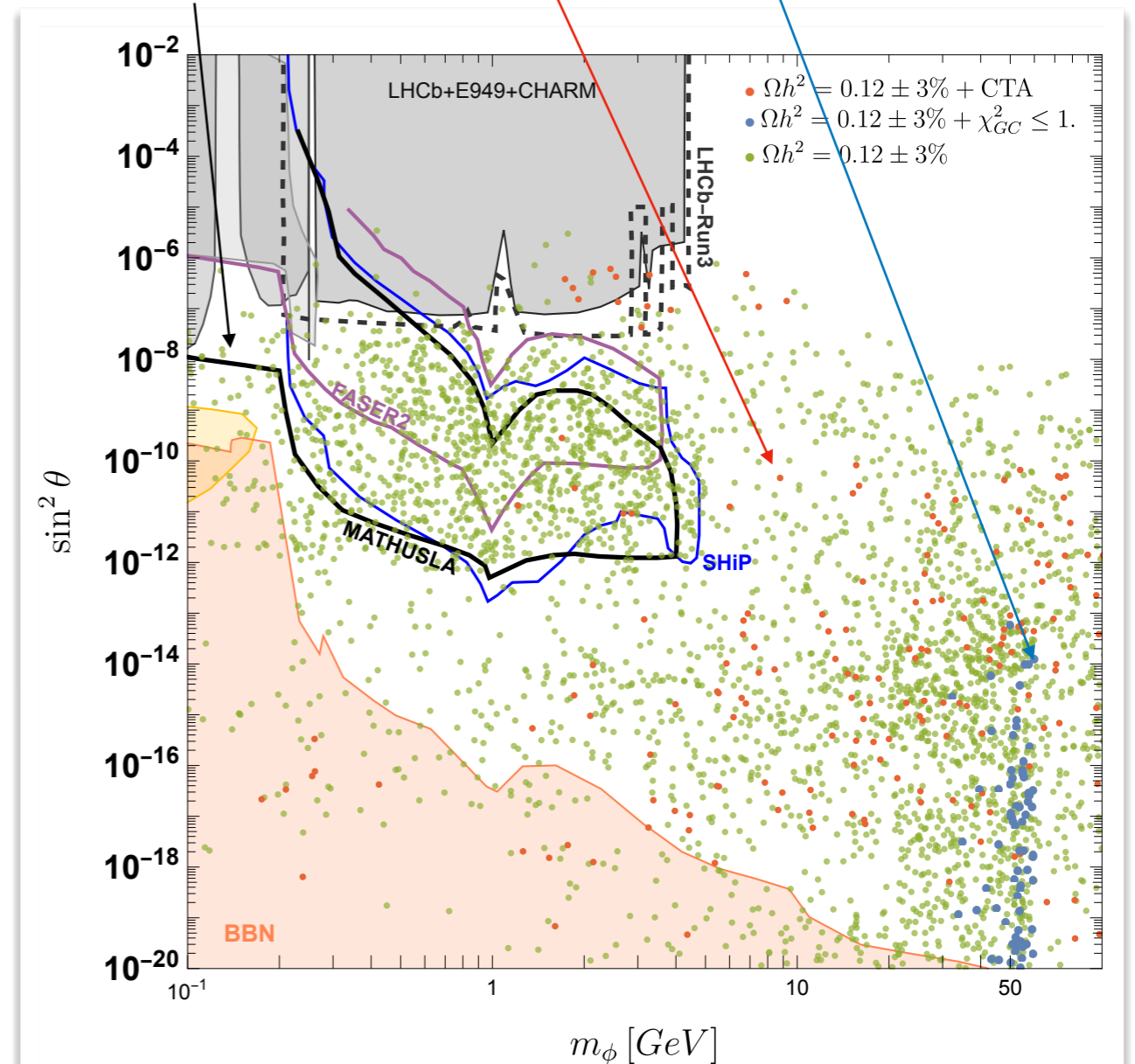
# SCAN RESULTS: ID AND FORWARD PHYSICS



Points giving good fit to GCE

Points within (optimistic) reach of CTA

within reach of MATHUSLA



All points satisfy relic density constraint

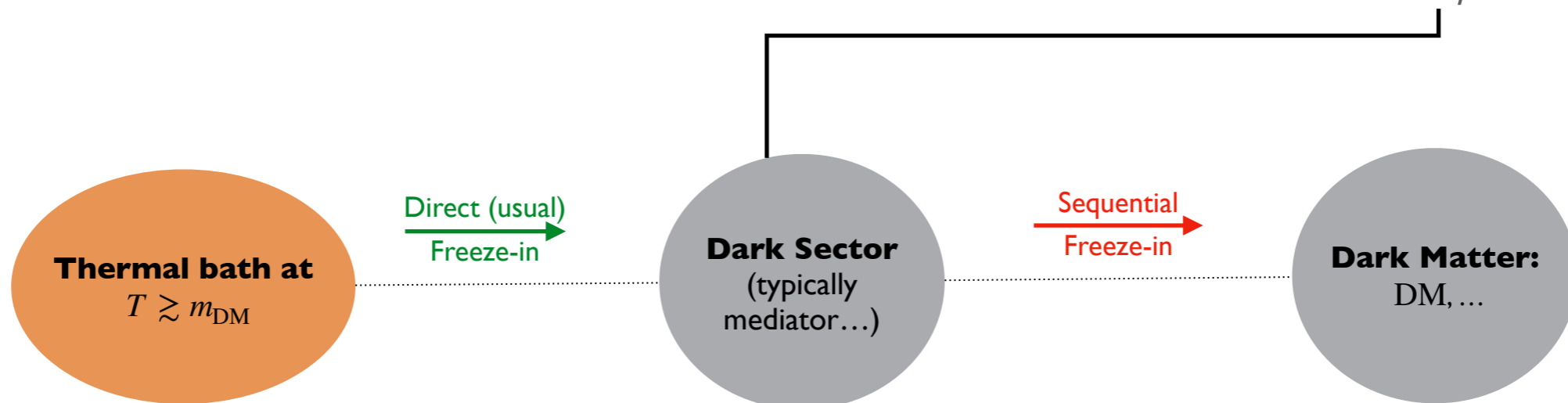
Scan driven towards regions that are covered by any of the experiments

# A SECOND LOOK ON $\Omega h^2$

The relic density was the main constraint of the scan. It was obtained by solving the Boltzmann equation for number densities of  $\phi$  and  $S$  (nBE) (as e.g. micrOMEGAs or DarkSUSY would)

But wait... isn't relic abundance (*freeze-in or freeze-out*) dependent on the  $T$  of the thermal bath it is produced from?

Which temperature is relevant for sequential freeze-in:  $T_{SM}$  or  $T_\phi$ ?

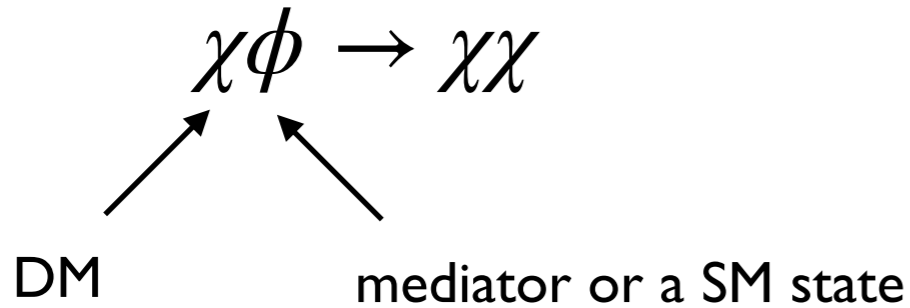


...OK, so it looks like we need to trace  $T_\phi$  as well!

# THIS IS REMINISCENT OF...

AH, Laletin 2104.05684  
(see also Bringmann et al. 2103.16572)

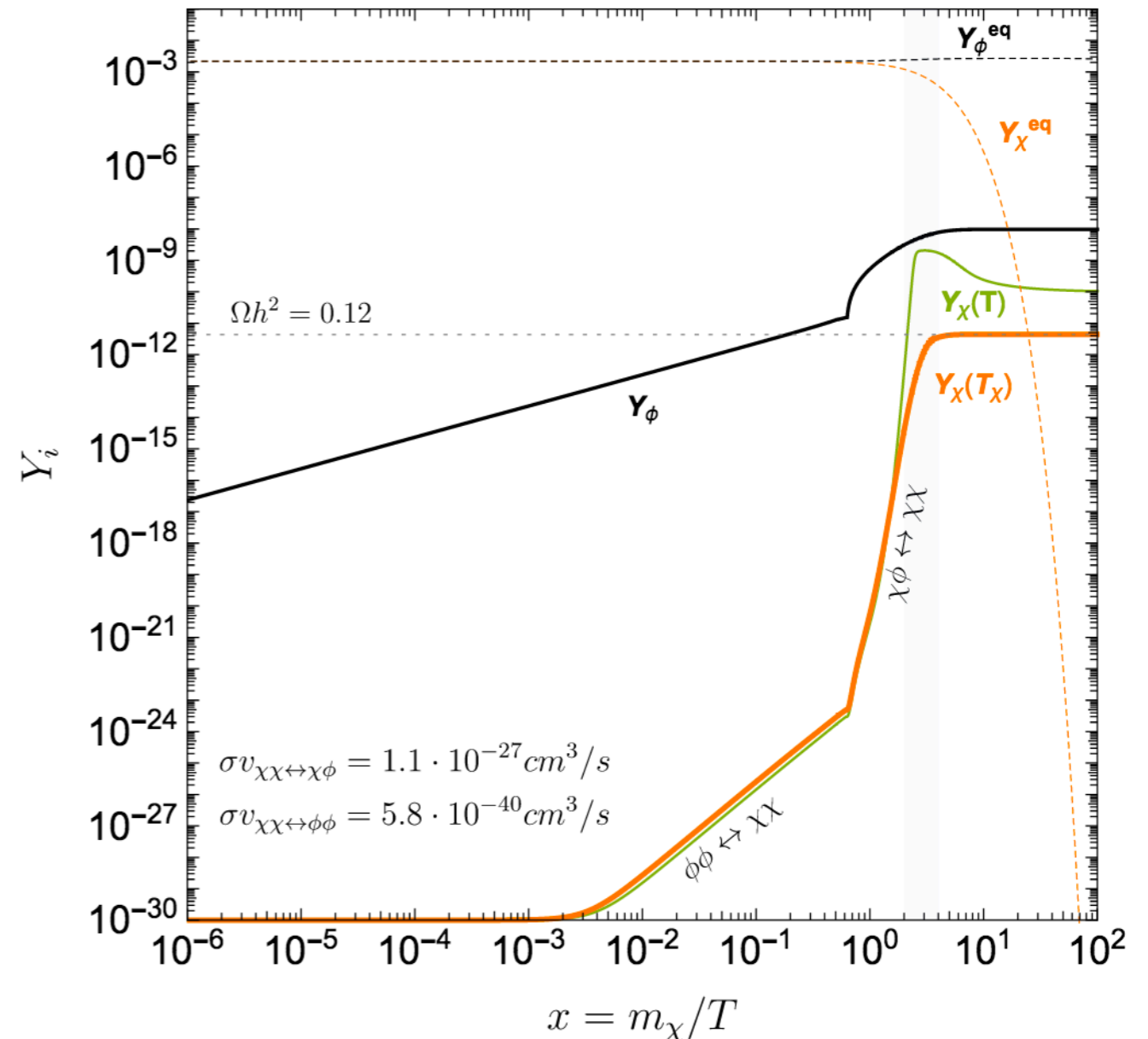
Consider process of production that is the **inverse of semi-annihilation**:



What is different?

(from the decay/annihilation freeze-in)

- The production rate is **proportional to the DM density**. (Smaller initial abundance  $\rightarrow$  larger cross section...)
- **Semi-production** modifies the energy of DM particles in a non-trivial way, so the **temperature evolution can affect the relic density**



# THERMAL RELIC DENSITY

## STANDARD APPROACH

Boltzmann equation for  $f_\chi(p)$ :

$$E (\partial_t - H \vec{p} \cdot \nabla_{\vec{p}}) f_\chi = \mathcal{C}[f_\chi]$$

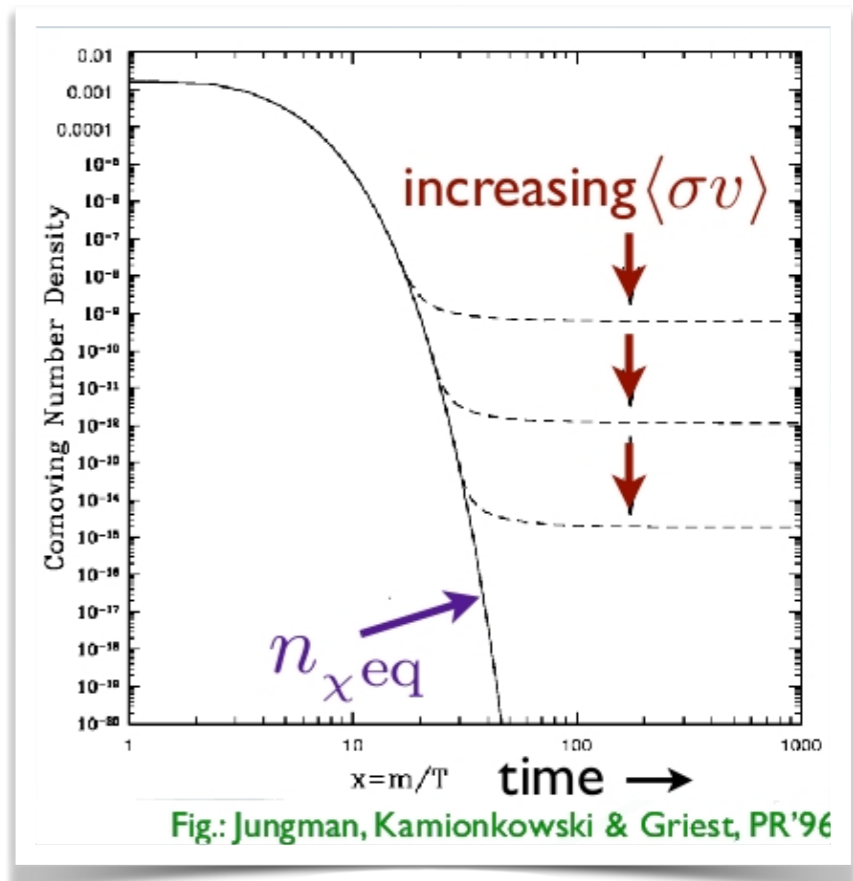


$$\frac{dn_\chi}{dt} + 3Hn_\chi = - \langle \sigma_{\chi\bar{\chi} \rightarrow ij} \sigma_{\text{rel}} \rangle^{\text{eq}} (n_\chi n_{\bar{\chi}} - n_\chi^{\text{eq}} n_{\bar{\chi}}^{\text{eq}})$$

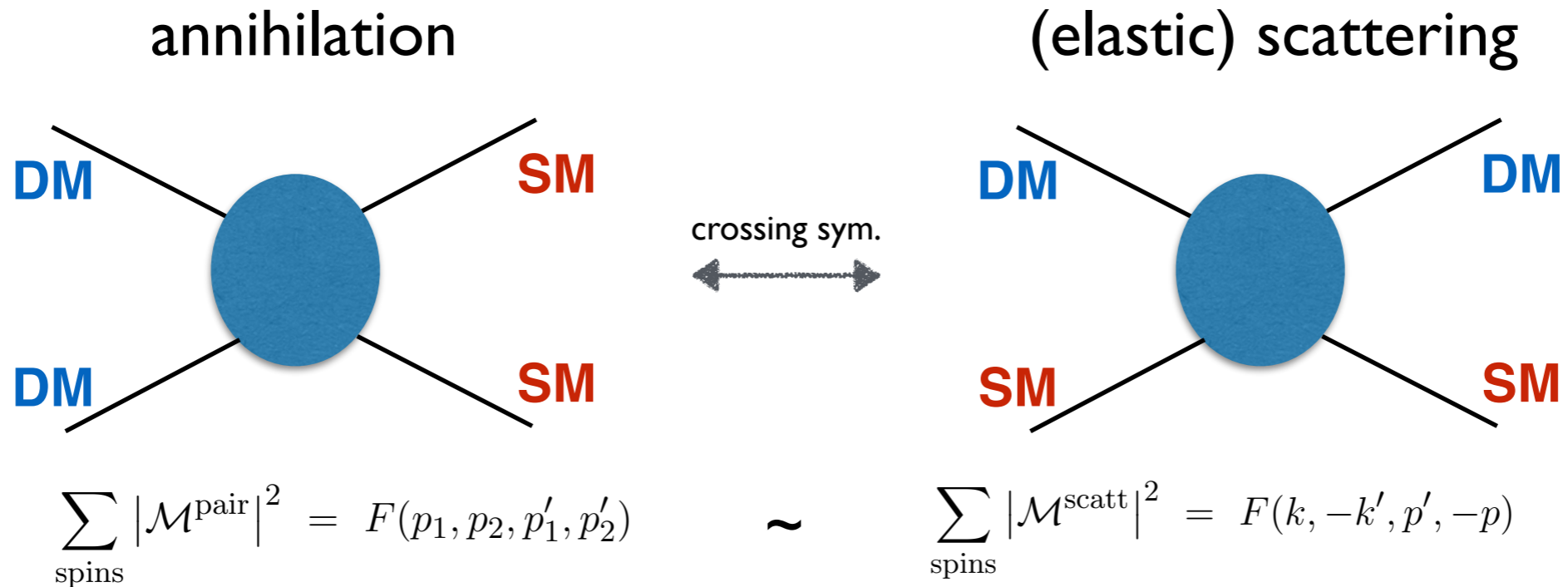


**Critical assumption:**  
kinetic equilibrium at chemical decoupling

$$f_\chi \sim a(T) f_\chi^{\text{eq}}$$



# FREEZE-OUT vs. DECOUPLING



Boltzmann suppression of **DM** vs. **SM**  $\Rightarrow$  scatterings typically more frequent  
 dark matter frozen-out but typically still kinetically coupled to the plasma

$$\tau_r(T_{\text{kd}}) \equiv N_{\text{coll}}/\Gamma_{\text{el}} \sim H^{-1}(T_{\text{kd}})$$

Schmid, Schwarz, Widern '99; Green, Hofmann, Schwarz '05

Two consequences:

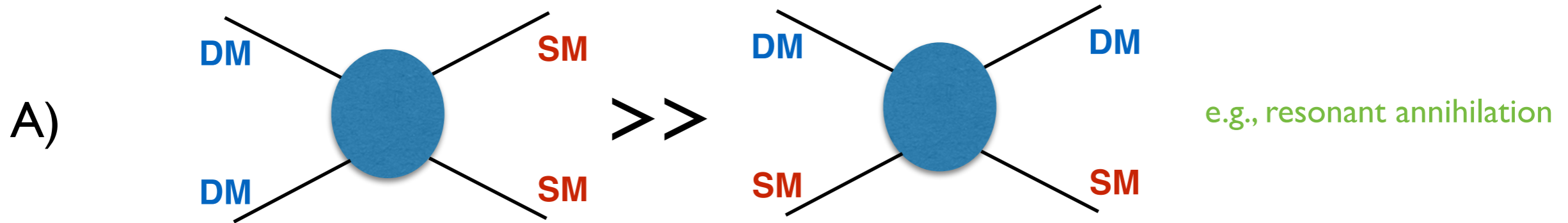
1. During freeze-out (chemical decoupling) typically:  $f_\chi \sim a(\mu) f_\chi^{\text{eq}}$
2. If kinetic decoupling much, much later: possible impact on the matter power spectrum  
 i.e. kinetic decoupling can have observable consequences and affect e.g. missing satellites problem

see e.g., Bringmann, Ihle, Karsten, Valia '16

# DEPARTURE FROM KINETIC EQUILIBRIUM?

A **necessary** and **sufficient** condition: scatterings weaker than annihilation  
i.e. rates around freeze-out:  $H \sim \Gamma_{\text{ann}} \gtrsim \Gamma_{\text{el}}$

Possibilities:



B) Boltzmann suppression of **SM** as strong as for **DM**  
e.g., below threshold annihilation (forbidden-like DM)

C) Scatterings and annihilation have different structure  
e.g., semi-annihilation, 3 to 2 models, ...

D) Multi-component dark sectors  
e.g., additional sources of DM from late decays, ...

# HOW TO GO BEYOND KINETIC EQUILIBRIUM?

All information is in the full BE:

both about chemical ("normalization") and kinetic ("shape") equilibrium/decoupling

$$E (\partial_t - H\vec{p} \cdot \nabla_{\vec{p}}) f_{\chi} = \mathcal{C}[f_{\chi}]$$

contains both scatterings and annihilations

Two possible approaches:

fBE

solve numerically  
for full  $f_{\chi}(p)$

have insight on the distribution  
no constraining assumptions

numerically challenging  
often an overkill

CBE

consider system of equations  
for moments of  $f_{\chi}(p)$

partially analytic/much easier numerically  
manifestly captures all of the relevant physics

finite range of validity  
no insight on the distribution

0-th moment:  $n_{\chi}$   
2-nd moment:  $T_{\chi}$

...

# PUBLIC TOOL!

Binder, Bringmann, Gustafsson, AH 2103.01944

## GOING BEYOND THE STANDARD APPROACH

- Home
- Downloads
- Contact



### Dark matter Relic Abundance beyond Kinetic Equilibrium

Authors: Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk

DRAKE is a numerical precision tool for predicting the dark matter relic abundance also in situations where the standard assumption of kinetic equilibrium during the freeze-out process may not be satisfied. The code comes with a set of three dedicated Boltzmann equation solvers that implement, respectively, the traditionally adopted equation for the dark matter number density, fluid-like equations that couple the evolution of number density and velocity dispersion, and a full numerical evolution of the phase-space distribution. The code is written in Wolfram Language and includes a Mathematica notebook example program, a template script for terminal usage with the free Wolfram Engine, as well as several concrete example models. DRAKE is a free software licensed under GPL3.

If you use DRAKE for your scientific publications, please cite

- **DRAKE: Dark matter Relic Abundance beyond Kinetic Equilibrium,** Tobias Binder, Torsten Bringmann, Michael Gustafsson and Andrzej Hryczuk, [arXiv:2103.01944]

Currently, a user guide can be found in the Appendix A of this reference. Please cite also quoted other works applying for specific cases.

**v1.0** « [Click here to download DRAKE](#)

(March 3, 2021)

<https://drake.hepforge.org>

### Applications:

DM relic density for  
any (user defined) model\*

Interplay between chemical and  
kinetic decoupling

Prediction for the DM  
phase space distribution

Late kinetic decoupling  
and impact on cosmology

see e.g., [1202.5456](#)

...

(only) prerequisite:  
*Wolfram Language (or Mathematica)*

\*at the moment for a single DM species and w/o  
co-annihilations... but stay tuned for extensions!

# SYSTEM OF CBE FOR $Y_i$ AND $T_i$

This we obtain through equations for the 0th and 2nd moment of the BE:

$$\frac{Y'_i}{Y_i} = \frac{m_i}{x\tilde{H}} C_i^0, \quad \text{where } y \equiv \frac{m_\chi T_\chi}{s^{2/3}} \text{ is a parameter that describes}$$

$$\frac{y'_i}{y_i} = \frac{m_i}{x\tilde{H}} C_i^2 - \frac{Y'_i}{Y_i} + \frac{H}{x\tilde{H}} \frac{\langle p^4/E_i^3 \rangle}{3T_i} \quad \text{the 'temperature' } T_\chi \equiv \frac{g_\chi}{3n_\chi} \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{E} f_\chi(p)$$

The collision term is also given by its moments:

$$C_i^0 \equiv \frac{g_i}{m_i n_i} \int \frac{d^3p}{(2\pi)^3 E_i} C[f_i], \quad C_i^2 \equiv \frac{g_i}{3m_i n_i T_i} \int \frac{d^3p}{(2\pi)^3 E_i} \frac{p^2}{E_i} C[f_i]$$

contains all scatterings and production/annihilation processes

In our model we got 4 equations for:  $Y_S, T_S, Y_\phi, T_\phi$

Implementation of such capability [together with fBE system, giving also evolution of the  $f(p)$ ]

is a part of update in the new version of **DRAKE2** 

## New features:

**Two-component** dark sectors  
(also with potentially unstable states)

Freeze-out & **Freeze-in**

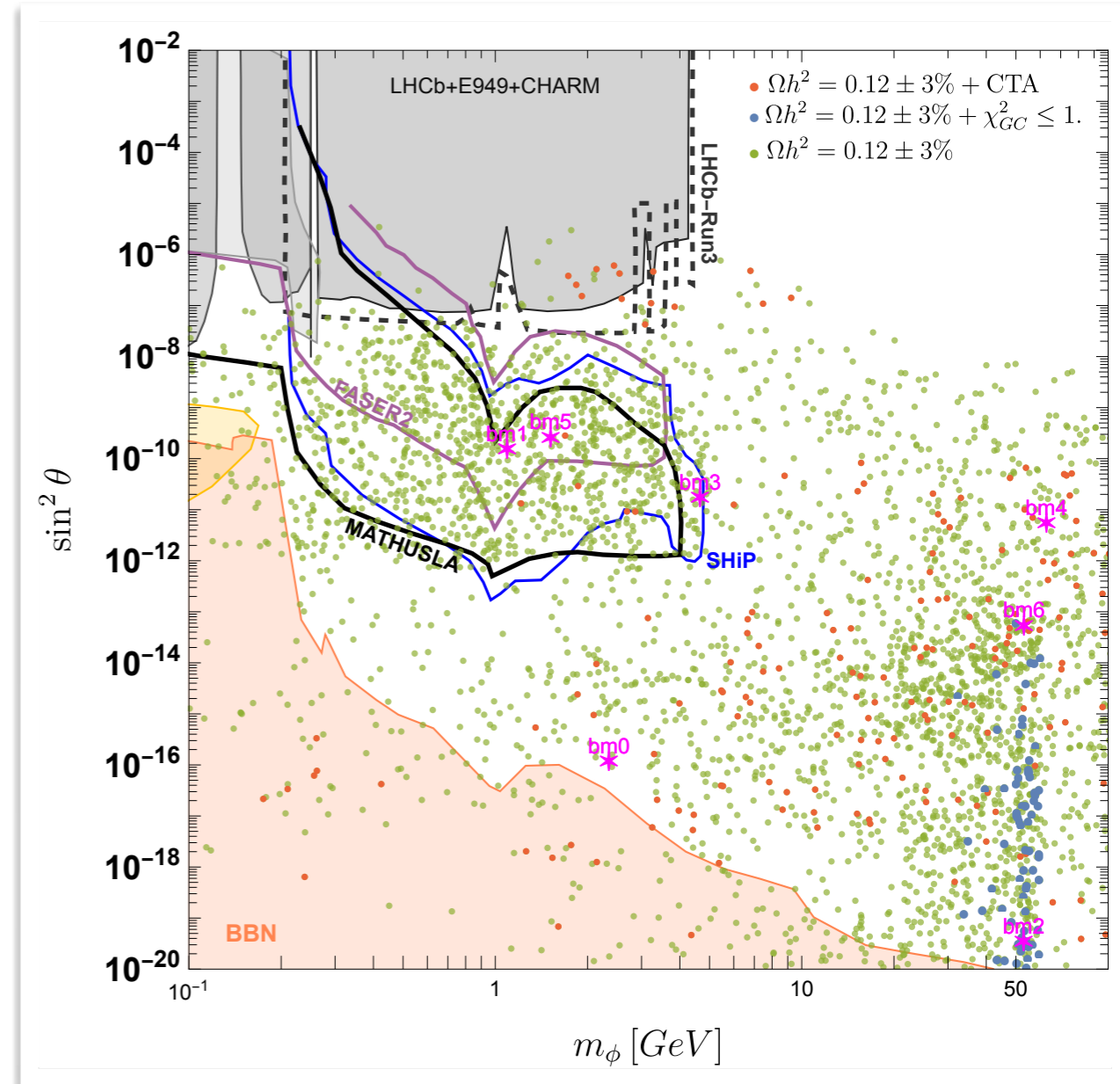
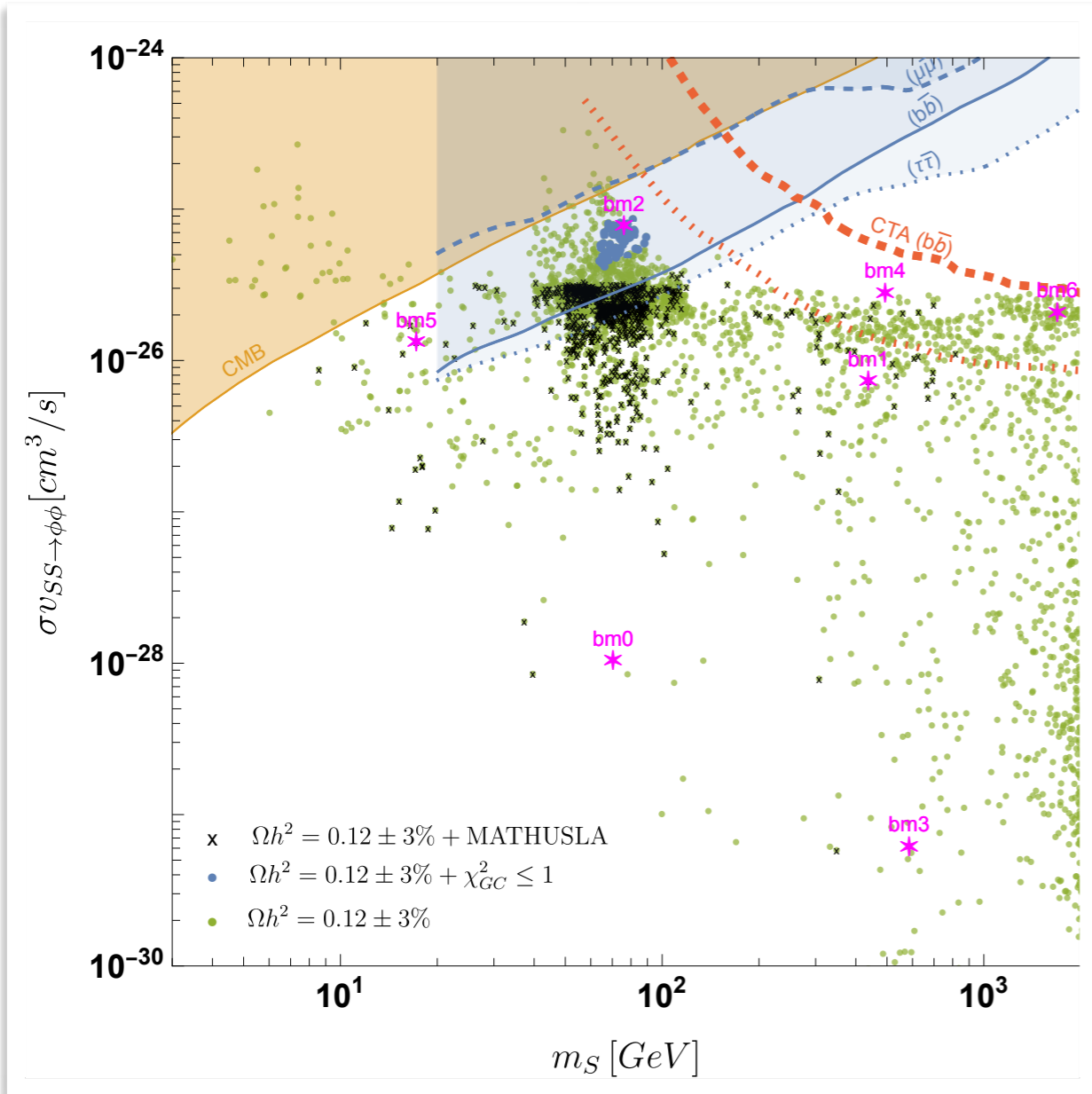
Automatic **model generation**  
[linking to **FeynRules etc.**]

## Improvements:

**Increased efficiency**  
[e.g. more extended use of  
compiled functions, parallelisation,  
matrix formulation]

Updated *user interface*

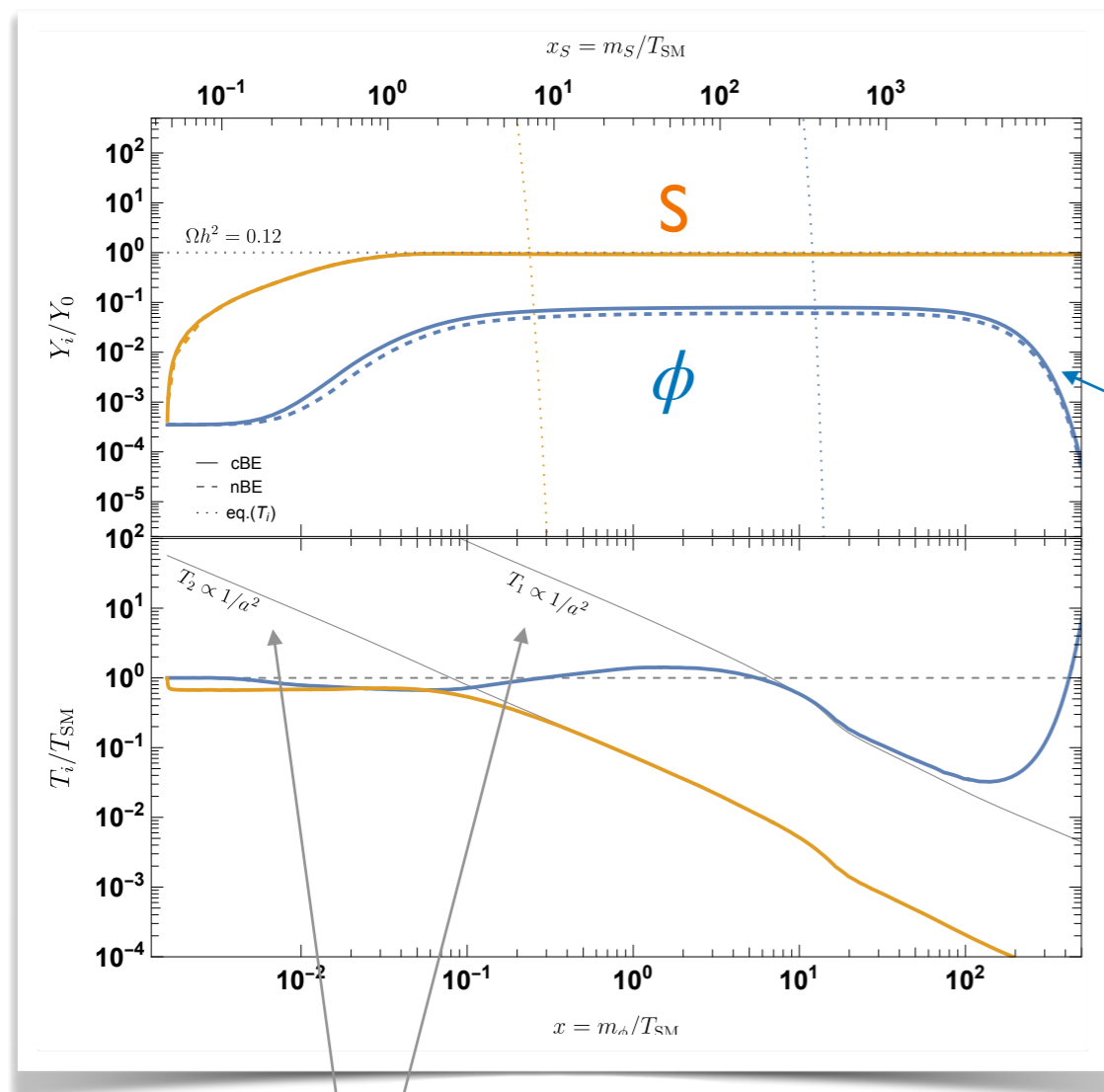
# BENCHMARKS



Name	$m_\phi$	$m_S$	$\theta$	$\lambda_{h\phi}$	$\lambda_{hS}$	$\lambda_{S\phi}$	$(\Omega h^2)_{\text{nBE}}$	$(\Omega h^2)_{\text{cBE}}$	change [%]	description
BM0	2.35	70.4	$1.09 \times 10^{-8}$	$1.67 \times 10^{-13}$	$5.98 \times 10^{-11}$	0.00298	0.113	0.110	-1.96	direct FI
BM1	1.09	438.	$1.24 \times 10^{-5}$	$3.56 \times 10^{-11}$	$3.72 \times 10^{-13}$	0.155	0.124	0.0205	-83.5	seq. FI/dark FO + MATHUSLA
BM2	53.0	76.1	$1.87 \times 10^{-10}$	$3.51 \times 10^{-7}$	$1.96 \times 10^{-11}$	0.104	0.115	0.0199	-82.7	dark FO + best GCE fit
BM3	4.66	586.	$4.15 \times 10^{-6}$	$8.62 \times 10^{-11}$	$4.32 \times 10^{-15}$	0.00603	0.0971	0.000883	-99.1	seq. FI
BM4	63.0	494.	$2.34 \times 10^{-6}$	$1.08 \times 10^{-15}$	$2.70 \times 10^{-6}$	0.344	0.0902	0.0503	-44.2	dark FO/co-decay + CTA
BM5	1.52	17.2	$1.62 \times 10^{-5}$	$1.30 \times 10^{-9}$	$4.46 \times 10^{-9}$	0.00823	0.110	0.0555	-49.5	co-decay + MATHUSLA
BM6	53.2	$1.69 \times 10^3$	$2.33 \times 10^{-7}$	$5.14 \times 10^{-8}$	$1.16 \times 10^{-7}$	1.01	0.119	0.0571	-51.9	dark FO + CTA

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yield (abundance)

Ratio of  $S$  and  $\phi$  temperatures to the SM plasma one

Simple point to keep in mind as a baseline

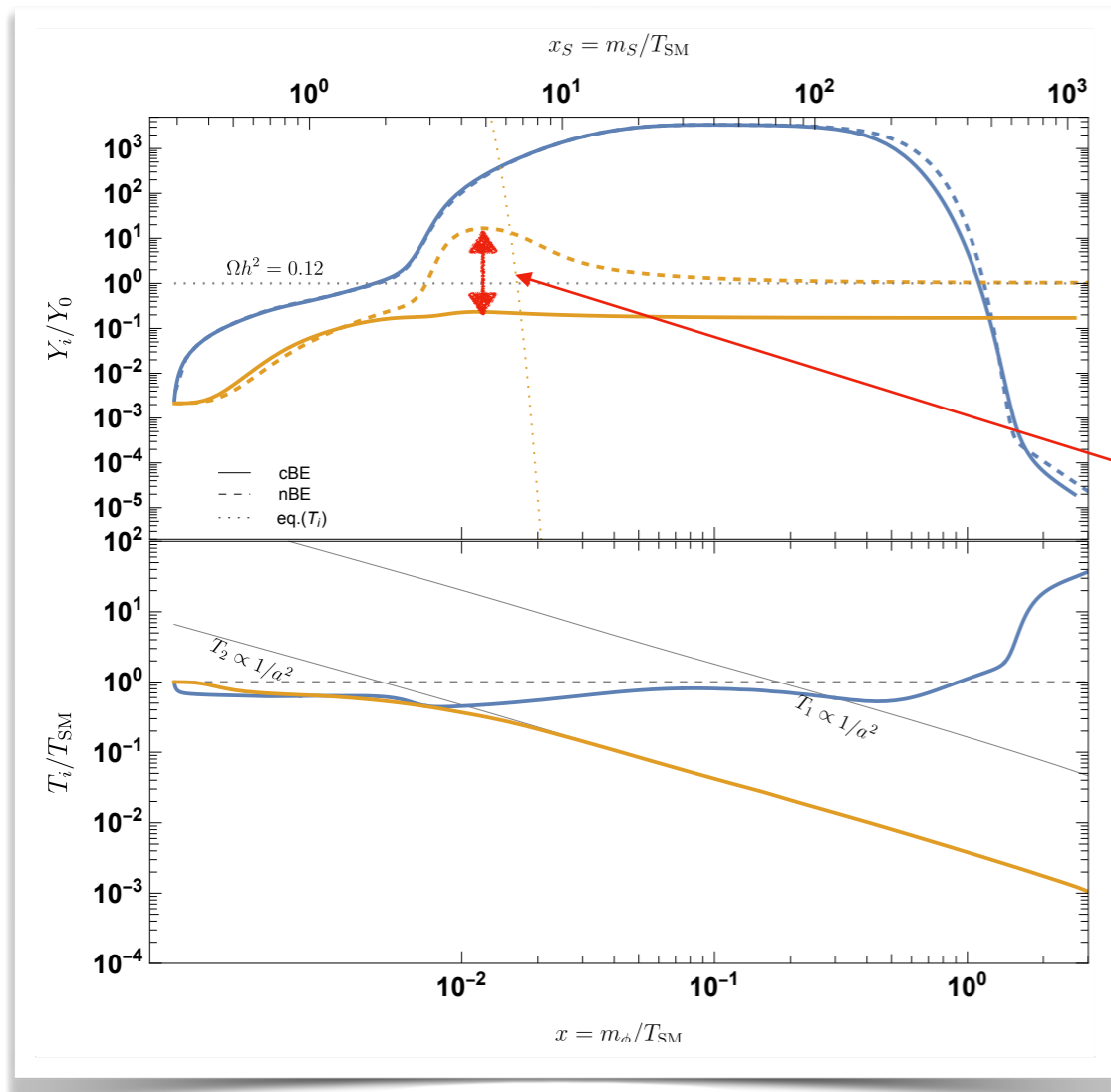
Relatively small  $\lambda_{S\phi}$  means both  $S$  and  $\phi$  evolve separately

In the end  $\phi$  decays

Very mild cBE effect

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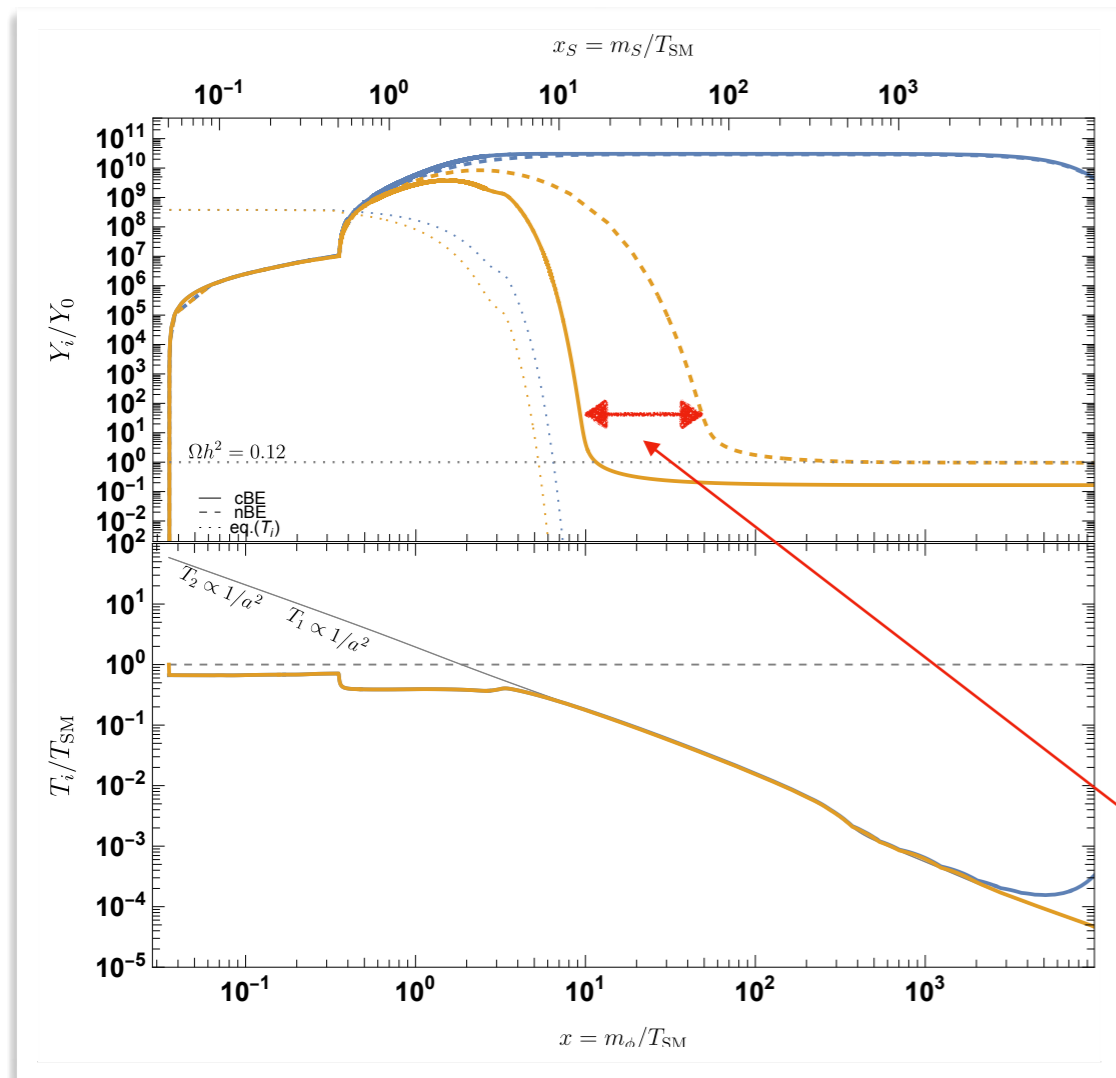
Hierarchy of  $\lambda_{h\phi} \gg \lambda_{hS}$  and  $m_S \gg m_\phi$  means freeze-in is sequential, followed by (mild) annihilation due to large  $\lambda_{S\phi}$

Large change due to cBE:  
lower  $T_\phi$  + large threshold from  $\phi$  to  $S$  suppresses sequential freeze-in!

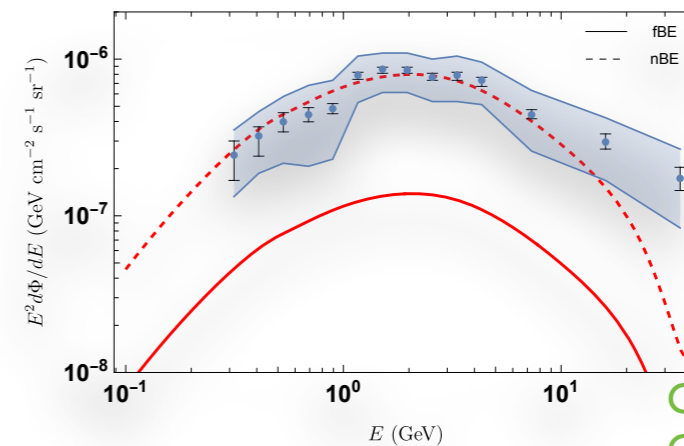
This point lies within reach of MATHUSLA, SHiP and FASER2

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Best fit point to the GCE found in the scan:



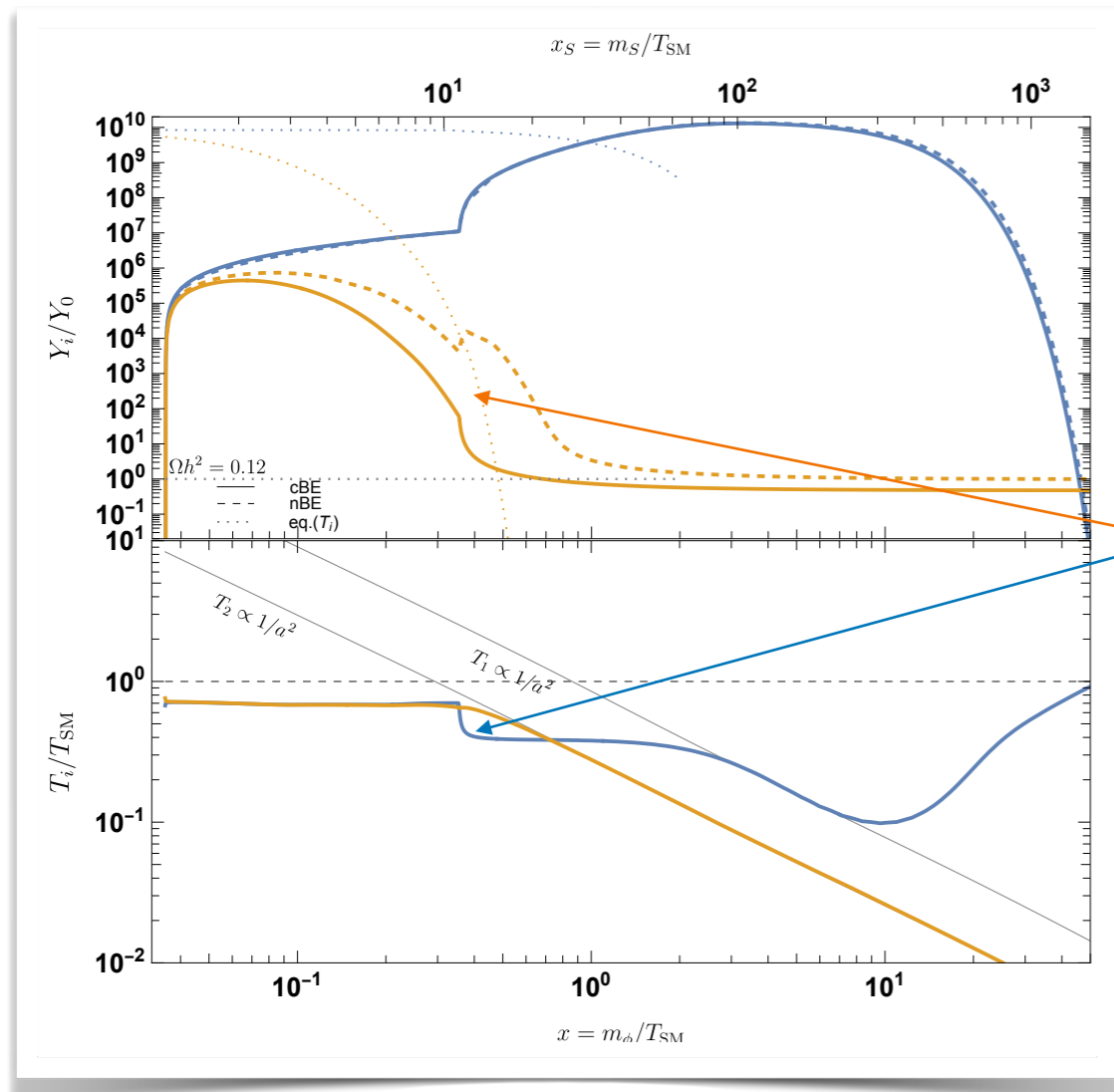
GCE analysis from Cholis et al. '22

Mostly dark freeze-out from a thermal bath with  $T_S \approx T_\phi < T_{SM}$

change in  $\Omega h^2$  due **sooner freeze-out**

# BENCHMARKS

Name	$m_\phi$	$m_S$	$\theta$	$\lambda_{h\phi}$	$\lambda_{hS}$	$\lambda_{S\phi}$	$(\Omega h^2)_{nBE}$	$(\Omega h^2)_{cBE}$	change [%]	description
BM0	2.35	70.4	$1.09 \times 10^{-8}$	$1.67 \times 10^{-13}$	$5.98 \times 10^{-11}$	0.00298	0.113	0.110	-1.96	direct FI
BM1	1.09	438.	$1.24 \times 10^{-5}$	$3.56 \times 10^{-11}$	$3.72 \times 10^{-13}$	0.155	0.124	0.0205	-83.5	seq. FI/dark FO + MATHUSLA
BM2	53.0	76.1	$1.87 \times 10^{-10}$	$3.51 \times 10^{-7}$	$1.96 \times 10^{-11}$	0.104	0.115	0.0199	-82.7	dark FO + best GCE fit
BM3	4.66	586.	$4.15 \times 10^{-6}$	$8.62 \times 10^{-11}$	$4.32 \times 10^{-15}$	0.00603	0.0971	0.000883	-99.1	seq. FI
BM4	63.0	494.	$2.34 \times 10^{-6}$	$1.08 \times 10^{-15}$	$2.70 \times 10^{-6}$	0.344	0.0902	0.0503	-44.2	dark FO/co-decay + CTA
BM5	1.52	17.2	$1.62 \times 10^{-5}$	$1.30 \times 10^{-9}$	$4.46 \times 10^{-9}$	0.00823	0.110	0.0555	-49.5	co-decay + MATHUSLA
BM6	53.2	$1.69 \times 10^3$	$2.33 \times 10^{-7}$	$5.14 \times 10^{-8}$	$1.16 \times 10^{-7}$	1.01	0.119	0.0571	-51.9	dark FO + CTA



Finally, a point within reach of CTA

Notice impact of  $h$  decay after EPWT:

(as  $m_\phi \sim m_h/2$  it lowers  $T_\phi$ )

this cooling suppresses  $\phi\phi \rightarrow SS$  while annihilation  $SS \rightarrow \phi\phi$  can proceed

# BENCHMARKS: SUMMARY

Name	$m_\phi$	$m_S$	$\theta$	$\lambda_{h\phi}$	$\lambda_{hS}$	$\lambda_{S\phi}$	$(\Omega h^2)_{\text{nBE}}$	$(\Omega h^2)_{\text{cBE}}$	change [%]	description
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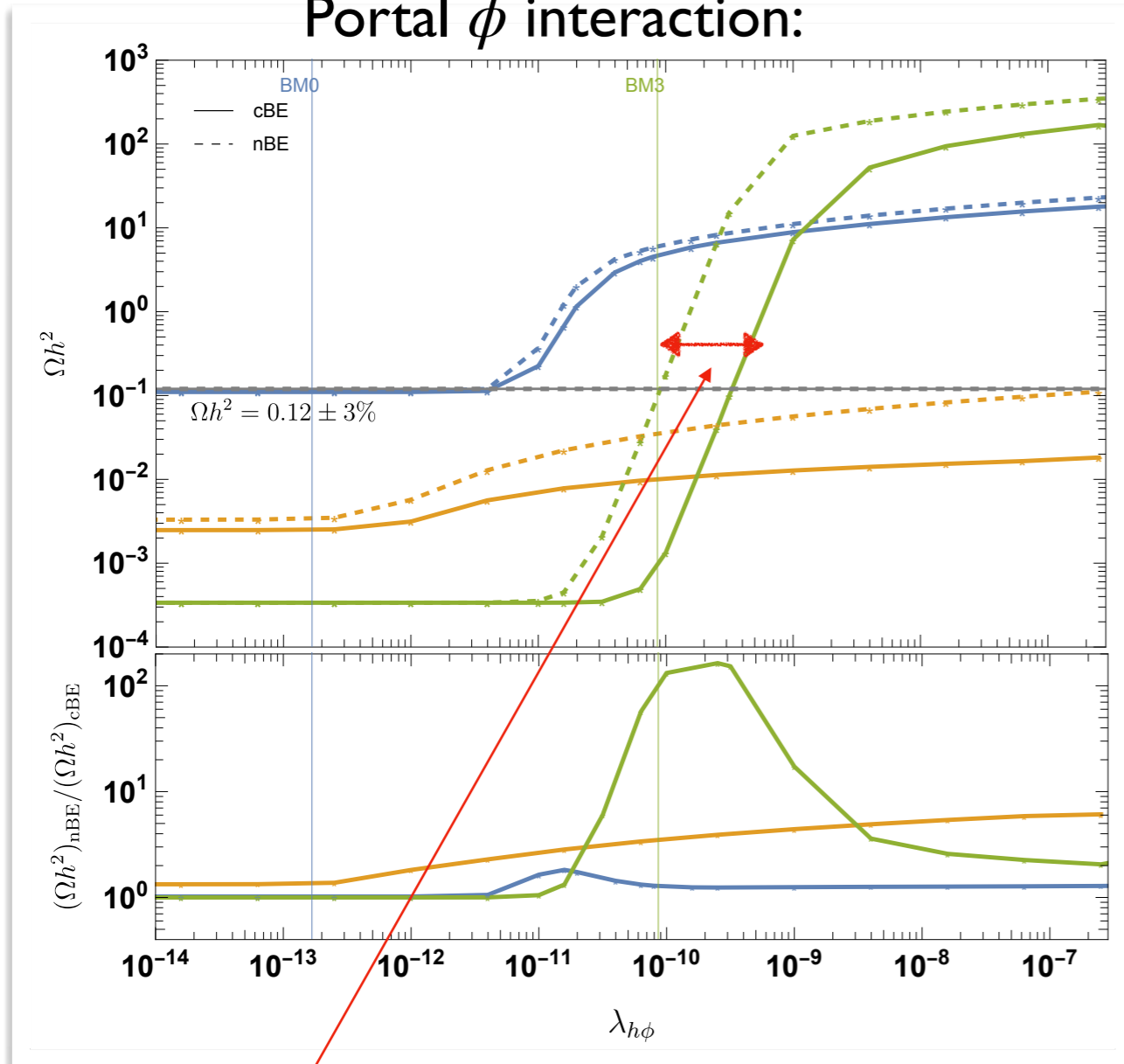
The model's parameter space spans over various production modes:

- direct & sequential freeze-in
- dark freeze-out
- co-decaying
- (and mixtures of these)

Effect of performing calculation at cBE level: from  $\sim \mathcal{O}(1\%)$  to  $> 100$

# DEPENDENCE ON THE COUPLINGS

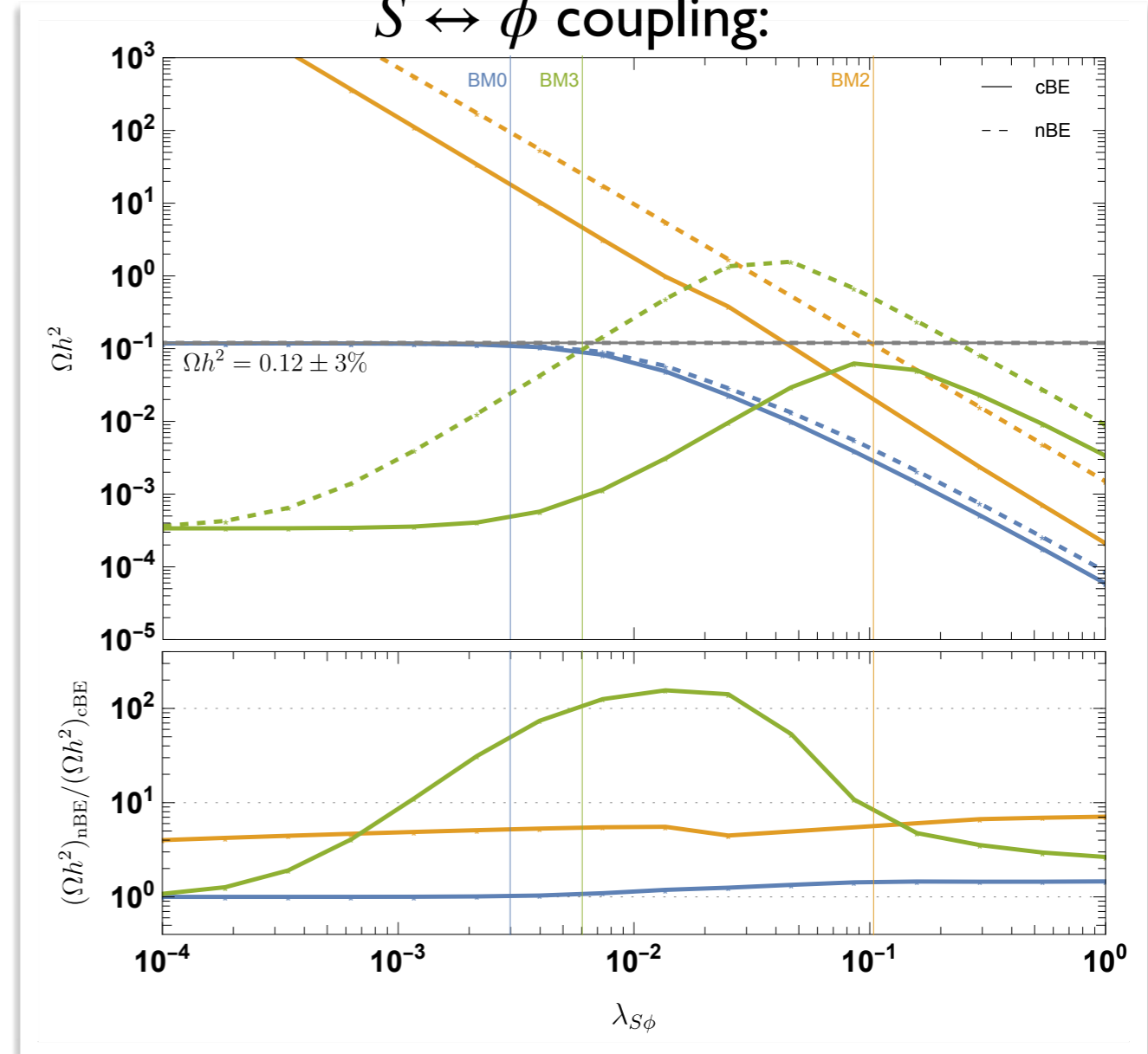
Portal  $\phi$  interaction:



Increasing  $\lambda_{h\phi}$  gives larger production (as expected)

Effect of cBE is the shift the required value by factor  $\mathcal{O}(1)$

$S \leftrightarrow \phi$  coupling:



3 different behaviours:

BM0 -  $\lambda_{S\phi}$  independent at first, then dark FO

BM2 - dark FO

BM3 - first (sequential) FI, then dark FO

# OTHER EXAMPLES...

Sequential freeze-in thus adds to the list of scenarios where **departure from LTE needs to be considered**:

Annihilation through a (narrow) resonance

Duch, Grządkowski '17; Binder, Bringmann, Gustafsson, A.H '17; Abe '21; Ala-Mattinen et al '22

Sub-threshold (e.g. forbidden DM)

Binder, Bringmann, Gustafsson, A.H 2103.01944; Liu et al '23; Aboubrahim et al. '23

Semi-annihilation and production

Kamada et al. '18; Cai, Spray '18; Hektor, AH & Kannike '19; AH & Laletin 2104.05684

Cannibal DM (freeze-out or freeze-in)

Herba et al '18; Cervantes & AH 2407.12104; Bernal, Cervantes, Deka, AH 2506.09155

Sommerfeld enhanced annihilation

Feng et al '10; Binder, Bringmann, Gustafsson, A.H 2103.01944

Two-component dark sectors (e.g. conversion-driven or co-decaying)

Beauchesne & Chiang 2401.03657; Chatterjee & AH 2502.08725

Freeze-out/freeze-in intermediate regime

Du et al. '22

SuperWIMP, WDM and Lyman- $\alpha$  limits

Decant et al. '22; AH & Laletin 2204.07078

...

# CONCLUSIONS

**1.** Freeze-in in multicomponent dark sectors (like sequential freeze-in) proceeds in a  $T$ -dependent way. This can alter the naive predictions by **more than an order of magnitude**. This is another example of importance of non-equilibration in dark matter production (as seen in some freeze-out scenarios)

**2.** A simple two scalar model with feeble couplings to SM can provide interesting phenomenology with cross correlation of ID & forward physics experiments

**3.** In recent years a **significant progress** in refining the relic density calculations in  to include **multicomponent case** & **freeze-in**

Thank you!

# BACKUP

# RELATIVISTIC OR NOT?

Relativistic reaction rate:

$$\Gamma_{a \rightarrow b} = \int \left( \prod_{i \in a} \frac{d^3 \mathbf{p}_i}{(2\pi)^3 2E_i} f(p_i) \right) \left( \prod_{j \in b} \frac{d^3 \mathbf{p}_j}{(2\pi)^3 2E_j} (1 + f(p_j)) \right) |\mathcal{M}_{a \rightarrow b}|^2 (2\pi)^4 \delta^4(p_a - p_b).$$

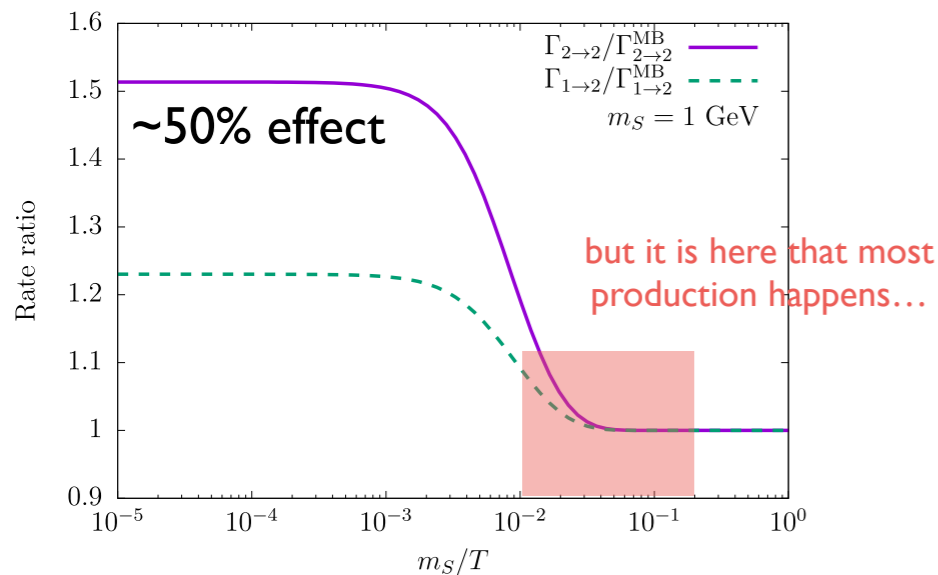
I) In freeze-out one (typically) takes Maxwell-Boltzmann distribution, should one use here:

$$f(p) = \frac{1}{e^{\frac{u \cdot p}{T}} - 1} \quad \text{instead?}$$

II) when relativistic, not obvious if  $(1 \pm f) \approx 1$  which poses a question of the **feedback of DM distribution** to the production rate

At early stages of evolution DM is very diluted allowing for such approx.

but when  $T \sim m$  this is less obvious...

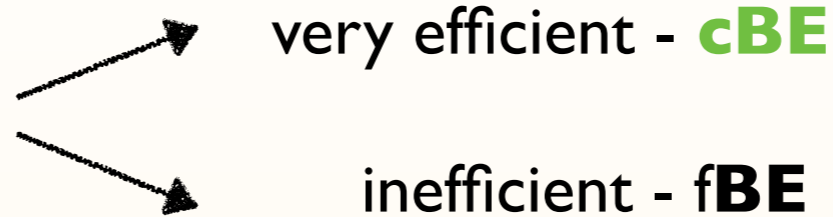


Lebedev, Toma 1908.05491 & subsequent works

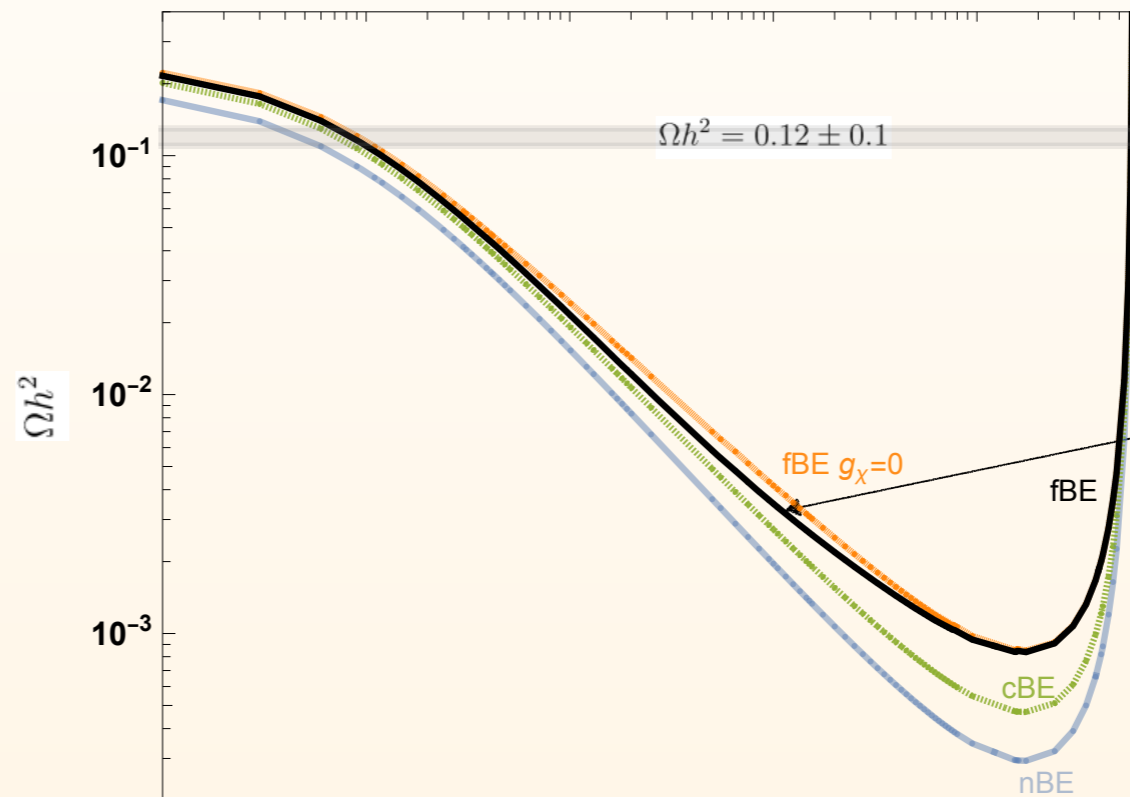
# CBE vs. FBE

WHICH IS MORE ACCURATE?! **A.H. & M. Laletin** [2204.07078](#)

They correspond to the opposite limits of **self-interaction strengths**:

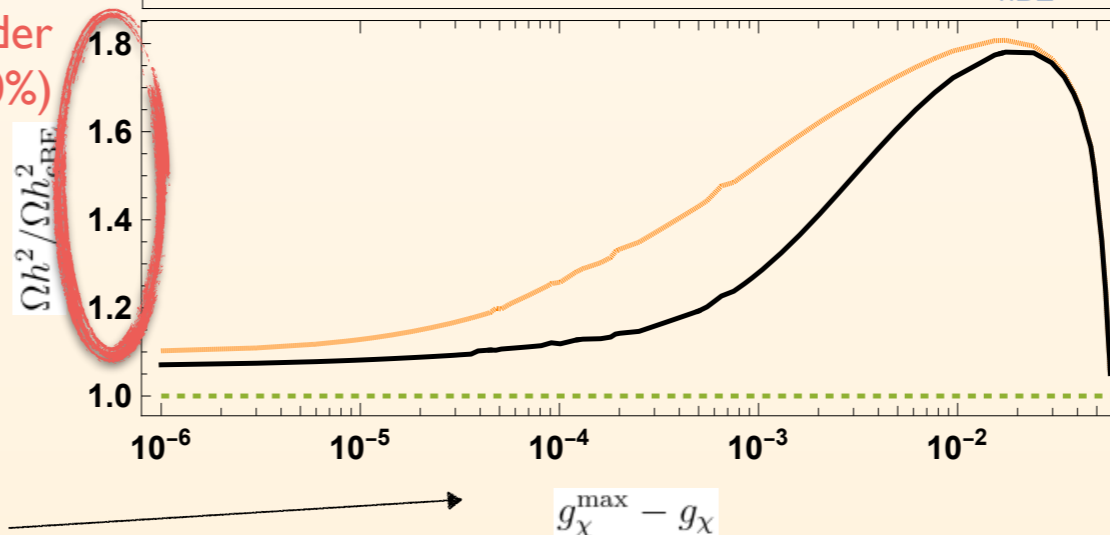


Which limit is closer to reality depends on the model, but it seems that fBE is typically more accurate, unless self-scattering is tuned up, e.g:



black line gives the result including self-scattering processes! (being between pure fBE and cBE)

difference of order  $O(10\%)$



coupling to the mediator; governs self-scatterings

# EXAMPLE EVOLUTION

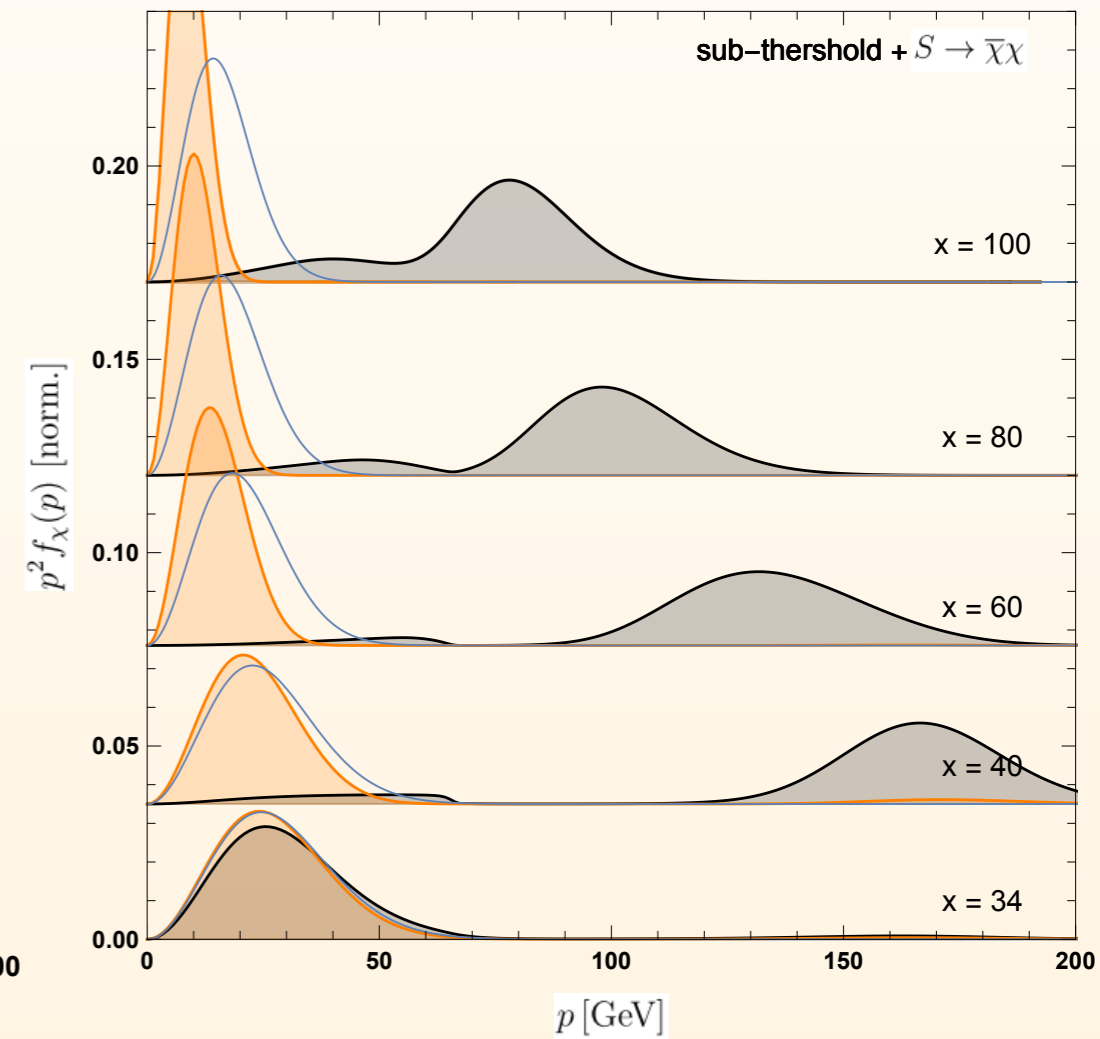
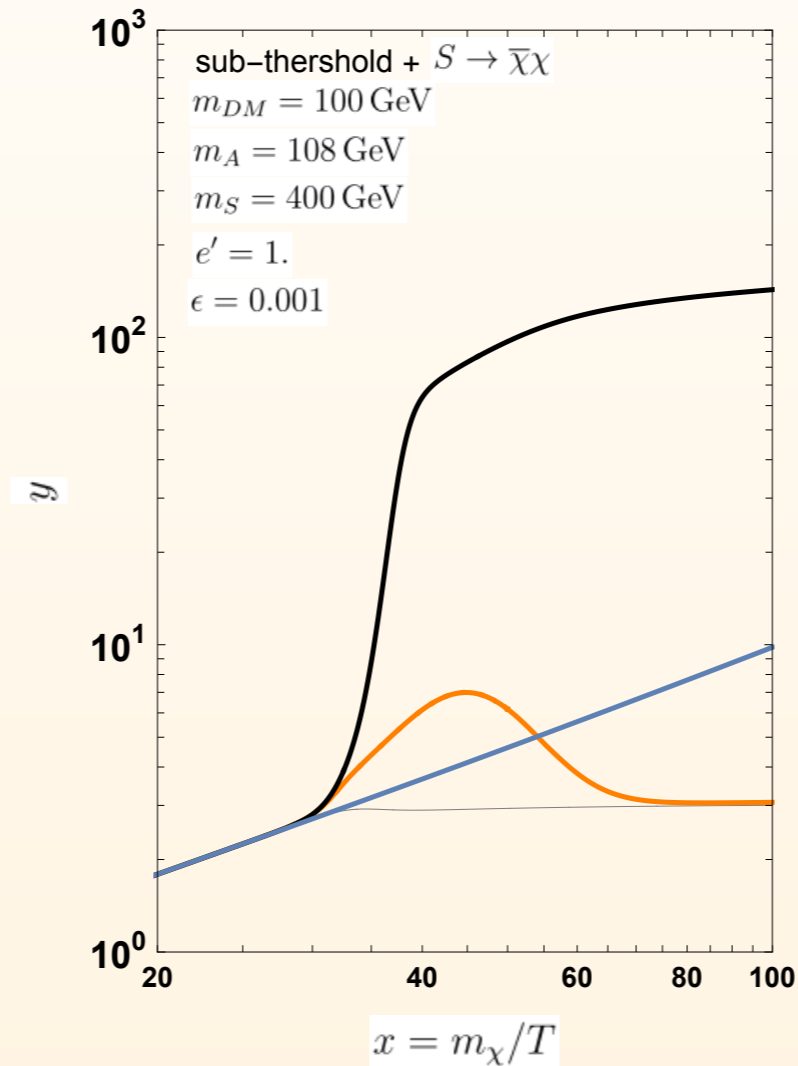
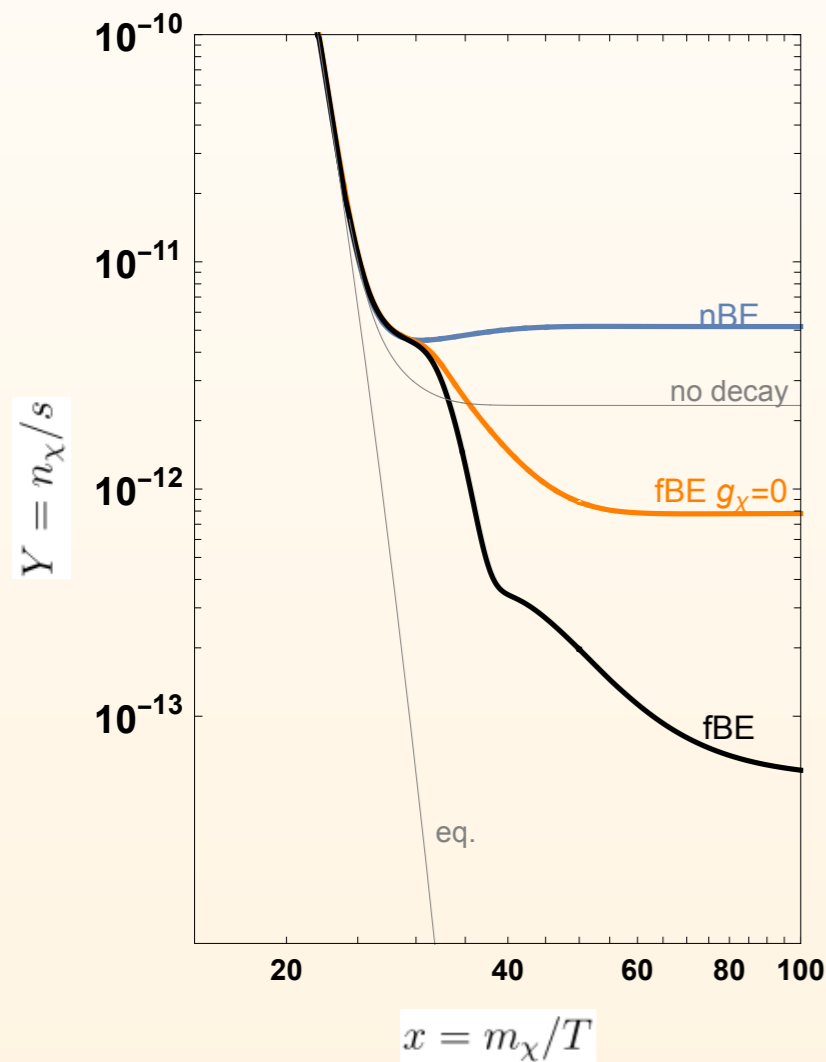
1) DM produced via:   
 - 1st component from **thermal freeze-out**   
 - 2nd component from **a decay  $\phi \rightarrow \bar{\chi}\chi$**

2) DM annihilation has a **threshold**   
 e.g.  $\chi\bar{\chi} \rightarrow f\bar{f}$  with  $m_\chi \lesssim m_f$

$Y \sim$  number density

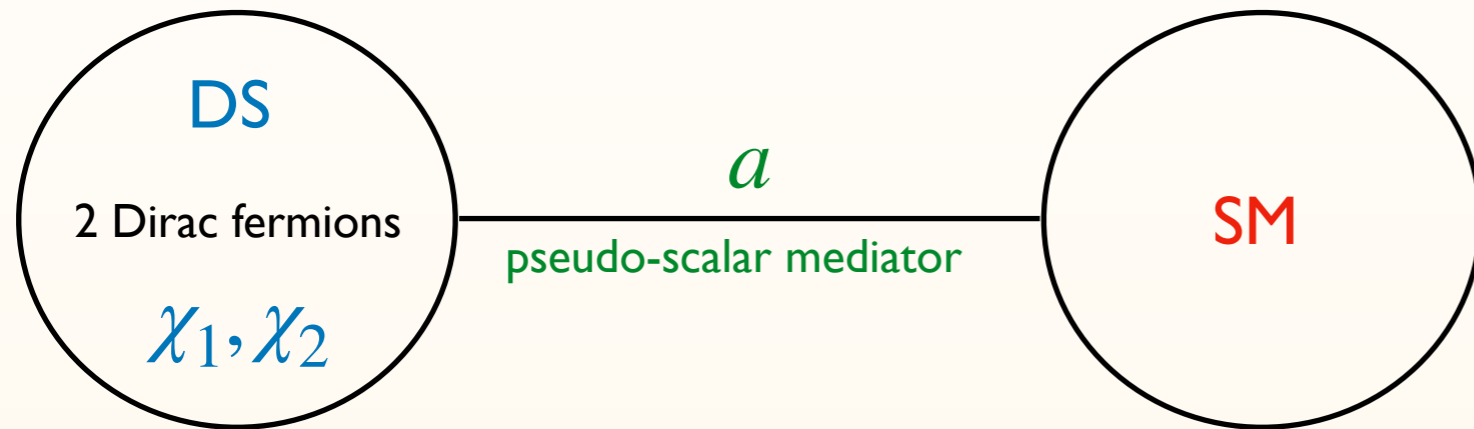
$y \sim$  temperature

$p^2 f(p) \sim$  momentum distribution



# RESULTS: THE MODEL

Let's take one of the simplest two-component DM models:



$$\mathcal{L}_{int} = - \sum_{i=1,2} i\lambda_i a \bar{\chi}_i \gamma^5 \chi_i - i\lambda_y \frac{m_f}{v} a \bar{f} \gamma^5 f$$

coupled directly to SM fermions in a MFV way

New fields:  $\chi_1, \chi_2, a$       New params:  $m_1, m_2, m_a$   
 $\lambda_1, \lambda_2, \lambda_y$

Parametrically:

$$\sigma_{11 \rightarrow SM} \sim \sigma_{1SM \rightarrow 1SM} \sim \lambda_1^2 \lambda_y^2$$

$$\sigma_{22 \rightarrow SM} \sim \sigma_{2SM \rightarrow 2SM} \sim \lambda_2^2 \lambda_y^2$$

$$\sigma_{11 \rightarrow 22} \sim \lambda_1^2 \lambda_2^2$$



Varying:

$$\lambda_1 \rightarrow \lambda_1 / c$$

$$\lambda_2 \rightarrow \lambda_2 / c$$

$$\lambda_y \rightarrow c \lambda_y$$

Keeps everything fixed, except conversions

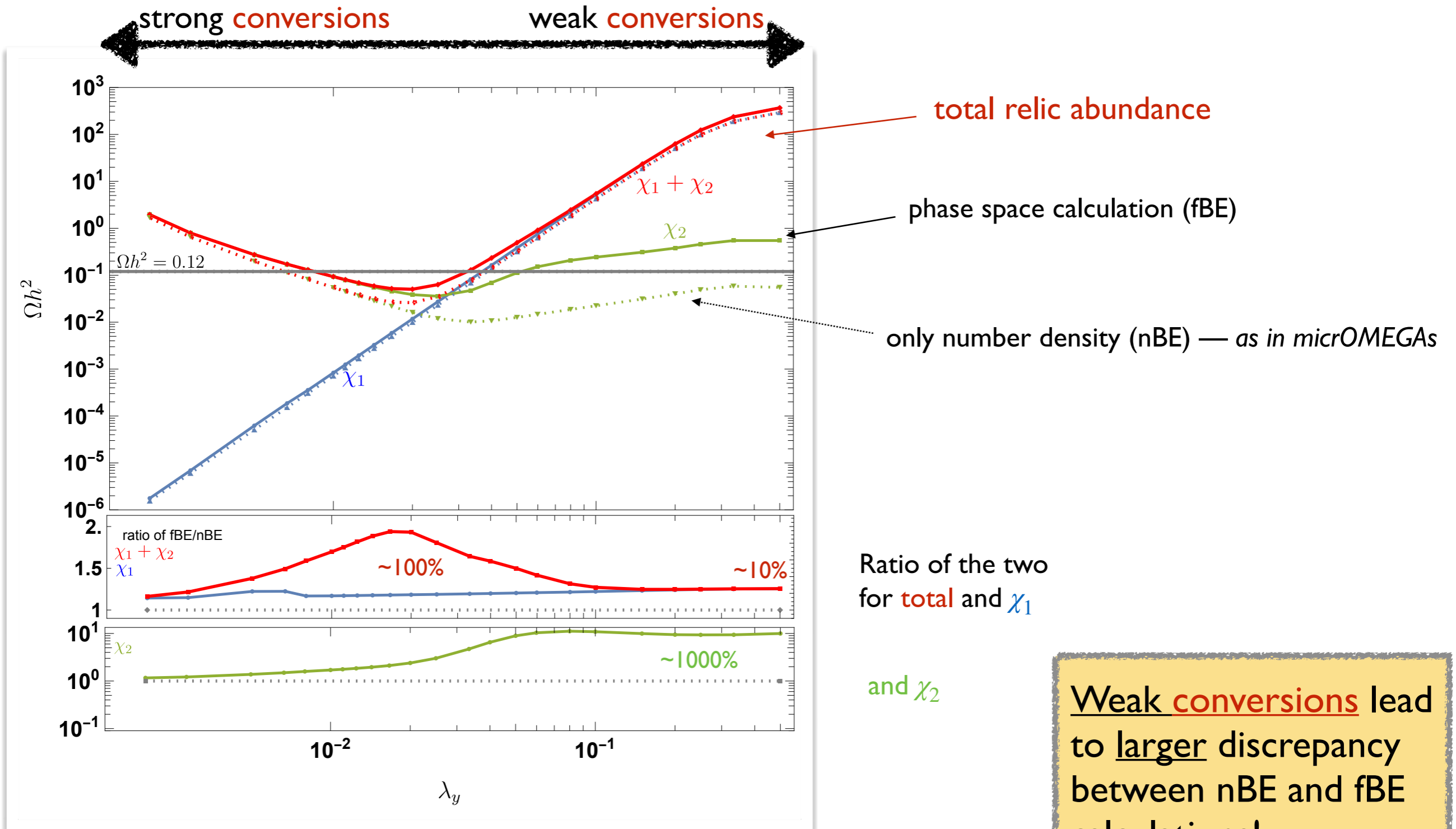
Main motivation (for models in the literature with pseudo-scalar mediator):

Evasion of the direct detection bounds... while giving strong signal in indirect detection, in particular **for explaining the Galactic Centre excess**

(see e.g. „Coy DM”)

# RESULTS: CONVERSION IMPACT

Varying:  $\lambda_1 \rightarrow \lambda_1/c$      $\lambda_2 \rightarrow \lambda_2/c$      $\lambda_y \rightarrow c \lambda_y$     Only **conversions** change!



**Weak conversions** lead to larger discrepancy between nBE and fBE calculations!